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Abstract

Social demand functions result from the budget constrained maximization of "social preferences" or "other regarding preferences." These preferences are non-selfish in the sense that they also depend on other consumers' wealth. This paper addresses the robustness to wealth externalities of the classical general equilibrium model with finite numbers of goods and consumers. The existence of equilibrium, the genericity of regular economies and, at those regular economies, the finite odd number of equilibria and the local continuity of equilibrium selection maps, and finally the identification (or diffeomorphism) of the equilibrium manifold with a Euclidean space are shown to be satisfied independently of the size of those wealth externalities provided total resources are variable.

Keywords: *social preferences, wealth externalities, general equilibrium, demand functions.*

JEL classification numbers: *C62, