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Abstract

This paper explores the interaction between a credit crunch and the maturity of government debt, focusing on its impacts on an economy with heterogeneous households. We find that an increase in debt maturity helps softening the economic slump that follows a credit crisis. We show that, immediately after the credit shock, there is an output drop of nearly 1% when the asset available has on average one quarter of maturity, while a contraction of only 0.6% follows when debt duration has three quarters. The rise of asset duration indirectly enhances the income effects unleashed by general equilibrium price dynamics, which benefits bondholders and thus softens the recession. On the other hand, an increase on debt duration impairs the improvement of wealth distribution on the long run. The main contribution this paper paper is to show that debt maturity is a key element to understand the magnitude of a recession driven by credit and its welfare consequences.

Keywords:Self Insurance, Incomplete Markets, Credit Crisis, Debt Maturity

JEL Classification:

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1 Introduction

The credit market plays a key role on smoothing income fluctuations. The existence of private debt instruments can be used by households as a buffer against negative idiosyncratic shocks. In an economy where the future income is uncertain, the degree of risk sharing will depend crucially on the borrowing constraints faced by individuals. The credit channel, in a large extent synthesized by the credit restriction, is therefore a determinant factor for the aggregate macroeconomic dynamics and wealth distribution.

The recent literature shows that there are many factors which affect the deleveraging process of indebted individuals, such as the existence of unemployment insurance, availability of health care assistance, the taxation code among others. This paper explores the role of debt instrument maturity on determining the dynamics of an economy with heterogeneous households that is affected by an abrupt credit crisis. After U.S. 2007/8 economic crises, there were massive efforts to clarify the connection between financial frictions and macroeconomic aggregates, in addition to its welfare implications. In this paper we investigate how the average bond duration interacts with an unexpected credit crisis in an incomplete markets environment.

More precisely, we study a Bewley-Hugget-Aiyagari economy where risk averse agents face idiosyncratic income shocks, choose consumption level, labor hours and savings. There is no aggregate shocks neither default is possible. There is, however, a financial friction that limits the individual risk sharing due to an exogenously defined credit constraint. In order to model long term debt we use the concept of perpetuity, first introduced by Woodford (2001), which can replicate a portfolio of higher than one period securities and also allows for a reasonable and tractable representation of government debt structure.

Along with the previous literature, this paper also aims to investigate how the interaction between credit constraints and debt duration affects the behavior of welfare both on aggregate level as well as in income groups of individuals. The total welfare variation is decomposed in its distributive and aggregate components, using the same methodology of Domeij and Heathcoate (2004).

Following Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012), on the simulated economy the credit shock consists in a sudden and exogenous tightening of the credit constraints designed to replicate the main features of the U.S. 2007/8 credit crisis. This abrupt shock forces households to adjust their balance sheet to the new liquidity conditions, which means that they need to deleverage. The nature of this exercise also is line with the evidence presented on Hall (2011), who argues that the unexpected tightening of household borrowing constraint was one of the main features of the recent U.S. crisis. Besides the steady state analysis of the macroeconomic aggregates before and after the credit shock, we focus on the transition dynamics for economies with distinct debt maturity and evaluate the path of the main variables until the new steady state. To the best of our knowledge, this paper is the first to quantitatively assess the importance of debt duration in a event of a credit shock, in a context of incomplete markets. The results of this exercise shows that the duration of the asset that is used as storage of value matters in determining the size and the length of the recession caused by the credit crunch.

Quantitative results of the exercises performed points that an increase in debt maturity can smooth the path of macroeconomic aggregates. This happens because a rise of bond duration intensifies the general equilibrium effect unleashed by the credit crisis. Bond prices becomes more interest rate sensitive when maturity increases, which potentializes the positive wealth effect for bondholders and deepens the negative balance sheet pressures faced by indebted ones. However, the indebted ones ex-ante show a more precautionary savings behavior, given the debt instrument.

We found that the credit shock triggers a recession of nearly 1% when asset has one quarter of maturity and the output contraction is reduced to 0.6% when debt duration has three quarters. An increase in debt duration also softens the undershooting experienced by the real interest rate. When debt has one quarter maturity the real interest rate drops to almost -2% (from an initial value of 2.5%), while when debt duration has three quarters the real interest rate only becomes slightly negative, decreasing from 3% to -0.2% in the fourth period after the initial impact of credit shock.

In terms of welfare, we show that, independently of bond duration, after a credit contraction there is a significant drop in the inequality of asset distribution. From the standpoint of consumption equivalent measure, the process of deleveraging endured by households results in an initial welfare loss, due to spending contraction. There is, in addition, subsequent important improvement in the economic resources distribution.

Overall, the precautionary motive that affects more intensely poor households is the main driver of the advances in income and wealth distribution. When bond maturity is one quarter, the wealth Gini Index falls 14.4%,while when debt duration has three quarters this inequality indicator decreases 13.5%. The analysis of the transition between steady states provides details on the short term dynamics, while it also preservers long term features, that is, larger reduction in inequality for shorter maturities. The lower the duration of assets available, the more urgent is the deleveraging pressures faced by individuals, because their debt exposition is higher. We find that when bond maturity has one quarter of duration the share of the population that becomes indebted is higher than the mass of households involved in debt when bond has duration of two or three quarters. Therefore, when there is an unexpected tightening in the credit limit the mass of individuals that need to make budget adjustments is also larger in the economy where bonds has one period maturity, which speeds up their deleveraging process.

Simultaneously, the general equilibrium effects that lowers interest rates, encourages richer households to reduce their asset positions. This decrease in asset holdings is the main mechanism behind the improvement of wealth distribution. The opposite is also true, i.e., the higher the asset duration, the lower is the increase in demand for bonds by a relevant share of poor holseholds, and therefore, smaller is the gain in terms of wealth distribution.

On the other hand, the long run impact of debt maturity on wage distribution increases with bond duration. When the maturity has one quarter, the wage Gini Index falls 2.2%, while when the debt duration has three quarters this welfare indicator decreases by 3.9%. This effect is derived by the use of the working hours as a buffer device against negative income shocks. However, as shown by quantitative results, the effective working hours rises more in the environment where bond duration is higher, which also makes the gains in wage distribution larger.

Therefore, we conclude that lower asset maturity enhances the gains in wealth distribution, while higher asset duration results in greater long run reduction in wage inequality. The short run transition dynamics for wealth Gini index also shows quicker convergence for lower bond maturities. In terms of income distribution, we found that a significant portion of the immediate decline in inequality of wages is temporary, being subsequently reversed, while the fall in inequality of wealth has a continuous and permanent nature.

Finally, we propose an extension of the original model designed to evaluate the role of government transfers in a credit recession. Taking into account that nearly a 75 percent of the fiscal expansion that took place in the U.S. in 2007/8 was due to increases in social transfers, as highlighted by Oh and Reis (2012), we evaluate the aggregate macroeconomic impact of a credit contraction in an environment where simultaneously occurs (i) a credit shock along with the lines of the one studied on the original model and (ii) a social program designed to replicate the features of the program that took place on 2007/08 recession period is implemented . We found that such kind of transfer program can significantly reduce the output slump that occurs in response of a credit tightening. Furthermore, the transfers provides an alternative liquidity channel which also softens the deleveraging process for all bond maturities.

The remainder of the paper is structured as follows: Section 2 presents a brief discussion of the related literature, Section 3 presents the economic environment and the model structure, Section 4 deals with calibration issues, Section 5 presents an analysis of the quantitative impact of credit shock and debt maturity in this economy, Section 6 discusses the welfare implications, Section 7 presents an extension of the main model which includes social transfers and, finally, Section 8 concludes the paper with our final remarks and considerations.

2 Related Literature

This section contains a summary of the literature on the economic impacts of a credit crisis in an incomplete markets framework and the role of long term debt on this environment. There is a vast and growing literature about these two themes separately, but not in the connection between them in the context of Aiyagari model. This paper helps to close this gap.

First of all, this paper is related to the long literature on the macroeconomic effects of financial frictions. From a consumer standpoint we have the classical role of borrowing constraints found on Huggett (1993) and Aiyagari (1994). Other papers that also explore the role of credit constraints in a similar environment are Guerrieri and Lorenzoni (2011), Hall (2011), Eggertsson and Krugman (2012) and Huo and Rios Rull (2015).

Financial frictions and wealth accumulation also plays a central role in recent contributions to the entrepreneurship literature in macroeconomics, such as Quadrini (2000), Angeletos and Calvet (2006), Cagetti and De Nardi (2006), Goldberg (2011), Buera et al. (2011), Jermann and Quadrini (2012), Buera and Shin (2013) and Buera and Shin (2015). These papers develop entrepreneurship models that shares many features with the class of Bewley-Hugget-Aiyagari model discussed on the following sections. In line to the qualitative analysis of a credit shock conducted on this paper, some of these works are focused on study how financial frictions shape the wealth distribution and others deals with the aggregate impacts of financial frictions.

Goldberg (2011) also analyzes precautionary saving and the scarcity of liquid assets. One important difference in relation to this work is that Goldberg (2011) emphasizes shocks to the borrowing constraints of entrepreneurs, whereas this paper focus on changes in the borrowing constraints of consumers. In an approach that is complementary to the one presented here, Jermann and Quadrini (2012) studies how shocks on financial sector affects firm's liquidity conditions and its developments in terms of macroeconomic aggregate dynamics.

Recentently, we have seen two usual ways of modeling credit shocks. There is an indirect approach adopted on papers like Del Negro et al. (2013), where the shock consists in abrupt drop of the resellability of financial assets in secondary markets, or Curdia and Woodford (2009), which allows for various types of disturbances that directly affect the financial intermediaries. In this paper the credit shock is a sudden reduction in the quantity of debt the household can borrow, in line with Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012). Since the onset of U.S. 2007/8 recession, Hall (2011) also has argued that the tightening of household borrowing constraint is an essential element in the understanding of the crisis. A difference of this paper to Eggertsson and Krugman (2012), is that borrowing and lending are motivated by idiosyncratic shocks, rather than differences in preferences.

This paper emphasizes the role of transition dynamics on assessing the impact of an economic shock as in Guerrieri and Lorenzoni (2011), Oh and Reis (2012), Buera and Shin (2013) and Buera and Shin (2015). Finally, empirical evidence presented in Mian and Sufi (2010) and Mian and Sufi (2011) support the quantitative results found on this paper. In particular, these papers exploit geographic variation across U.S. counties in the degree of leverage since 2006. The geographic variation proxies well for the borrower heterogeneity that is present in the theoretical models. Consistent with the deleveraging intuition, these studies show that highly levered U.S. counties were the driving force behind sharp drops in consumption during the downturn. Regarding the persistence of deleveraging process, Reinhart and Reinhart (2010) find that financial crises are followed by large and long-lasting declines in the ratio of domestic bank credit to GDP. These empirical evidence are used as reference to the credit crunch modeled in this paper.

As in Challe et al. (2013), here the existence of uninsured idiosyncratic shocks generate bond exchange, as traders are willing to buy bonds for precautionary purposes purchase from agents willing to sell them in order to smooth consumption and from the government. This feature of the model is particularly intensified after the credit shock, once indebted households must repurchase their debt. These authors study how the existence of a portfolio with assets of different maturities affects the shape of the yield curve and concludes that combining aggregate risk, idiosyncratic risk and positive net supplies of bonds allows for the existence of a liquidation risk premium over the long end of yield curve. They also find that, under certain conditions, the increase on average maturity also increases the average interest rates, a result that also is found in this paper.

Debortoli et al. (2014), in an approach that focus the question of commitment on fiscal policy, provides additional motivation in favor of high debt duration. These authors argues that borrowing long term provides the government with a hedging benefit since the value of outstanding government liabilities declines when short-term interest rates rise. On their perspective, the degree of commitment with a given fiscal policy plays a key role on determining the optimal maturity structure of government debt. The stronger is the commitment, the larger is the duration of optimal debt.

Faraglia et al. (2014) argues that taking into account bonds with maturity higher than one year is essential to introduce in the model an additional dynamics that emerges after an exogenous shock. According to these authors, the government uses both tax policy and debt management to soften the impacts of adverse innovations in thebusiness cycles. In this paper fiscal policy is not an active tool for stabilization of the business cycle, however, quantitative results supports the assessment that the existence of long term bonds smoothens the economic cycles.

To sum up, the articles mentioned above emphasize that the existence of long-term bonds is important to the government fiscal balance and to reduce the uncertainty of the tax regime imposed on the families. This work performs a complementary approach in a sense that it takes into account the the importance of long-term bond as a source of liquidity to the indebted households facing a credit crunch.

3 The Economic Environment

The model follows the standard framework proposed by Bewley-Hugget-Aiyagari, i.e., it incorporates the basic features of the environment proposed by Huggett (1993), Aiyagari (1994), Den Haan (1996), Krusell et al. (2011) and Guerrieri and Lorenzoni (2011) with the addition of public debt whose maturity is allowed to be higher than one period.

We consider a discrete time and infinite horizon economy, with idiosyncratic income shocks and sequential asset trade subject to an exogenous defined borrowing constraint. The economy is populated by a government and a *continuum* of measure 1 infinite lived households, that are indexed by $i \in \mathrm{I} = [0, 1]$.

3.1 Households

Formally, consider a discrete time economy where there is a *continuum* of infinitely-lived and ex-ante identical households, which have additively separable preferences over a sequence of non-durable[1] consumption goods $c_i \equiv \{c_{i,t}\}_{t=0}^{\infty}$, where $c_{it} \geq 0$, and from leisure plans $l_i \equiv \{l_{i,t}\}_{t=0}^{\infty}$, $l_{it} \geq 0$, where t denotes time. These individuals maximizes an expected discounted utility of the form:

$$U\left(c_i, l_i\right) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, l_{i,t}\right), \tag{1}$$

where $\beta \in (0, 1)$ is the subjective time discount factor and the operator E_0 denotes the expectation conditional on the information available at date $t = 0$. The period utility function $u : \mathbb{R}_{+}^{2} \to \mathbb{R}$ is a C^2 function assumed to be strictly increasing, strictly concave and continuously differentiable, with $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Each period, the individual receives as endowment a unit of time, which is allocated between leisure and work, represented by $h_{i,t}$, such that $l_{i,t} = 1 - h_{i,t}$. The households also faces an exogenous borrowing constraint denoted by $\phi \geq 0$, which means that there is credit restrictions that limits the ability of individuals to insure their income shocks.

We assume that each household face an uninsurable idiosyncratic risk modeled as a stochastic process that determines the level of their productivity z_{it}. Equivalently the

[1] A perishable consumption good that cannot be used as store of value.

value of their their labor income is $w_t h_{i,t} z_{i,t}$, where w_t represents the aggregate wage rate on date t. . I assume that the productivity process is independent and identically distributed across households and that it follows a finite state Markov Chain with transition probabilities given by $\mathrm{P}(z', z) = \Pr\{z_{t+1} = z' | z_t = z\}$, where z and $z' \in \mathcal{Z} \equiv \{z^1, z^2, ..., z^{N-1}, z^N\}$. The first productivity state z^1 is set to be 0, and denotes a situation where individual is unemployed. There are no aggregate shocks.

Each household produces, at every period, the non-durable consumption good using a linear technology represented by,

$$y_{i,t} = z_{i,t} h_{i,t}, \tag{2}$$

where $y_{i,t}$ denotes the output produced by individual i on period t.

3.2 Assets and Market Structure

There is no insurance market for the idiosyncratic risk. Hence, markets are incomplete and the only source of insurance against future productivity uncertainty is wealth accumulation through the risk-free asset, the use of working hours or borrowing. Households self-insure to smooth consumption and, as consequence, a precautionary motive for savings arise. Agents are borrowed constrained and ϕ is a liquidity restriction. This paper is not intended to model the sources of the debt limit. I assume that it exists as a feature of the economic environment, following the classical Bewley-Hugget-Aiyagari approach, however it is set in order to pose no risk of intentional default. When $\phi > 0$ the household is allowed to become indebted, which can be understood as a situation where it issues bonds to other households. Individuals can save or borrow at the same aggregate interest rate, denoted by r_t, and there is no possibility of default on their obligations.

The only saving instrument available on this economy is a risk-free government bond with maturity denoted by $\mathrm{M} \geq 1$, issued by the public sector. Through this asset individuals self-ensures against the idiosyncratic risk. In order to overcome the problem of dimensionality of the state space[2], we follow Woodford (2001), Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chen et al. (2012), Broner et al. (2013), and Berriel and Bhattarai (2013) and model long term bonds as perpetuities with decaying coupon payments where the rates of decayment mimic differences in maturity of these assets.

In this formulation the government issues perpetuities, denoted by b_t, with coupon payments that decay geometrically[3] at the constant rate $\rho \in [0, 1]$. A bond with decayment

[2] If I would have assumed that government could issue zero coupon bonds that mature j periods ahead, it would have been necessary to keep track of bonds issued from periods $t-1$ to $t-n$.

[3] Note that if $\rho = 1$, the perpetuity b_t is a consol and if $\rho = 0$ we are back to the case of one period

factor ρ issued on period t pays a coupon equal to $\rho^j b_{t+j}$ in the period $t+j$, with $j \geq 0$. Defaulting is not an option for the government. Defining B_t as the economy's outstanding payments position from all previous and current bonds issued, we have, at every date t,

$$B_t = \sum_{j=1}^{t} \rho^{j-1} b_{t-j} = b_{t-1} + \rho b_{t-2} + \cdots + \rho^{t-1} b_0 \tag{3}$$

where b_0 is given. As one can see, at every period the set of this general instrument can be interpreted as a portfolio of infinitely many bonds whose weights are given by $\rho^{j-(t+1)}$, with $j > t$. Indirectly, the parameter ρ defines the maturity of this portfolio. Is also worth to note that B_t follows a recursive structure given by,

$$B_t = \rho B_{t-1} + b_t \tag{4}$$

This expression is very useful to write the household borrowing constraint in a more concise manner. Treating this as the outstanding stock of the perpetuity we have a convenient way of dealing with long maturity bonds which dramatically reduces the state space once it is only necessary to keep track of the total number of bonds issued and not the number of bonds issued in each period.

Government Budget Constraint The government imposes on all individuals a lump sum tax, denoted by $\tau_t \geq 0$ and provides an unemployment assistance denoted by $v_t > 0$ on date t. Therefore the effective lump sum tax $\tilde{\tau_{it}}$ is defined as,

$$\tilde{\tau_t}\left(z_{i,t}\right) = \begin{cases} \tau_t, & if\ z_{i,t} > 0 \\ \tau_t - v_t, & if\ z_{i,t} = 0 \end{cases} \tag{5}$$

Households can use their net after-tax income to purchase consumption goods or bonds. The government chooses the aggregate supply of perpetuity that will be issued every period $b_t \geq 0$, a lump sum tax denoted by $\tau_t \geq 0$ and an unemployment benefit $v_t \geq 0$. There is no public spending other than the debt service and the unemployment benefit transfer. Let Zm_t denotes the price of the perpetuity, and the government budget constraint will be given by,

$$Zm_t b_t + \tau_t = B_{t-1} + v_t u$$

or alternatively, using Equation 4, we have,

bond, which is a particular case of this more general structure.

$$Zm_t B_t + \tau_t = B_{t-1}\left(1+\rho Zm\right) + v_t u \tag{6}$$

where $u = Prob(z_{it} = 0)$. Following Guerrieri and Lorenzoni (2011), when studying the transition dynamics we assume that bond supply and the unemployment benefit will be held constant on its initial steady state level. On that case, the lump-sum tax will adjust in order to keep the government budget constraint balanced.

Household Budget Constraint The borrowing constraint of the individual $i \in I$ will be given by,

$$c_{i,t} + Zm_t b_{i,t} + \tau_t \leq \sum_{j=1}^{\infty} \rho^{j-1} b_{i,t-j} + w_t z_{i,t} h_{i,t} + I_{\{z_{i,t}=0\}} v_t \tag{7}$$

One advantage of assuming that the entire stock of long-term government bonds consists of perpetuities of this type is that the price in period t of a bond issued s periods ago $Zm_t(s)$ is a function of the coupon and the current price $Zm_t\left(s\right) = \rho^s Zm_t$. This relation allows us to rewrite the household budget constraint in a more convenient recursive formulation. One bond of this type that has been issued $s-1$ periods ago is equivalent to ρ^{s-1} new bonds. Considering the individual $i \in I$, the no arbitrage condition at time $t-1$ implies that the his portfolio value is given by,

$$Zm_{t-1} B_{i,t-1} = \sum_{s=1}^{\infty} Zm_{t-1}\left(s-1\right) b_{i,t-s}$$

$$Zm_{t-1} B_{i,t-1} = \sum_{s=1}^{\infty} \rho^{s-1} Zm_{t-1} b_{i,t-s}$$

$$B_{i,t-1} = \sum_{s=1}^{\infty} \rho^{s-1} b_{i,t-s}$$

Therefore, the individual budget constraint can be rewritten as,

$$c_{i,t} + Zm_t b_{i,t} + \tau_t \leq B_{i,t-1} + w_t z_{i,t} h_{i,t} + I_{\{z_{i,t}=0\}} v_t$$

Using the relation established on Equation 4, we have that,

$$c_{i,t} + Zm_t B_{i,t} + \tau_t \leq \left(1+\rho Zm_t\right) B_{i,t-1} + w_t z_{i,t} h_{i,t} + \mathrm{I}_{\left\{\mathrm{z_{i,t}=0}\right\}} v_t \tag{8}$$

Prices, Yield to Maturity and Duration The price of a new debt issued is the discounted sum of the value of the promised future payments. I define the yield-to-maturity on each bond as the implicit constant interest rate at which the discounted value of the bond's coupons equal its price. Its exactly the concept of internal rate of return of a cash flow. Denoting the yield-to-maturity by ytm, the perpetuity price Zm_t on date t will be given by,

$$Zm_t = \sum_{j=1}^{\infty} \frac{\rho^{j-1}}{(1+ytm)^j} = \frac{1}{\rho} \sum_{j=1}^{\infty} \left(\frac{\rho}{1+ytm} \right)^j = \frac{1}{1+ytm-\rho} \tag{9}$$

and conversely,

$$ytm = \frac{1}{Zm_t} + \rho - 1 \tag{10}$$

Finally, for a given $\rho \in [0,1]$ I define the duration of debt issued at each date as the weighted average of the time until each coupon payment[4], with the weights determined by the fraction of the bond's value on each payment date, that is,

$$\mathrm{M} = \frac{1}{Zm_t} \sum_{n=1}^{\infty} n \frac{\rho^{n-1}}{(1+ytm)^n}$$

Solving it with known results form geometric series results in,

$$\mathrm{M} = \frac{1+ytm}{1+ytm-\rho} \tag{11}$$

3.3 Equilibrium Definition

In the presented framework, the aggregate state of the economy is given by the joint distribution of households, denoted by Ψ, over the idiosyncratic productivity status z, and the asset holdings b. Therefore, state variables can be summarized by a vector $(z, b; \Psi)$. Furthermore, individuals of this economy know that this distribution evolves according to:

$$\Psi' = \mathcal{Q}(\Psi),$$

where $\mathcal{Q}$ represents a transition function to the current aggregate state into the next period joint distribution. In practice, $\mathcal{Q}$ is found by combining the households optimal decision policies for bond holdings with the exogenous markov process for productivity[5].

[4] This is the duration concept was originally presented on Macaulay (1938).

[5] The function $\mathcal{Q}((z,b), \mathcal{Z} \times \mathcal{B})$ is defined as the probability that an individual with current state(z,b),

Let $\bar{b}$ be the maximum bond holdings in the economy, assuming that these upper bound exists. Define the compact set $\mathcal{B} = \left[-\underline{b}, \bar{b}\right]$. Using the set of productivity states $\mathcal{Z}$, let state space $\mathcal{S}$ be the cartesian product $\mathcal{Z} \times \mathcal{B}$. Let the σ- algebra $\Sigma_{\mathcal{S}}$ be defined as $\mathfrak{B}_{\mathcal{B}} \otimes \mathcal{P}(\mathcal{Z})$, where $\mathfrak{B}_{\mathcal{B}}$ is the Borel σ-algebra on $\mathcal{B}$ and $\mathcal{P}(\mathcal{Z})$ is the power set of $\mathcal{Z}$. The space $(\mathcal{S}, \Sigma_{\mathcal{S}})$ is a mensurable space and $\mathcal{S}$ is a typical subset of $\Sigma_{\mathcal{S}}$. Given that, we have all elements needed to state the equilibrium definition.

Definition. *Given a Markov Chain* $(\mathrm{P}, \mathcal{Z})$, *an initial bond holdings position* $b_0 \equiv (b_{i,0})_{i\in I}$, *a stationary recursive competitive equilibrium is a set of decision rules denoted by the functions* $\{\mathrm{V}, g_b, g_c\}$, *with* $\mathrm{V} : \mathcal{S} \to \mathbb{R}$ *as a value function and* $g_b : \mathcal{S} \to \mathbb{R}$, $g_c : \mathcal{S} \to \mathbb{R}_+$ *as saving and consumption policy functions, a government policy* $\{\tau, v\}$, *a law of motion* $\mathcal{Q}$, *a measure of households* Ψ^* *and a price* Zm, *such that:*

1. *For each* $i \in I$, *given the price* Zm, *the government policy* $\{\tau, v\}$, *the initial condition* (b_0), *the policy functions* $\{g_b, g_c\}$ *and the value function* V *solve the following household decision problem, in its recursive formulation:*

$$\mathrm{V}\left(z, b\right) = \underset{\{c,h,b'\}}{max} \left\{ u(c, h) + \beta \sum_{z' \in Z} P_{zz'} \mathrm{V}\left(z', b'\right) \right\} \tag{14}$$

$$s.t.: \quad c + Zmb' + \tau = \left(1 + \rho Zm\right) b + zh + \mathrm{I}_{\{z=0\}} v \tag{15}$$

$$c \geq 0, \;\; h \in [0, 1]\,, \;\; b' \geq -\phi \tag{16}$$

where $\mathrm{I}\{.\}$ *is an indicator function that takes* 1 *if* $z = 0$ *and* 0 *otherwise.*

2. *Labor supply and asset demand is obtained aggregating over households:* $N = \int_{\mathcal{S}} zh\left(z, b\right) d\Psi^*$ *and* $B^d = \int_{\mathcal{S}} g_b\left(z, b\right) d\Psi^*$*;*

3. *The bond price* Zm *clears the bond market* $\int_{\mathcal{S}} g_b\left(z, b\right) d\Psi^* = \mathrm{B}$*;*

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moves to any point in the space $\mathcal{S}$ on the next period. Formally this function is stated as $\mathcal{Q} : \mathcal{S} \times \Sigma_{\mathcal{S}} \to$ [0, 1], with,

$$\mathcal{Q}\left(\left(z, b\right), \mathcal{Z} \times \mathcal{B}\right) = \mathcal{I}_{\{g_b(z,b) \in \mathcal{B}\}} \sum_{z' \in \mathcal{Z}} p(z', z), \tag{12}$$

where $\mathcal{I}_{\{.\}}$ is an indicator function, which takes 1 if its argument is true and 0 otherwise, and $g_b\left(z, b\right)$ is the optimal saving policy. Then $\mathcal{Q}$ is a transition function and the associated next period state distribution is given by,

$$\Psi\left(\mathcal{Z} \times \mathcal{B}\right) = \int_{\mathcal{Z} \times \mathcal{B}} \mathcal{Q}\left(\left(z, b\right), \mathcal{Z} \times \mathcal{B}\right) \Psi\left(dz, db\right) \tag{13}$$

The algorithm used to compute the distribution function is presented with further details on Appendix A1 Section.

4. *The government budget is balanced:* $\tau = \mathrm{B}\left(1 + Zm\left(1-\rho\right)\right) + \upsilon u$*;*

5. *For all* $(\mathcal{Z} \times \mathcal{B}) \in \Sigma_S$*, the transition function is consistent with agent's optimal decisions, in the sense that is generated by the optimal decision rules and by the law of motion of the shock. The invariant probability measure satisfies,* $\Psi^* = \mathcal{Q}\left(\Psi^*\right)$*.*

4 Calibration

The model is calibrated in order to be consistent with known features of U.S. wealth distribution and specific data moments before the credit shock. This section is divided in two parts. The first, presents preferences parameters and the second describes in detail the stochastic process used to model the productivity shocks.

Preference Parameters The calibration strategy is in large extent in accordance with the incomplete markets literature. A brief summary, with description of parameters, its targets and/or the sources used for calibration is presented on the Table 1. Considering that the purpose of this paper is evaluate the role of asset maturity on macroeconomic aggregate and price dynamics in an event of a credit crisis, on the different exercises performed on the next sections asset price parameters were calibrated for three distinct maturities. For this reason the field "Value" of Table 1 is divided in three columns, in which each of them has parameters associated to a given maturity. Along the text $\mathrm{M_i}$, with $i \in \{1,2,3\}$, represents the economy where the bonds has maturity of i periods.

Time is discrete, measured in quarters. The households has CRRA type preferences, given by,

$$u\left(c,h\right) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma \frac{\left(1-h\right)^{1-\psi}}{1-\psi} \tag{17}$$

where σ is the intratemporal elasticity of substitution, the parameter ψ is the curvature of utility from leisure, directly related to the Frisch elasticity of labor supply, and γ is the coefficient of leisure in utility. The coefficient of relative risk aversion parameter σ, set to be equal 4, a value that is commonly used on this literature. In line with the evidence found on Chang and Kim (2006), Fuster et al. (2007), Floden (2008) and Kimball and Shapiro (2008) the parameter ψ was chosen so that the average Frisch elasticity of labor supply is 1. The parameter γ was chosen so that the average worked hours for employed workers are 40% of their time endowment, in line with the evidence reported on Nekarda and Ramey (2013). When the bond debt has 1 quarter of maturity, the target for the intertemporal discount factor, β, is the quarterly interest rate of 0.625%. This target represents the average of the real Fed Funds rate over the last 50 years. In the case of an asset which has average maturity of 2 quarters, the discount factor is chosen to yield a yearly interest rate of 0.70%, and when the average maturity occurs in three quarters, the interest target is set on 0.75%. The targets for the yield to maturity of the perpetuity with two and three periods were chosen based on the average interest rate[6] of the 2 and 3 quarter treasury bonds, respectively, in the last 50 years according to the data of the constant yield to maturity price curve, daily informed by U.S. Treasury Department.

Table 1: Summary of Parameters and Calibration Targets

Parameter	Description	Value			Target/Source[1]
		M_1	M_2	M_3	
β	Discount factor	0.9777	0.9862	0.9886	ytm $\in \{0.625\%, 0.70\%, 0.75\%\}$
σ	Coefficient of relative risk aversion	4.000	4.000	4.000	General literature
ψ	Curvature of utility form leisure	1.8816	1.9999	2.0801	$\mathrm{F}_{\mathrm{U}_c} = \frac{\partial h}{\partial w}\frac{w}{h}\vert U_c = 1$
γ	Coefficient in leisure on utility	12.487	9.6826	8.6589	$\int g_h(z,b)\left(1 - 1_{\{z=0\}}\right) d\Psi = 0.4$
ρ_z	Persistence of productivity shock	0.967	0.967	0.967	Floden and Linde (2001)
σ_θ^2	Variance of productivity shock	0.0179	0.0179	0.0179	Floden and Linde (2001)
ϕ_1	Credit Constraint	1.7542	2.3901	2.9396	$\frac{D}{Y} = 0.18$
ϕ_2	Credit constraint (after the shock)	0.9759	1.3488	1.6929	$\frac{D}{Y} = 0.08$
v	Unemployment benefit	0.1685	0.1766	0.1797	40% of labor income
ρ_1	Perpetuity parameter	0.00	-	-	Debt duration: $\mathrm{M} = \mathrm{M}_1$
ρ_2	Perpetuity parameter	-	0.5035	-	Debt duration: $\mathrm{M} = \mathrm{M}_2$
ρ_3	Perpetuity parameter	-	-	0.6716	Debt duration: $\mathrm{M} = \mathrm{M}_3$
δ_ϕ	Time length of credit crunch	8	8	8	Non-default condition
B	Stock of bond supply	2.6963	2.8259	2.8758	Liquid Asset/ GDP =1.6

Notes: This parameters were calibrated for initial steady state, except for debt constant after credit shock.

The debt limit parameter, ϕ, was chosen to replicate the U.S. household's balance sheet on 2006, one year before the financial crisis of 2007/8. The target for ϕ is the debt to GDP ratio of 0.18, which is a good figure of private non-residential household debt observed on 2006, according to Federal Reserve's Flow of Funds report. On the quantitative exercises that will be performed on Sections 5 and 6, the credit crunch consists of an exogenous contraction on the limit for household debt by 10 percentage points, reducing the debt to GDP ratio to 0.08 on the final steady state. The parameter δ_ϕ, that will be discussed in details on Section 5, controls the speed of convergence during the transition to the new steady state and is chosen to avoid that indebted households became unable to repay their obligations. Along the transition to the new steady state, the same preference parameters calibrated for the initial steady state are used.

Fiscal Policy Parameters Regarding fiscal policy, we calibrate the unemployment benefit parameter following the work of Shimer (2005) and Chetty (2008), also used by Guerrieri and Lorenzoni (2011), and set it as 40% of average labor income. The inclusion of an unemployment insurance mechanism is supported by the empirical and

[6]Refers to the daily average of the last ten years.

theoretical literature, which founds evidences that many job loosers cannot smooth consumption fully due to failures in credit and insurance markets. The transition between employment and unemployment was also estimated by Shimer (2005), which used *Current Population Survey* (CPS) data. Taking into account that the time is measured in quarters, I chose the same parameters used on Guerrieri and Lorenzoni (2011), that is, the probability of become unemployed given that the individual is employed is given by $\Pi_{eu} = Prob(x' = u|x = e) = 0.057$, and the probability of become employed starting from an unemployment situation is given by $\Pi_{ue} = Prob(x' = e|x = u) = 0.882$. The lump-sum tax is chosen to keep the government budget constraint balanced.

In the same spirit of the debt parameter, the stock of bond supply B was also chosen to replicate the U.S. household's balance sheets on 2006. For the bond supply level, the target chosen was the liquid asset[7] to GDP ratio, according to the data available on Federal Reserve's Flow Funds Report. The parameter ρ_m, with $m \in \{1, 2, 3\}$, is calibrated to match the duration of perpetuity. Along the transition, ρ_m is adjusted in order to ensure that the maturity of perpetuity is held constant. The total supply of liquidity is kept constant throughout the transition, so that the government budget constraint is balanced by lump-sum tax.

In Section 7 we perform an additional policy experiment, in which the credit contraction is offset by a policy of targeted transfers implemented by the government, in the same spirit of Oh and Reis (2012). The rationale of parameter's choices, its targets and other particularities will be discussed on that Section.

4.1 Income Process

The labor endowment shocks are calibrated to replicate the typical dynamics of individual wealth and earnings in the U.S. economy. The survey on this topic is generally based on PSID[8] or SCF[9] data, which consists in a panel about individual wage, income, assets and other particular information. There are several articles estimating processes for wages. In this sense, is worth to mention the work of Floden and Linde (2001), Pijoan-Mas (2006), Krueger and Perri (2006) and Blundell and Preston (2008). The typical set up followed on these papers considers a wage distribution net of fixed heterogeneity. The hourly wage rate is given by the following process,

$$\omega_{i,t} = \theta s_{i,t} + \nu_i + z_{i,t} + \upsilon_{i,t}, \qquad with\ \upsilon_{i,t} \sim \mathrm{N}\left(0, \sigma_u^2\right) \tag{18}$$

where $\omega_{i,t}$ is the logarithm of wage received by individual i on time t, $s_{i,t}$ is a vector of observable characteristics, ν_i reflects an individual i unobserved fixed component, $u_{i,t}$

[7]Defined as the sum of all deposits plus securities.
[8]Panel Study of Income Dynamics
[9]Survey of Consumer Finances.

is an homocedastic error term normally distributed and $z_{i,t}$ is the logarithm of specific component to wage formation of individual i on time t. This idiosyncratic component is used in this literature as an approach to the productivity shock. Following Chang and Kim (2007), its assumed that this component evolves according to an AR(1) process, as follows,

$$z_{i,t} = \rho_z z_{i,t-1} + \theta_{i,t}, \qquad \textit{with } \theta_{i,t} \sim \mathrm{N}\left(0, \sigma_\theta^2\right) \tag{19}$$

where the parameter ρ_z measures the persistence of productivity shock $z_{i,t}$, with $\mathrm{Var}(\theta) = \frac{\sigma_\theta^2}{1-\rho_z^2}$, given that $|\rho_z| < 1$. Estimates for ρ_z varies in a range between 0.9 and 0.99. This paper follows the results found on Floden and Linde (2001), which estimates $\rho_z = 0.967$ and $\sigma_\theta^2 = 0.018$ using data from PSID. This continuous stochastic process for income is discretized using Tauchen (1986) methodology. It is assumed that that there are 13 different productivity states for individuals, i.e., $dim(\mathcal{Z}) = 13$, including the status $z^1 = 0$, interpreted as an unemployment shock.

Table 2 presents a summary of wealth and income distribution generated by the model in the initial steady state and also the statistics found on data collected by the Survey of Consumer Finances[10]. Income is defined as all kinds of labor revenues before taxes and wealth as the net worth of households. The model matches reasonably well both wealth as income distribution, although it tends to overestimate lower quintiles. As other important feature from the data, the model captures the fact that wealth is more unequally distributed than income[11]. Under the calibration presented on this Section, wealth Gini index found on the initial steady state, when bond duration has one period, is 0.79 and the earnings Gini index is 0.39. Despite of underestimating the fifth quintile and overestimate the first quintile of wealth distribution, the model seems to be able to generate a good representation of the American wealth distribution, even in environment of lump-sum taxes. In regard of income distribution, the model also underestimates the fifth quintile, but performs relatively well in lower earnings quintile.

The impact of asset duration on wealth and income distribution is explored in details on Section 6.

4.2 Solution Method

To solve the model, we use a policy function iteration algorithm improved by endogenous grid interpolation, that is described in detail in Appendix A and B. In order to evaluate the aggregate dynamics of a credit crisis, I have also computed the transition of economy between the stationary distributions of the pre-credit shock and the post credit

[10] This data refers to the 1998 Survey of Consumer Finances and is provided on Rodriguez et al. (2002).

[11] To see more on this topic, refer to Rodriguez et al. (2002).

Table 2: Wealth and Earnings Distribution in Model and Data

Wealth (%)						
Quintile	1st	2nd	3rd	4th	5th	Gini
Model (M_1)	−9.1	−1.3	14.3	34.1	62.1	0.789
Model (M_2)	−8.6	1.9	14.1	33.3	59.2	0.745
Model (M_3)	−8.0	3.5	14.0	32.7	57.8	0.719
Data	−0.3	1.3	5.0	12.2	81.7	0.803

Data Source: Rodriguez et al. (2002)

Income (%)						
Quintile	1st	2nd	3rd	4th	5th	Gini
Model (M_1)	5.6	10.1	15.0	24.7	44.7	0.377
Model (M_2)	4.1	8.6	14.2	25.7	47.4	0.426
Model (M_3)	3.6	8.0	13.9	26.2	48.3	0.443
Data	4.6	6.6	8.3	13.9	66.6	0.553

Data Source: Rodriguez et al. (2002)

Note: The data on this table represents shares of each quintile of total wealth and income, respectively. This data refers to wealth and income distribution for initial steady state.

tightening steady states. The extra difficulty of this exercise is that factor prices and the distribution of individuals over asset holdings and labor income change during the transition. In order to evaluate the welfare effect of this experiment, we used known measures of wealth and income inequality as well as consumption equivalent welfare measures that will be described in details on Section 6. Technical details about welfare computations is presented on Appendix C.

5 Policy Experiment

The policy experiment we propose evaluates the behavior of the economy described above in the aftermath of an adverse credit shock when debt maturity is higher than one period, which is often what is used by the recent literature. This exercise assumes that a negative credit shock is defined as an exogenous and unexpected reduction of the borrowing limit ϕ that makes credit more restricted. The objective is to indirectly induce a deleveraging process resulting in a drop of the debt to GDP ratio supported by empirical evidence presented in Mian and Sufi (2010), Mian and Sufi (2011) and Reinhart and Reinhart (2010), among others. As in Eggertsson and Krugman (2012), the motivation for this exercise is that admissible levels for leverage, here represented by the debt limit ϕ, changes over time. An extended period of steady economic growth or rising asset prices encourages relaxed attitudes towards risk-taking and leverage. External forces, however, can induce a behavioral change, sometimes abruptly. Guerrieri and Lorenzoni (2011) performs a similar exercise to replicate the credit shock observed after 2007/8 recession.

Specifically, the credit limit is reduced from ϕ_1 in the initial steady state to ϕ_2 in the end of the credit crunch, where $\phi_1 > \phi_2$, in a process that takes Γ periods to fully develop[12]. We assume that from the moment in which the credit crunch happens, the whole adjustment path of the credit limit is completely known for all individuals.

On each period, starting at $t = 1$, the borrowing limit ϕ_t follows a linear adjustment path given by,

$$\phi_t = max\left\{\phi_2, \phi_1 - \delta_\phi t\right\} \tag{20}$$

where the parameter δ_ϕ represents the speed of credit conditions adjustment. In order to preserve the basic features of the model and at the same time make the numerical solution more tractable, it is assumed that the number of periods necessary to the credit limit be reduced from ϕ_1 to ϕ_2 is the smallest number of periods such that the set of individuals who come into debt default throughout the process have zero measure. I chose δ_ϕ so that $\delta_\phi = \frac{(\phi_1 - \phi_2)}{\Gamma}$, with

$$\Gamma = argmin\left\{t \in \mathbb{Z}_{++} | \text{Prob}\left(b_{i,t+1}\left(z\right) < -\phi_t\right) = 0, \forall\, i \in \mathrm{I}, z \in \mathcal{Z}\, and\, t \leq \Upsilon\right\} \tag{21}$$

In fact, the credit limit that makes individuals indifferent on breaking the credit agreement or not can be thought as an endogenous element, as shown by Abraham and Carceles-Poveda (2010). In their analysis, this decision fundamentally depends on the individual productivity status. In this experiment we are abstracting from the possibility of default by matching the parameter ϕ so as to respect the debt limit of individuals, preventing involuntary defaults, that would take place if the borrowing constraint of highly indebted households were reduced too fast. With that in mind, we will keep the focus on macroeconomic dynamics that emerge from the model in response to the shock and study the role of the asset maturity.

5.1 Quantitative Results

Steady State Analysis The main steady state results are presented on Table 3. For each maturity, the equilibrium bond supply remains in the same level of the original steady state. However, the tightening of the debt limit in the second steady state implies in the increase on bond demand, which leads to a rise of asset price and in a drop of yield to maturity.

[12] Γ refers to the number of periods required to the credit limit decrease from ϕ_1 to ϕ_2 so that the group of individuals who come into default have zero measure. The number of periods required for the convergence of the numerical algorithm that solves the transition between the two steady states is $\Upsilon = 150$.

As shown in sixth line of Table 3, the drop on the interest rate is more intense when the maturity of the saving instruments is lower. As can be seen from Equations 9 and 11, the bond price Zm is an increasing function of maturity. Therefore, as shown in seventh row, even with a lower drop of interest rate the bond price rises more for higher durations, which results in stronger positive income effects for savers[13]. The lump-sum tax falls consistently between steady states, as can be seen on fifth row of Table 3, because the increase on bond's price has a positive net effect over the government budget constraint. Hence, the lump-sum tax needed to keep the government budget balanced decreases. When bond has one quarter of duration, the rise on its price increases government revenue, but does not affect its liabilities. On the other hand, when asset duration is higher than one period, the perpetuity's discount factor implies that the increase on bond price will also affect the outstand value of government debt that will mature in the future[14]. Therefore, the positive net effect of the rise on bond's price over government budget becomes lower as bond maturity increases. For this reason the fall of lump-sum tax is more intense when the asset maturity is lower.

In terms of macroeconomic aggregates, the crisis results in a small decrease on the output and consumption since the reduction in liquidity provided by credit induces a precautionary behavior of households. The drop on aggregate consumption and output is slightly less intense as the bond maturity gets higher, as shown on the first and second rows of Table 3. Since not all families are able to form enough precautionary savings to face the scarcer liquidity, households increase their labor supply in order to smooth eventual low productivity shocks. The labor supply rises more when the asset maturity is lower, as shown on third line of Table 3, which in great extent reflects the more intense deleveraging process endured by poorer individuals, as will be explored with further details on Section 5.2.

Table 3: Steady State Results

	Steady State 1			Steady State 2			%[1]		
	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
Y	0.4213	0.4216	0.4493	0.4206	0.4410	0.4489	−0.17%	−0.14%	−0.09%
H	0.3707	0.3672	0.3632	0.3734	0.3695	0.3650	0.73%	0.62%	0.51%
B	2.6963	2.8259	2.8758	2.6963	2.8259	2.8758	-	-	-
τ	0.0270	0.0501	0.0752	0.0201	0.0411	0.0662	−25.63%	−18.04%	−14.66%
ytm	0.0063	0.0070	0.0075	0.0036	0.0054	0.0065	−41.71%	−22.20%	−13.96%
Zm	0.9938	1.9861	2.9771	0.9964	1.9922	2.9869	0.26%	0.31%	0.33%

Notes: [1] These columns presents the percentage variation between steady states to a given maturity.

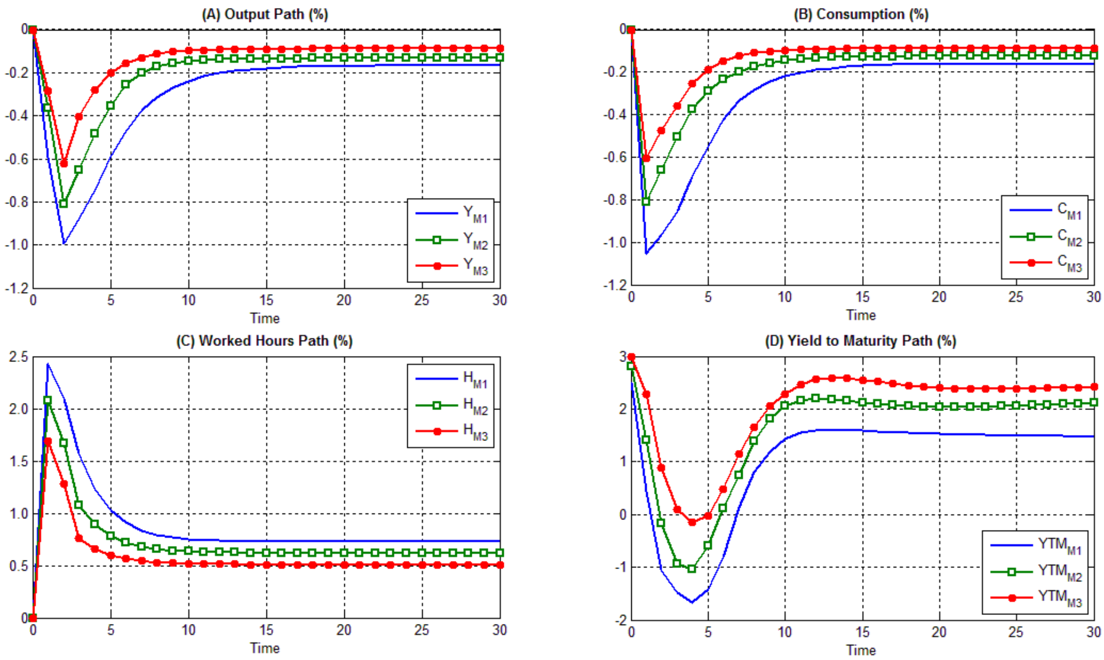
[13]As will be shown next, the difference of interest rate paths, that is a function of asset maturity, is a key element on explaining the dynamics of macroeconomic aggregates.

[14]Alternatively, one may think that the fall in interest rate reduces debt service payments without affecting the price of bonds sold by the government. When bond duration increases the drop on debt service becomes lower.

Transition Dynamics On this subsection will be presented the transition dynamics between the initial steady state, where the borrowing constraint is calibrated to keep the debt to GDP ratio at 0.18, and the final steady state, where the debt to GDP ratio is reduced to 0.08. The main results are reported on Figure 1, where $\mathrm{M_i}$, with $i \in \{1, 2, 3\}$, represents the economy where the bonds has maturity of i periods[15].

In general terms, the behavior of aggregate macroeconomic variables when $\mathrm{M} = \mathrm{M_1}$ are very similar to what is presented on Guerrieri and Lorenzoni (2011), both in terms of level and dynamics, however, is quite clear that as the bond maturity increases the impact of the credit crunch becomes less intense.

Figure 1: Aggregate Variables Transition Paths



Note: Output, Consumption and Worked Hours are expressed in terms of percent deviation from the initial steady state. The Yield to Maturity is in annual terms.

On the Panel A, we see that when $\mathrm{M} = \mathrm{M_1}$ the output drop becomes more pronounced on the third period, which results in a GDP contraction of about 1% when compared to the level that was observed on the initial situation. The new steady level is reached around the fifteenth period after the beginning of the crisis. When $\mathrm{M} = \mathrm{M_2}$, the trough occurs on the same period, when the output contracts nearly 0.8%, but the new steady state level is achieved in the twelfth period. Finally, when $\mathrm{M} = \mathrm{M_3}$, the trough of recession also occurs on third period, when the output shows a accumulated drop of approximately 0.6%. However, the recovery is completed on the tenth period. Therefore, we have two main facts associated with the output dynamics when asset maturity is higher than one quarter: first, the cumulative impact becomes smaller, and second, the recovery is faster as bond's maturity increase. These factors are also observed on other macroeconomic

[15]This notation for maturity will be used on the text from now on, unless if expressly indicated otherwise.

aggregates.

On Panel B is presented the evolution of aggregate consumption. Although the equilibrium steady state consumption is equal to the product level, along initial periods of the transition path we can see that there are some minor differences, once a sharper drop occurs immediately after the credit shock. This temporary imbalance between the aggregate output and consumption is largely explained by the tightening of credit conditions, that imposes to the families the need to reduce debt levels quickly. As a result, the consumption reacts faster than the effective labor supply. Once again it can be noticed that the introduction of assets with more than one quarter of maturity has the effect of softening the fall of the aggregate consumption. After the initial adjustment household consumption starts a recovery path that is supported by the fall on lump-sum taxes, once the rise on bond prices loosens the government budget constraint. In addition, the wealth effect generated by the appreciation of the assets of non-indebted households also acts as an important element to support the recovery on aggregate consumption. This effect is stronger when $\mathrm{M} = \mathrm{M}_3$ even with a softer decrease on interest rate. This is the case since the duration makes the price overshooting in this scenario more intense.

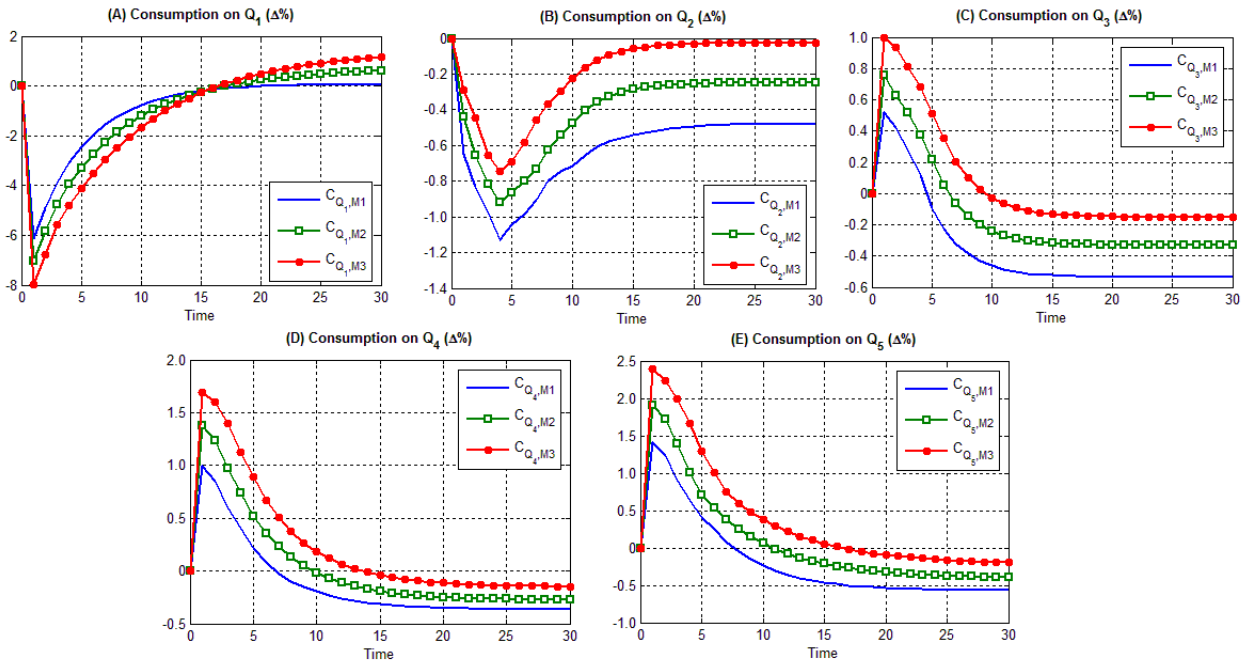
On Panel C, which shows the evolution of worked hours, we can see that the increase in supply of labor hours is also used in order to soften budget pressures faced by families. This aggregate mirrors the dynamics of consumption, though in a more intense fashion. A shorter debt duration, implies in a sharper is the initial increase of labor supply.

Finally, Panel D shows the dynamics of yield to maturity for the three cases under analysis. The dynamics is similar to the one presented on Eggertsson and Krugman (2012), which states that a credit shock can push the economy to a zero lower bound for interest rates. Although the relevant yield decline in the initial periods, that takes place in all cases due to the need of reducing the debt stock, and, therefore, the need of increasing the net asset demand, again, we see that the increase on asset maturity softens this process. As will be discussed on Subsection 5.2, the consolidated deleveraging pressure is lighter for $\mathrm{M} = \mathrm{M}_2$ and $\mathrm{M} = \mathrm{M}_3$, which makes the yield to maturity less responsive to the credit shock as bond duration increases. The trough of interest rate path lies between the third and forth period, independently of bond maturity, which resembles output dynamics

5.2 Transmission Mechanisms of a Credit Shock When There is Long Term Debt

On this subsection, our goal is tounderstand the mechanisms which makes the increase on asset maturity soften the recession. For this purpose, we analyze the household's decisions in groups of individuals (cross section) ordered by their asset holdings. The families on the economies under study were divided on 5 different wealth quantiles, so that each of them is identified by Q_i, with $i \in \{1, 2, 3, 4, 5\}$. As usual in this type of analysis quantiles are ascending ordered, so that wealthier individuals in terms of assets are represented by higher quintile levels.

Figure 2: Consumption on Quintiles



Note: On all panels the paths for consumption, for all cases, are reported in terms of accumulated percentage deviation in relation to the initial steady state.

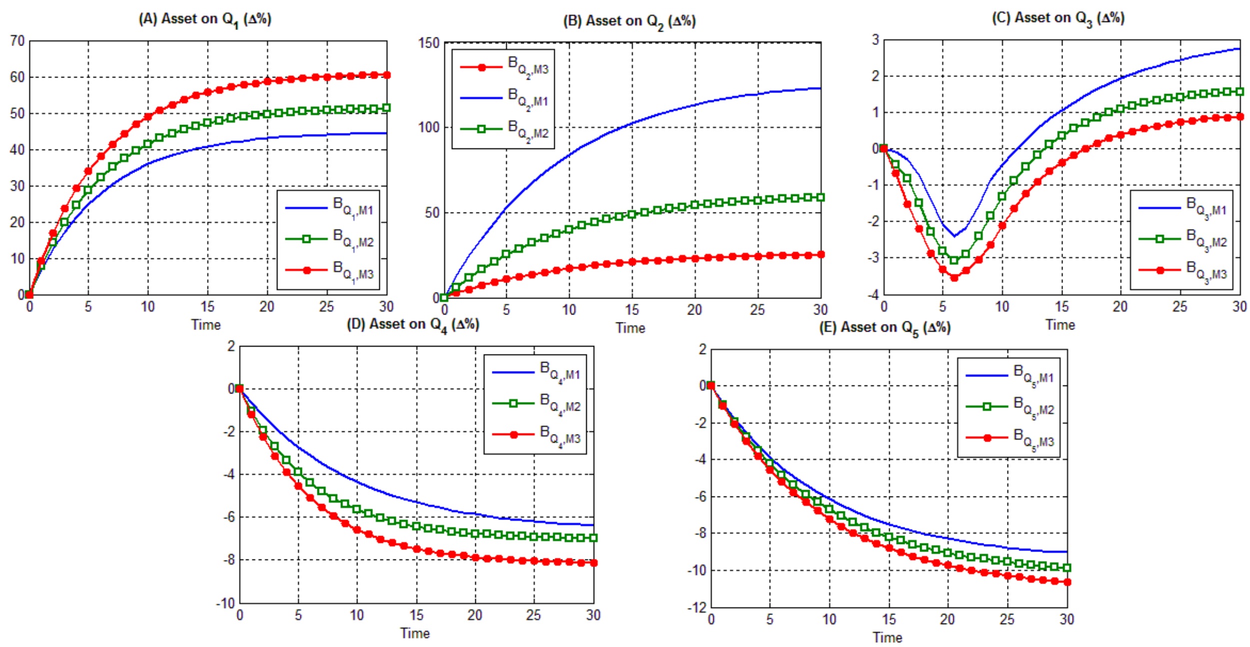
The basic transmission mechanisms of a sudden shock on debt limit are the general equilibrium forces that work through the price system and the asset reallocation among individuals. Note that total liquidity level provided by the government is held fixed in $\bar{B}$ along all the transition to the new steady state for all different asset maturity[16]. As credit becomes scarcer, there is an increase in the demand for assets due to debt repayment, which causes a sharp rise in its prices and a fall in the interest rate (or yield to maturity). There are important particular factors driving the behavior of individuals, depending on their wealth position. At lower levels of wealth, Q_1 and Q_2, where individuals are more leveraged and less sensitive to the asset price, there is a sharp cut on consumption, as shown on Panels A and B of Figure 2, and simultaneously a strong increase in labor supply, as can be seen in Panels A and B of Figure 4. For these individuals extra income from wage earnings may enable them to pay for existing debt and/or to raise their level of precautionary savings. This movement is more intense when bond maturity is higher, since the price effect of the drop in interest rate becomes stronger as maturity increases. With this, the indebted households of Q_1 experience a more severe budget tightening, which results in the need of cutting consumption level more significantly. In Q_2, we have some differences, since these individuals became indebted when bond has maturity of

[16]The equilibrium bond supply, for each steady state and maturity is shown on Table 3.

one quarter, but do not have debt when it has a higher duration. In the latter case, such individuals are benefited by the increase in bond price, which allows for a smoother consumption adjustment. As the asset maturity increases, this mechanism is enhanced.

The net increase in demand for savings by poorer individuals is offset by the reduction on asset demand by the richer, a movement that in higher quantiles is strongly heighten by the rise on asset prices. The positive wealth effect experienced by this group results in a drop of asset demand, which frees up space on their budget and leads to an increase in consumption. This movement, that starts on the range of middle wealth households, becomes more intense for the richer ones. This dynamics is properly described on Panels C, D and E of Figure 2

Figure 3: Asset on Quintiles



Note: On all panels the paths for asset holdings are presented in percent deviations from the initial steady state accumulated along the time.

We highlight the movement that happens on Q_4 and Q_5, where the consumption increase just after the credit shock is more intense for higher bond maturities. As will be shown on Figure 3, the asset sell off in this group gets stronger as the asset duration increases.

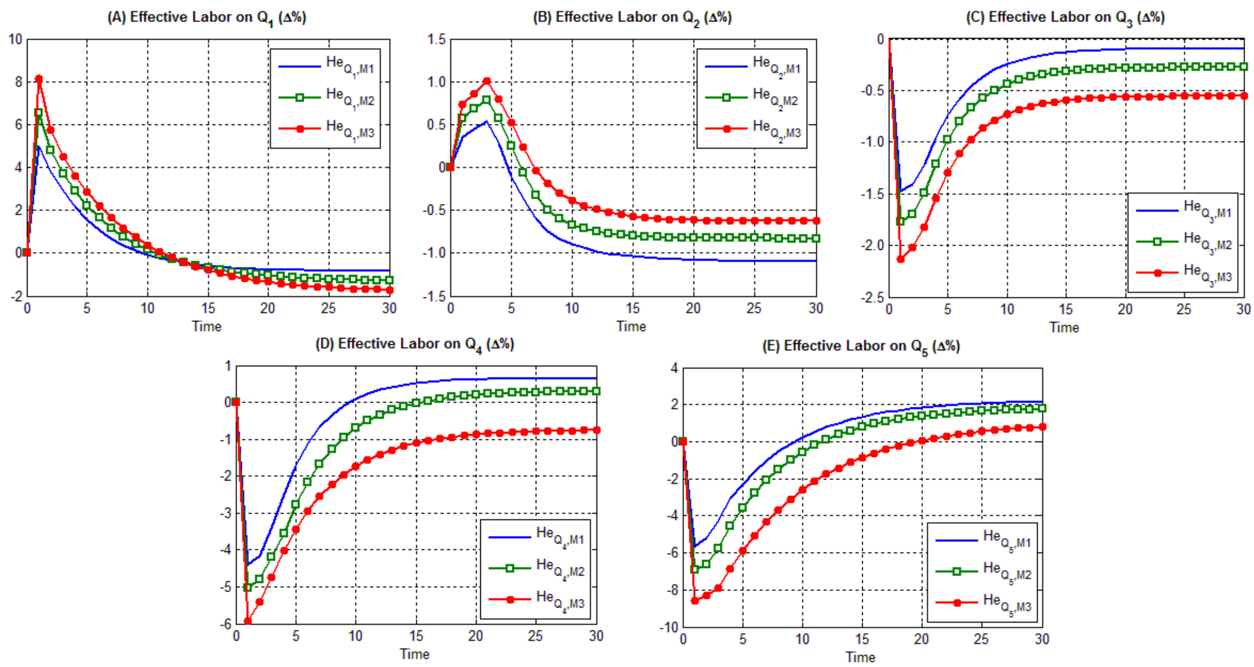
The price stimulus is stronger when debt maturity increases, which makes wealthier individuals more willing to reduce their asset positions when perpetuity duration is longer. The distance from the borrowing constraint also helps to explain this reaction: for this group, the mechanical impulse of deleveraging is negligible, unlike what occurs on Q_1 and Q_2 where the balance sheet adjustment pressure is the main driver of expenditure dynamics. Mirroring the consumption behavior, there is also a reduction in labor supply of these individuals, as can be seen on Panels C, D and E of Figure 4.

Complementing this analysis, Figure 3 exhibits the behavior of household savings for the five wealth quintiles along the transition to the new steady state. On the Panels A and B, which depicts the behavior of families on Q_1 and Q_2, there is a strong movement of deleveraging with debt repayment and increase on precautionary savings. Given the persistence of the productivity shock, deleveraging is the only viable alternative to poorer individuals. Individuals of Q_1 remain with negative net worth after the transition to the new steady state, regardless of the asset maturity, but the families of Q_2 end up carrying positive net savings in the case of $M = M_1$[17].

On Q_1, we notice that the deleveraging process is more intense when the maturity of debt is higher, for the general equilibrium reasons already exposed. The stronger increase on bond prices motivates a faster repayment of financial obligations of families on this group. On Q_2, in its turn, the deleveraging process is more intense when $M = M_1$. This happens because when assets has maturity of one quarter, households on this quantile have a higher willingness on taking debt positions. On the other hand, when maturity is longer than one quarter, individuals of Q_2 preventively accumulate savings to move away from credit limit and to avoid the higher risks of unexpected fluctuations of bond prices.

On Q_3, after an initial period of savings reduction , families accumulate assets and end up with an increase in their wealth. The individuals on Q_4 and Q_5, reduce their savings due to the rise in bond prices (or equivalently, because of the fall on yield). This wealth effect makes them sell part of its assets to poorer households. In short, as shown in the Panel A, increasing the maturity of assets makes the deleveraging process of the poorer families starker, while Panels D and E shows that the increase in maturity sharpens the reduction in the stock of assets of the rich agents.

Figure 4: Effective Labor Supply on Quintiles



Note: For all panels, the transition paths of the effective labor supply, for all cases under analysis, are presented in percentage deviation from the initial steady state accumulated along the time.

[17] When $M = M_2$ and $M = M_3$ individuals of Q_2 has positive bond savings even before the credit shock.

Figure 4 shows the behavior of the effective labor supply, measured as percentage deviation from the values observed in the initial steady state, for individuals that belongs to the five wealth quantiles under study. As already mentioned, the supply of effective labor rises among poorer individuals, those on Q_1 and Q_2, at least until the tenth period. This dynamics is represented in Panels A and B. After this initial boost the supply of labor hours gradually decreases to the level of the new steady state. Conversely, the effective labor supply undergoes a sharp initial drop among richer individuals in Q_3, Q_4 and Q_5, as can be seen on Panels C, D and E, a movement which also lasts a little less than ten periods. Afterwards, it starts a progressive recovery path toward the equilibrium level of the second steady state, which on Q_5 results in modest advance with regard to the level of effective supply of labor observed before the credit shock.

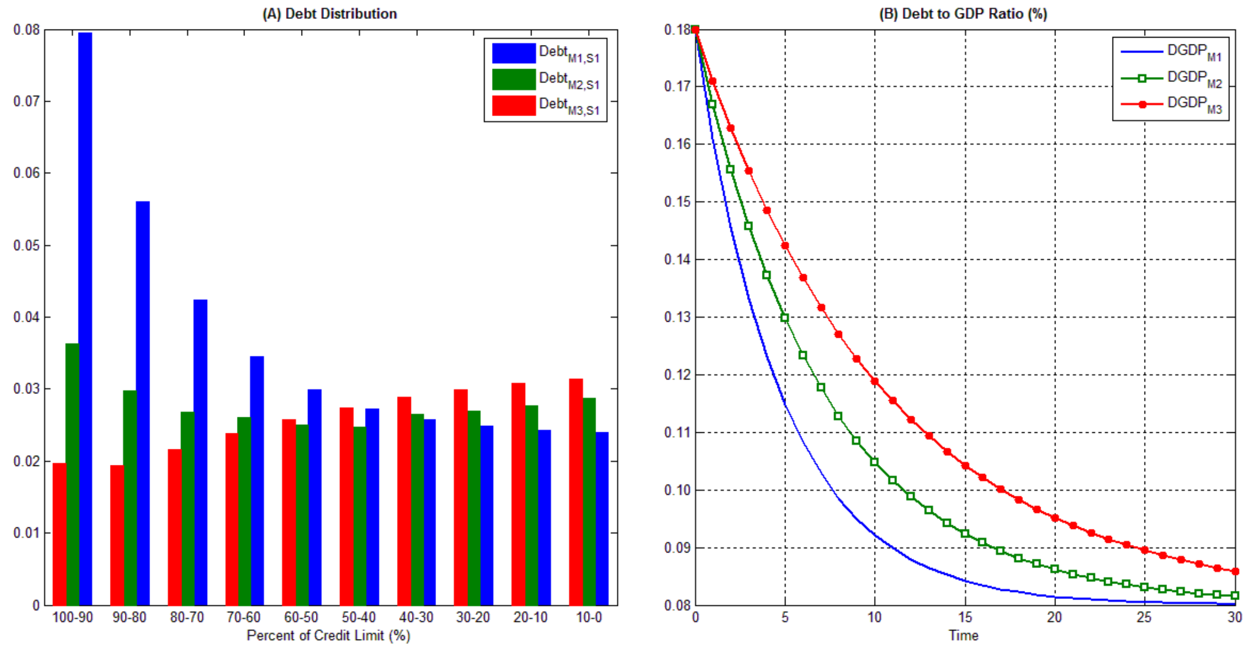
Also in this figure, we notice that the longer the debt duration is, the more effective labor supply rises among individuals of Q_1 and Q_2. This pattern reflects that an increase in maturity deepens the balance sheet pressures faced by poorer individuals due to the more intense general equilibrium effects on bond price. Finally, individuals that belongs to Q_3, Q_4 and Q_5, who originally had positive asset holdings before the credit crunch, and therefore are benefit by the increase in bond prices, experiences a drop on its labor supply. Is worth to note that this drop becomes more intense as wealth quantile rises. The reduction on labor supply in the group of rich agents is sharper for higher bond maturity since the positive wealth effects increases with asset duration. This impact is particularly beneficial for more productive households, who understands the loosening on their budget as a low risk opportunity of rising consumption. The increase in bond duration boosts the growth of net asset demand by the poorer individuals, which, as a consequence, causes the wealthy ones to speed up the reduction of their bond position. The positive wealth effects, which occurs specially on the first periods after the credit crunch, makes richer individuals more prone to temporarily cut working hours when maturity increases. Naturaly this reduction is larger in Q_4 and Q_5.

To sum up, there is a dichotomic behavior in quantile effective labor response that reflects the forces that drives the economy after such unexpected change on liquidity conditions. As presented on the paragraphs above, the first two wealth quintiles reflect more clearly the mechanical deleveraging process that arises after the shock, while the superior quintiles reacts to an endogenous wealth increase through the price system. In terms of debt maturity,an increasing on bond duration makes the economy more exposed to this two forces.

Finalizing, we dedicate the following paragraphs to exam some facts related to the debt dynamics.

As already explored, when the assets are perpetuities with more than one quarter of maturity, indebted individuals experience a stronger general equilibrium pressure for

Figure 5: Debt Distribution and Debt to GDP Ratio



Note: On the legend, $\mathrm{M_i}$ represents the economy where the asset has maturity $i \in \{1, 2, 3\}$.

deleveraging. In the opposite direction, these households face a less intense mechanical pressure of deleveraging, as they use to be farther from the credit limit than individuals of the economy where asset maturity is lower. This fact is illustrated on Panel A of Figure 5, where is presented the distribution of assets of individuals who become indebted on economies with $\mathrm{M} = \mathrm{M}_1$, M_2 and M_3. In this Panel, the horizontal axis is divided in groups that denotes shares of total credit limit[18] and the vertical represents percentage of indebted households. In our simulations we found that 36.8% of households assume some kind of debt when $\mathrm{M} = \mathrm{M}_1$, 27.8% when $\mathrm{M} = \mathrm{M}_2$ e 24.9% when $\mathrm{M} = \mathrm{M}_3$. Panel A shows that, when $\mathrm{M} = \mathrm{M}_1$, about 8% of all individuals that becomes indebted uses a range between 90% and 100% of the total credit limit of the economy. The percentage of individuals in each range decreases as the credit utilization level is reduced. On the other hand, when $\mathrm{M} = \mathrm{M}_3$, we see that the percentage of individuals who uses between 90% and 100% of the credit limit is less than 2%. In addition, the percentage of individuals who become exposed to debt increases when the share of credit limit utilization is lower. There is an inverse relation between maturity and debt exposition. This occurs because households knows that oscillations in the price of bonds are more harmful to their budget when asset duration is higher.

On Panel B of Figure 5, we present the dynamics of debt to GDP ratio for each economy along the 30 first periods of transition. Debt to GDP ratio falls more smoothly as the bond maturity increases. As a consequence of the features of the debt distribution, the higher exposition to debt makes the individuals in the economy where $\mathrm{M} = \mathrm{M}_1$ more

[18] This representation of debt distribution on the initial steady state were chosen because the credit limit varies according to asset duration.

sensitive to changes on borrowing constraint. This result may seem inconsistent with the fact that, as seen in Panel A of Figure 3, the price effect makes the initial deleveraging impulse stronger in the economy where $\mathrm{M} = \mathrm{M}_3$. However, it should be noted that in the economy where $\mathrm{M} = \mathrm{M}_3$ deleveraging occurs only in Q_1, while in the economy where $\mathrm{M} = \mathrm{M}_1$ there is deleveraging both on Q_1 and Q_2. In other words, the speed of deleveraging is higher when $\mathrm{M} = \mathrm{M}_1$ because the group of individuals reducing the level of debt is greater in this economy.

In short, the results presented on this section follows the same reasoning: even with different paths for interest rates, the quantile outcomes in terms of consumption, labor supply and asset accumulation are derived from the fact that an increase in maturity intensifies the positive wealth effect for savers, on the one hand, and exacerbate the financial burden experienced by indebted individuals on the other. The mechanical reduction in credit limit, in its turn, is a more relevant transition driver for lower bond maturities because the household debt exposition is higher in this case.

6 Welfare Implications of a Credit Shock

In this section, we present a quantitative analysis of welfare. The presentation of the results is divided into two parts. The first, deals with the evaluation of the welfare variation between steady states and the second presents the welfare behavior along the transition from the initial steady-state, where the credit restriction is calibrated such that the debt to GDP ratio is 0.18, to the final steady state, where the debt to GDP ratio decreases to 0.08.

Welfare Measures The methodology used to evaluate the welfare gains is based on Domeij and Heathcoate (2004), which decomposes the total consumption equivalent variation, denoted by Δ_{T}, into an aggregate component, denoted by Δ_{A}, and a distributional component, defined as Δ_{D}. We quantify the welfare gains by evaluating the welfare between steady states and also along the transition path. In the same tradition of Lucas (1987), Floden and Linde (2001) and Alonso-Ortiz and Rogerson (2010), we consider an utilitarian mechanism of social welfare evaluation, where the total welfare variation, in terms of consumption equivalent, is the constant percentage increase in the consumption in a non-reform situation needed to equalize the welfare in the post-reform economy. Let $\mathtt{NR}$ denote an interest variable in a given period when there is no reform, here an economy where credit to GDP ratio remains in the level of the initial steady state, and $\mathtt{R}$ be the notation for a variable after the reform, that is, when credit is getting tighter. Let $x = (z, b) \in \mathcal{Z} \times \mathcal{B}$ be a point in the individual state space of the economy. Therefore, the total welfare in terms of consumption equivalent is defined as the constant $\Delta_{\mathrm{T}} \in \mathbb{R}$

that solves,

$$\int_{\mathcal{Z}\times\mathcal{B}} \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u \left(c^{\mathtt{R}}_{i,t+j}\left(x_{i,t+j}\right), h^{\mathtt{R}}_{i,t+j}\left(x_{i,t+j}\right) \right) \right\} d\Psi\left(x_{i,t}\right) =$$

$$\int_{\mathcal{Z}\times\mathcal{B}} \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u \left(\left(1+\Delta_{\mathrm{T}}\right) c^{\mathtt{NR}}_{i,t+j}\left(x_{i,t+j}\right), h^{\mathtt{NR}}_{i,t+j}\left(x_{i,t+j}\right) \right) \right\} d\Psi\left(x_{i,t}\right) \tag{22}$$

Following Domeij and Heathcoate (2004), I assume that in the absence of aggregate distortions, the relationship between the consumption of a given individual $x \in \mathcal{Z} \times \mathcal{B}$ on period t, represented by $c_{i,t}\left(x_{i,t}\right)$ and the aggregate consumption observed in the economy on the same period, denoted by C_t, must be the same before and after the reform. In other words, the following relation must holds,

$$\frac{c^{\mathtt{R}}_{i,t}\left(x_{i,t}\right)}{C^{\mathtt{R}}_t} = \frac{c^{\mathtt{NR}}_{i,t}\left(x_{i,t}\right)}{C^{\mathtt{NR}}_t} \tag{23}$$

The aggregate component of the total welfare variation is measured by the constant percentage change on consumption of individuals that are in a non-reform situation, that is needed to ensure that the equivalence relation presented on equation 23 is respected. Therefore, the aggregate welfare component is the $\Delta_{\mathrm{A}} \in \mathbb{R}$ that solves,

$$\int_{\mathcal{Z}\times\mathcal{B}} \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u \left(c^{\mathtt{NR}}_{i,t+j}\left(x_{i,t+j}\right) \frac{C^{\mathtt{R}}_t}{C^{\mathtt{NR}}_t}, h^{\mathtt{R}}_{i,t+j}\left(x_{i,t+j}\right) \right) \right\} d\Psi\left(x_{i,t}\right) =$$

$$\int_{\mathcal{Z}\times\mathcal{B}} \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u \left(\left(1+\Delta_{\mathrm{A}}\right) c^{\mathtt{NR}}_{t+j}\left(x_{i,t+j}\right), h^{\mathtt{NR}}_{t+j}\left(x_{i,t+j}\right) \right) \right\} d\Psi\left(x_{i,t}\right) \tag{24}$$

The distributional welfare component, denoted by Δ_D, is calculated as a residual of the following equation,

$$\left(1+\Delta_{\mathrm{T}}\right) = \left(1+\Delta_{\mathrm{A}}\right)\left(1+\Delta_{\mathrm{D}}\right) \tag{25}$$

More details about the computation of welfare measures are found on the Appendix C.

Results The main results of this Section will be presented on the following paragraphs.

Steady State This section analyzes the long run implications of the credit shock discussed on this paper. We focus on the comparison of aggregate measures of income and wealth distribution between the non-reform and the reform steady state for each debt duration. In terms of welfare, we compute the wage and wealth Gini index, denoted by

$Gini_{\mathrm{W}}$ and $Gini_{\mathrm{B}}$, respectively, and also the total welfare variation and its subcomponents, as defined and discussed on the previous paragraphs. The Table 4 reports the main results.

Table 4: Long Run Effects of a Credit Shock

	Steady State 1			Steady State 2			%[1]		
	M_1	M_2	M_3	M_1	M_2	M_3	M_1	M_2	M_3
$Gini_{\mathrm{B}}$	0.7896	0.7449	0.7188	0.6755	0.6412	0.6213	−14.45%	−13.92%	−13.55%
$Gini_{\mathrm{W}}$	0.3857	0.4339	0.4507	0.3772	0.4207	0.4332	−2.20%	−3.04%	−3.88%
Δ_{T}		-			-		1.92%	1.54%	1.22%
Δ_{A}		-			-		−0.14%	−0.09%	−0.07%
Δ_{D}		-			-		2.06%	1.63%	1.29%

Notes: [1] These columns presents the percentage variation between steady states to a given maturity.

In general, the results confirms the dynamics described in Subsection 5.2. In an environment where credit is scarce inequality is reduced bond in income and wealth among individuals. We can also observe that in first line, from eighth column on, where is shown the wealth distribution, the reduction in inequality becomes less intense as the maturity of asset increases. The opposite outcome is observed in the second line, from eighth column on, which presents the wage distribution between steady states.

The improvement of the wealth distribution is a result of the deleveraging of poorer individuals combined with the asset sell-off by richer ones. This effect loses intensity as the asset maturity increases, since in this scenario the household debt exposure is lower, which reduces the momentum of the deleveraging process, as shown in the end of Subsection 5.2. The same reasoning also can be used to explain the variation of the distributional component of welfare, presented on the fifth line, from eighth column on. As the aggregate component is small, the distributional component explains almost all the variation of welfare in terms of equivalent consumption.

The aggregate component of welfare, falls in the environment of tighter credit. This is explained by the reduction on output of the economy, which in its turn also decreases aggregate consumption. The growth rates of aggregate consumption and the aggregate component of welfare variation are very close in all maturities analyzed.

Higher the maturity softens the drop in consumption. The Gini index for wages falls when the economy goes from the first to the second steady state because the constraints in the credit market put pressure on individuals to use working hours as a precautionary tool to protect them against productivity shocks. As seen previously, the increase in maturity of assets helps to potentialize the general equilibrium stimulus that operates in the economy. As a response of poor households concerned with their budget balance, the growth rate of effective labor supply gets higher as the bond maturity increases, which, therefore, intensifies the overall fall of Gini index for wages.

As will be shown in the following paragraphs, the assessment of the welfare dynamics along the transition keeps the same basic features of the static welfare comparison between steady states presented here, that is, a credit tightening results in a significant and permanent improvement on wage and wealth distribution.

Transition Dynamics In this section, we analyze the transitional paths arising from a credit shock when the debt limit ϕ follows the course determined by Equation 20. Once more, the object of the analysis is the temporal dynamics of wage and wealth Gini index and the variation of the welfare in terms of consumption equivalent.

The transitional dynamics of Gini index in the three alternative maturities after the shock is shown in Figure 6. In general, the assessment of Gini index dynamics results on findings that are similar to the ones presented in the previous subsection, that is, the tightening in credit conditions results in a significant improvement in distribution of wealth and wages. However, some important qualifications need to be made.

According to Panel A of Figure 6, the wealth Gini index presents a steady declining trend along all the transition path, however, an analysis of Panel B of the same figure shows that after a sharp initial drop, the wage Gini index presents a gradual increasing path that leads it to level that is around 2% below the level of the initial steady state when $\mathrm{M} = \mathrm{M}_1$, 3% below when $\mathrm{M} = \mathrm{M}_2$ and 4% below[19] when $\mathrm{M} = \mathrm{M}_3$. In the first period after the credit crunch, the wage Gini index drops around 5% when bond has one quarter of maturity, and nearly 6.3% when the bond maturity is 3 quarters. Therefore, we have that a significant portion of the decline in inequality of wages is temporary, being subsequently reversed, while the fall in inequality of wealth has a continuous and permanent nature. This behavior ilustrates the precautionary nature of working hours in the most severe fase of credit crunch.

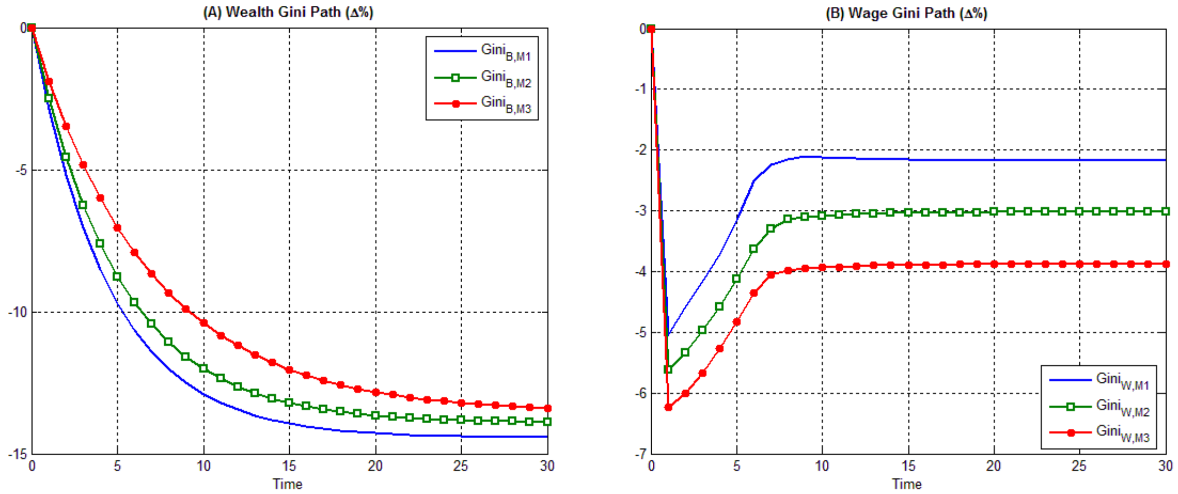
As already discussed, the transition dynamics for the Wealth Gini Index shown on Panel A of Figure 6 reflects the fact that the deleveraging loses intensity as bond maturity increases. For this reason, when $\mathrm{M} = \mathrm{M}_1$ the fall of $Gini_{\mathrm{B}}$ is faster.

The Panel B of Figure 6 shows that the new equilibrium level of $Gini_{\mathrm{W}}$ is reached around the eighth period, proportionally faster than $Gini_{\mathrm{B}}$, which reaches the second steady state level after the twentieth period. Households reducing their debt level need to increase the supply of labor with more intensity when bond prices are close to its peak. But as price transition is completed around the tenth period, as shown on Panel D of Figure 1, individuals also start to normalize their labor supply faster. In its turn, deleveraging is a slow process.

Thus, despite the downturn experienced in production, as discussed in Subsection 5.1, there is an important improvement in the distribution of wealth and wages among house-

[19]These results are near to the values presente on second line of Table 4.

Figure 6: Gini Index Path



Note: The Wealth Gini Index on Panel (A) and the Wage Gini Index on Panel (B) are resented in terms of percentage deviation from the initial steady state.

holds. The transition approach enable us to see some subtleties which makes clear that the intensity of deleveraging process is the main driver of the overall improvement of wealth distribution and that the speed of price adjustment is the key element for the evaluation of wage inequality dynamics.

Table 5: Short Term Impact on Welfare Components

	$t=1$		
	M_1	M_2	M_3
Δ_{T}	-13.2%	-11.9%	-10.1%
Δ_{A}	-12.1%	-10.4%	-8.2%
Δ_{D}	-1.1%	-1.5%	-1.9%

Notes: [1] These columns presents the percentage variation between steady states to a given maturity.

The main responsible for the sharp decrease of the total welfare variation is the aggregate component, which recedes approximately 12% when $\mathrm{M}=\mathrm{M}_1$, 11% when $\mathrm{M}=\mathrm{M}_2$ and 8.5% when $\mathrm{M}=\mathrm{M}_3$ just after the initial contractionary impact on the borrowing limit. This movement follows the fall of aggregate consumption and the increase on labor supply, as can be seen in Panels B and C of Figure 1. This dynamics becomes less intense as the deleveraging of households loses momentum, causing the aggregate component of the welfare to return to something near its initial level nearly after the tenth period. The lower level of aggregate consumption in the second steady state is what leads to a negative aggregate welfare effect in the long run, as argued on the steady state analysis performed in the beginning of this subsection.

7 The Role of Transfers on Credit Crisis

On this Section, we evaluate the role of a targeted transfer program on the above studied environment of an adverse credit shock. Oh and Reis (2012) argue that the largest part of the fiscal expansion that took place in the developed world between 2007 and 2009 occurred due to increase of social transfers. Using annual data from National Income and Product Accounts (NIPA), they estimate that medical care, retirement and disability, unemployment assistance and other similar rubrics accounts for about half of the increase in social transfers in United States.

On this section, we perform some modifications on the model presented above in order to capture the role of transfers as a buffer of a sudden reduction in credit limit. As a criteria for the targeted transfers, we follow Oh and Reis (2012) and design a transfer function which is able to extends the reach of unemployment insurance policy used on the original model (defined on Equation 5). Every household remains paying lump-sum taxes $\tau > 0$, unemployed ones (those with $z_{it} = 0$) remain able to access the unemployment insurance program, but now some individuals receive transfers that depends on their productivity status. This is done through the following transfer function, inspired by Oh and Reis (2012),

$$\mathrm{T}(z) = \gamma_z \left(1 - \frac{z}{\bar{z}}\right) \mathrm{I}(z \leq \bar{z}) \tag{26}$$

where $\mathrm{I}(z \leq \bar{z})$ is an indicator function that takes 1 if the condition $z \leq \bar{z}$ is true and 0 otherwise. The parameter γ_z measures the resources transferred to the individual with the worst productivity status. As in Oh and Reis (2012) the upper threshold for the individual productivity $\bar{z}$ is calibrated to replicate the features of U.S. tax and transfer system and also aims to keep transfers bounded above. For this purpose , we used the Congressional Budged Office (CBO) data between 2003 and 2007. According to CBO, individuals only became net tax payers[20] in the 5th quintile of income distribution. Considering the features of labor policy function, we found that the 75th percentile of idiosyncratic shock distribution[21] presents a good match to the empirical evidence and therefore is used as a target to the parameter $\bar{z}$. Finally, the parameter γ_z is calibrated to match the total transfers to GDP ratio net from unemployment insurance in the U.S. economy between 2003 and 2007, which is intended to replicate the pre-crisis profile of transfers.

On the transition exercise performed here, there will be simultaneously the credit tightening originally described by the Equation 20 and an exogenous and unexpected increase

[20] This refers to the total amount of taxes paid by each group of individual less government transfers.

[21] This number is different to the 95th percentile used in Oh and Reis (2012), because on their work there is also health transfers and the labor supply is indivisible.

on the parameter γ_z that will last while the debt limit is adjusting. The rise on γ_z is defined in order to replicate the behavior of the government transfers between 2007 and 2009. Along that period, the social transfers to GDP ratio (net from unemployment assistance) increased from 11% to 14%, according to NIPA data. On first step, γ_z^1 was calibrated when the credit limit is in the initial steady state, ϕ_1, and after we choose γ_z^2 under the credit constraint of the final stead state, that is, ϕ_2. On the second step, I assume that γ_z^1 converges to γ_z^2 through the following exogenous, fully known and linear path given by,

$$\gamma_z\left(t\right) = min\left\{\gamma_z^2, \gamma_z^1 + \delta_\gamma t\right\} \tag{27}$$

were δ_γ is an adjustment parameter defined as $\delta_\gamma = \frac{\left(\gamma_z^1 - \gamma_z^2\right)}{\Gamma}$ and Γ is defined as in Equation 21.

In the model, with the adjustments presented on the above paragraphs, the government budget constraint becomes,

$$Zm_t B_t + \tau_t = B_{t-1}\left(1 + \rho Zm\right) + \sum_{z\in\{z_2 \ldots \bar{z}\}} \mathrm{T}\left(z\right) u\left(z\right) + v_t u\left(0\right) \tag{28}$$

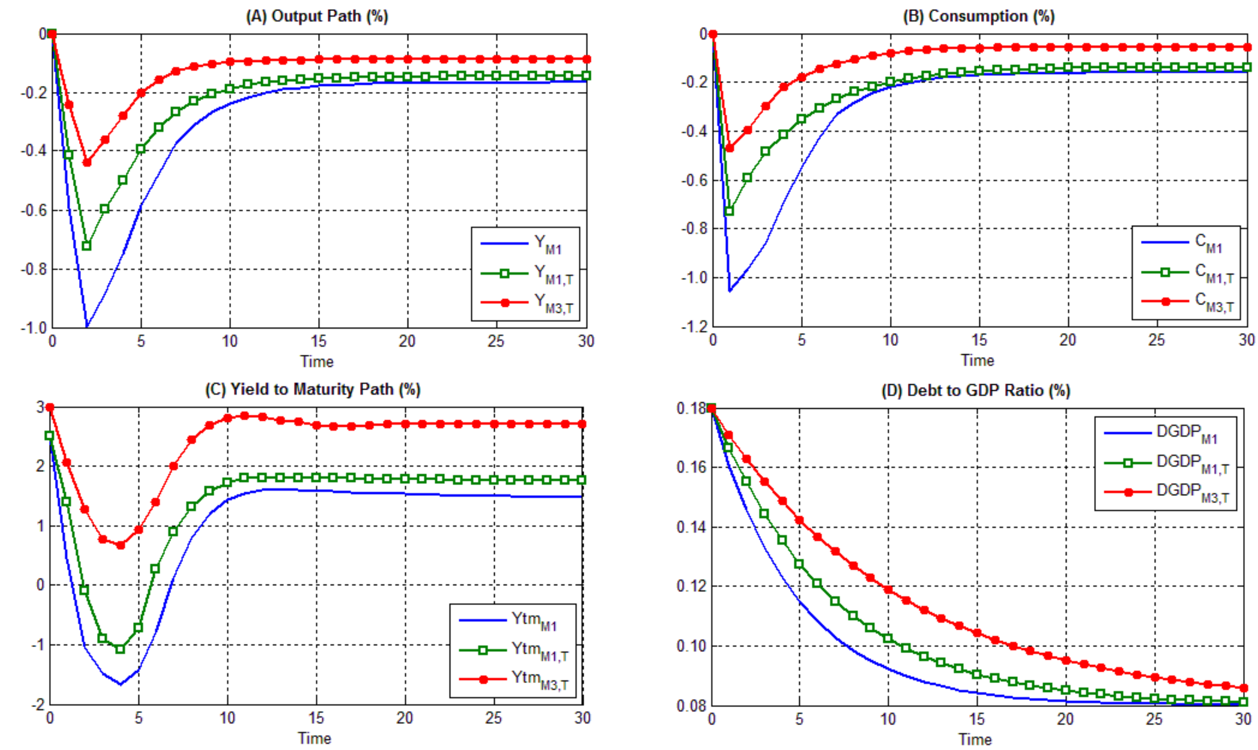
where $u\left(z\right) = Prob(z_{it} = z)$ and $u\left(0\right) = Prob(z_{it} = 0)$. There is also a little modification on the household budget constraint, which is now represented by,

$$c_{i,t} + Zm_t b_{i,t} + \tau_t \leq \left(1 + \rho Zm_t\right) b_{i,t-1} + w_t z_{i,t} h_{i,t} + \mathrm{I}_{\left\{\mathrm{z_{i,t}=0}\right\}} v_t + \mathrm{T}\left(z_{i,t}\right) \tag{29}$$

Results The main results of this exercise are presented in Figure 7, where we show the equilibrium paths for output, consumption, yield to maturity and debt to GDP ratio along the transition. The solid blue line represents the transition paths of the original exercise, where there is only unemployment benefit assistance, the green squared line represents the situation where, besides the unemployment benefit, the government implements a broad social program of targeted transfers and issues only one quarter debt and the red dotted line represents the same public policy previously described, but here government issues perpetuity which has the average maturity of three quarters. It is worth noting that on this exercise the total outstanding securities of the economy remains at the same level observed on the initial equilibrium steady state along transition. Therefore, there is no public bond issuance nor any type of direct injection of liquidity to offset the contraction of credit limit.

Concerning the price, shown on Panel C, a comparison of the trajectory of the yield to maturity on the original exercise and the results observed after the transfer program reveals that asset prices rise more in the first case. On the trough of the original exercise,

Figure 7: Targeted Transfer Policy and Maturity



Note: Output, Consumption and Worked Hours are expressed in terms of percent deviation from the initial steady state. The Yield to Maturity is in annual terms.

which occurs on the fourth period, the real interest rate approaches -2%, in annual terms. With the transfer program and $M = M_1$, the trough occurs on the second period, when the real yield to maturity reaches nearly -1%, and in the case of $M = M_3$, the trajectory is even smoother, with a trough on the fifth period, when the yield to maturity falls to 1%. An analysis of the policy functions and the individual behavior on wealth quintiles reveals that a more generous policy of transfers, implemented by the government during the period of credit decline, provides liquidity to the families that are closer to the borrowing constraint, softening, thus, the deleveraging process. The increase in bond duration reinforces the initial effect of transfers, in a sense that the rise of wealth effect is also a reason of smoother impact. The panel D shows that the decline rate of the debt to GDP ratio is a little less intense when government increases the transfers on the case where $M = M_1$, and becomes considerably smoother when $M = M_3$.

The panel B of Figure 7 shows that with the reduction in budget pressures experienced by individuals, especially those on lower wealth quintiles, the contraction of consumption is lower with the transfer policy. The increase on asset maturity makes the aggregate consumption less volatile. On the other hand, the richer individuals, upon which relies the heavier burden of transfers, become more cautious on reducing asset level, when compared to what is observed in the original exercise (where there is only unemployment benefit). The cost of transfer policy also restricts the positive wealth effect of bond price for such individuals, which results in a lower consumption expansion relative to what was observed in the environment of original exercise.

Finally, the original precautionary motive is reinforced by the new transfer payments, which discourage them on cutting labor hours aggressively. Thereby, as the distribution of assets becomes more skewed to the levels where there is positive asset holdings, we have now a larger mass of individuals reducing reducing less their labor supply. It is also worth noting that due to the design of the transfer program, the share of individuals who least reduce the labor supply consists of higher productivity workers. Therefore, this kind of program also mitigates the fall of the effective labor supply, and, as a consequence, softens the output drop, as shown in Panel A. For comparison, when bond has one period of maturity the through of output constraction is reduced from 1 to 0.7%. When bond maturity is three quarters, the sharpest output drop is reduced from -0.6%, as can be seen on Panel A of Figure 1, to -0.45%.

8 Conclusion

This paper investigates the effects of a credit shock in an environment of incomplete markets where assets can have duration higher than one period. I found that a liquidity crisis caused by a sudden reduction of the credit limit leads to sharp short term drop on the output, but the duration of the asset that is used as storage of value matters in determining the size and the length of that recession. Quantitative results points that an increase in debt maturity can smooth the adjustment path of macroeconomic aggregates. This occurs because the rise of asset duration indirectly enhances the income effects that works through the general equilibrium channel, which benefits bondholders and thus softens the recession. Summarizing, there is a dichotomic behavior in quantile response to the change on liquidity conditions that drives the economy after such unexpected credit shock. As presented on the sections above, the first two wealth quintiles reflects more clearly the mechanical deleveraging process that arises after the shock, while the superior quintiles reacts to an endogenous stimulus related to the general equilibrium forces that works through the price system. In terms of debt maturity, it can be said that increasing the duration makes the economy more exposed to the second force.

In standpoint of welfare, the exercises performed shows that after a credit contraction occurs a significant drop on the inequality of wealth distribution and also a meaningful progress in terms of income distribution. Although the process of deleveraging endured by households results in an initial welfare loss, due to contraction of consumption, there is an important improvement in the economic resources distribution, which promotes a continuous increase on the welfare of the families. Finally, we found that a transfer program designed to match the features of the 2007 to 2009 fiscal expansion in U.S. can significantly reduce the output drop that occurs in response of a credit tightening.

The transfers reinforces the liquidity channel which also works with the asset maturity softening the deleveraging process for all bond maturities.

Appendix

A. Steady State

This appendix presents the structure of algorithm used to compute the steady state equilibrium of simulated economies. First, was contructed a grid of asset holdings, with equidistant grid points for b_j and values in the interval $[-\phi, 30]$, and a grid for idiosyncratic productivity level, which has 13 points. The continuous stochastic process for productivity was discretized using Tauchen (1986) methodology. After, for a given initial interest rate r_0 and a tax policy $\{\tau, \mathrm{B}\}$, the individual policy functions $g_c(z,b)$, $g_b(z,b)$ and $g_h(z,b)$ were computed by Euler equation iteration, with endogenous gridpoints method. The next step, is compute the distribution of assets for households, which is done through the algorithm presented on the following subsection. With distribution function and decision rules, the aggregate consumption, asset holdings and effective labor supply are found using $C = \int_{\mathcal{Z}\times\mathcal{B}} g_c(z,b)\, d\Psi$, $B^d = \int_{\mathcal{Z}\times\mathcal{B}} g_b(z,b)\, d\Psi$ and $H = \int_{\mathcal{Z}\times\mathcal{B}} z g_h(z,b)\, d\Psi$, respectively. With these aggregates, parameters were adjusted to match the targets proposed on the Section 4 and $\{\tau, \mathrm{B}\}$ were chosen to balance the government budget constraint. We defined the excess of asset demand function as $\Phi(r) = B^d - \mathrm{B}$ and repeated successively the steps described above until find a root for the function $\Phi(r)$ for a given metric, or equivalently, until finding the fixed point of the asset to labor ratio. For each iteration, the parameter ρ, which controls bond maturity was updated.

Distribution Function

After compute the individual policy function $g_b(z,b)$, the invariant distribution function was obtained through the following procedures: (1) Set a grid on the asset space finer than the one used to compute the optimal decision rules, (2) Initialize the distribution function with the equal distribution, (3) Compute the inverse of the policy function $g_b(z,b)$, $g_b^{-1}(z,b) = b$ over the grid and (4) Iterate on the equation,

$$\Psi_{m+1}(z',b') = \sum_{z' \in Z} \mathrm{P}(z',z)\Psi_{m+1}\left(z, g_b^{-1}(z,b')\right) \tag{30}$$

on gridpoints (z',b'), with $b' \geq -\phi$ until $\Psi(.)$ converges.

Since the agents with low wealth may want to accumulate debt equal or exceeding $-\phi$, $g_b(z,b)$ may not be invertible when $g_b(z,b) = -\phi$. For this reason, we defined $g_b(z,-\phi)$ as the maximum b such that $g_b(z,b) = -\phi$. Furthermore, the computation of $g_b(z,b)$ involves linear interpolation to speed up the process[22].

[22] We use linear interpolation for the computation of $g_b(z,b)$ on $b_j < b < b_{j+1}$, as $g_b(z,b)$ is obtained for a grid with larger spacing.

B. Transition Dynamics

This appendix describes the algorithm used to compute the transitional dynamics induced by the credit shock described on the Section 5. The starting point for that is the final steady state and its associated objects. The method is the backward induction which exploits the recursivity of Euler equation. First of all, we chose a time period Υ large enough to ensure that after the shock the economy is near to its final steady state. On the exercises reported on Section 5.2 we used $\Upsilon = 200$. Second, we set a path for interest rate $\{r_t\}_t$, assuming that $r_t = r_0$, $\forall t \in \{0, ..., \Upsilon\}$. After that, we set the consumption policy function on period Υ to be $g_{\Upsilon,c}(z, b) = g_c(c, h)$, and then, using the Euler equation and labor supply decision rule, calculate the sequence $\{g_{t,c}(z, b), g_{t,h}(z, b)\}_{t=0}^{\Upsilon}$ going backward from $t = 200$ to $t = 0$ through the method of endogenous gridpoints. The next step is compute the sequence of distribution functions $\{\Psi_t\}_t$, going forward, from $t = 0$ to $t = \Upsilon$, starting at $\Psi_0 = \Psi$. After that we aggregate the asset policy function for each time period to obtain $\{B_t\}_{t=0}^{\Upsilon}$ and update the interest rate the linear rule $r_t^{(m)} = r_t^{(m-1)} - \epsilon(B_t - \mathrm{B})$, where m represents iteration. Choosing a ϵ small enough, after some iterations the bond market clears for all $t \in \{0, ..., \Upsilon\}$. The parameter γ, which controls the duration of government bond, is kept constant at the initial stedy state level along the transition.

Bibliography

References

Abraham, A. and Carceles-Poveda, E. (2010). Endogenous trading constraints with incomplete asset markets. *Journal of Economic Theory*, 145(974-1004).

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk, and aggregate saving. *Quarterly Journal of Economics*, 109(3):659–684.

Alonso-Ortiz, J. and Rogerson, R. (2010). Taxes, transfers and employment in an incomplete marktes model. *Journal of Monetary Economics*, 57(8):949–958.

Angeletos, G.-M. and Calvet, L.-E. (2006). Idiosyncratic production risk, growth, and the business cycle. *Journal of Monetary Economics*, 53(6):1095–1115.

Arellano, C. and Ramanarayanan, A. (2012). Default and the maturity structure in sovereign bonds. *Journal of Political Economy*, 120(2):187–232.

Berriel, T. C. and Bhattarai, S. (2013). Hedging against the government: A solution to the home asset bias puzzle. *American Economic Journal: Macroeconomics, American Economic Association*, 5(1):102–134.

Blundell, R., P.-L. and Preston, I. (2008). Consumption, inequality and partial insurance. *American Economic Review*, 98(5):1887–1921.

Broner, F., Lorenzoni, G., and Schmukler, S. L. (2013). Why do emerging economies borrow short term? *Journal of European Economic Association*, 11(1):67–100.

Buera, F., Kaboski, J. P., and Shin, Y. (2011). Finance and development: A tale of two sectors. *American Economic Review*, 101(5):1964–2002.

Buera, F. and Shin, Y. (2013). Financial frictions and the persistence of history: A quantitative exploration. *Journal of Political Economy*, 121(2):221–272.

Buera, F. and Shin, Y. (2015). Productivity growth and capital flows: The dynamics of reforms. *Mimeo.*

Cagetti, M. and De Nardi, M. C. (2006). Entrepreneurship, frictions, and wealth. *Journal of Political Economy*, 114(5):835–870.

Challe, E., Le Grand, F., and Ragot, X. (2013). Incomplete markets, liquidation risk, and the term structure of interest rates. *Journal of Economic Theory*, 148(6):2483–2519.

Chang, Y. and Kim, S. B. (2006). From individual to aggregate labor supply: A quantitative analysis based on a heterogeneous agent macroeconomy. *International Economic Review*, 47(1):1–27.

Chang, Y. and Kim, S. B. (2007). Heterogeneity and aggregation: Implications for labor market fluctuations. *American Economic Review*, 97(05):1939–1956.

Chen, H., Curdia, V., and Ferrero, A. (2012). The macroeconomic effects of large scale asset purchase programmes. *Economic Journal*, 122(564):F289–F315.

Chetty, R. (2008). Moral hazard versus liquidity and optimal unemployment insurance. *Journal of Political Economy*, 116(2):173–234.

Debortoli, D., Nunes, R., and Yared, P. (2014). Optimal government debt maturity. *National Bureau of Economic Research*, (20632).

Del Negro, M., Eggertsson, G., Ferrero, A., and Kiyotaki, N. (2013). The great escape? a quantitative evaluation of the fedï¿œs liquidity facilities. *Federal Reserve Bank of New York Staff Reports*, 520.

Den Haan, W. J. (1996). Heterogeneity, aggregate uncertainty, and the short-term interest rate. *Journal of Business and Economic Statistics*, 14(4):399–411.

Domeij, D. and Heathcoate, J. (2004). On the distributional effects of reducing capital taxes. *International Economic Review*, 45(2):523–554.

Eggertsson, G. B. and Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach. *Quarterly Journal of Economics*, 127(3):1469–1513.

Faraglia, E., Marcet, A., and Scott, A. (2014). Modelling long bonds: The case of optimal fiscal policy. *C.E.P.R. Discussion Papers*, (9965).

Floden, M. (2008). Aggregate savings when individual income varies. *Review of Economic Dynamics*, 11:70–82.

Floden, M. and Linde, J. (2001). Idiosyncratic risk in the united states and sweden: Is there a role for government insurance? *Review of Economic Dynamics*, 4:406–437.

Fuster, L., Imrohoroglu, A., and Imrohoroglu, S. (2007). Elimination of social security in a dynastic framework. *Review of Economic Studies*, 74(1):113–145.

Goldberg, J. (2011). A model of deleveraging. *Mimeo.*

Guerrieri, V. and Lorenzoni, G. (2011). Credit crises, precautionary savings, and the liquidity trap. *NBER Working Papers*, 17583.

Hall, R. (2011). The long slump. *American Economic Review*, 101(2):431–469.

Hatchondo, J. C. and Martinez, L. (2009). Long-duration bonds and sovereign defaults. *Journal of International Economics*, 79(1):117–125.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969.

Huo, Z. and Rios Rull, J.-V. (2015). Tightening financial frictions on households, recessions, and price reallocations. *Review of Economic Dynamics*, 18(1):118–139.

Jermann, U. and Quadrini, V. (2012). Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–71.

Kimball, M. S. and Shapiro, M. D. (2008). Labor supply: Are the income and substitution effects both large or both small? *NBER Working Paper 14208*.

Krueger, D. and Perri, F. (2006). Does income inequality lead to consumption inequality? evidence and theory. *Review of Economic Studies*, 73(1):163–193.

Krusell, P., Mukoyama, T., and Smith, A. A. (2011). Asset prices in a hugget economy. *Journal of Economic Theory*, 146(3):812–844.

Macaulay, F. R. (1938). Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in united states since 1856. *New York, Columbia University Press*.

Mian, A. and Sufi, A. (2010). Household leverage and recession of 2007 to 2009. *IMF Economic Review*, 58:74–117.

Mian, A. and Sufi, A. (2011). House prices, home equity-based borrowing, and the u.s. household leverage crisis. *American Economic Review*, 101(5):2132–2156.

Nekarda, C. J. and Ramey, V. A. (2013). The cyclical behavior of the price-cost markup. *Working Paper 19099, NBER*.

Oh, H. and Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59:50–64.

Pijoan-Mas, J. (2006). Precautionary savings or working longer hours? *Review of Economic Dynamics*, 9(2):326–352.

Quadrini, V. (2000). Entrepreneurship, saving and social mobility. *Review of Economic Dynamics*, 3(1):1–40.

Reinhart, C. M. and Reinhart, V. R. (2010). After the fall. *Proceedings Economic Symposium - Jackson Hole*, pages 17–60.

Rodriguez, S. B., Diaz-Gimenez, J., Quadrini, V., and Rios-Rull, J.-V. (2002). Updated facts on the us distributions of earnings, income, and wealth. *Quarterly Review*, 17:2–35.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181.

Woodford, M. (2001). Fiscal requirements of price stability. *Journal of Money Credit and Banking*, 33:669–728.

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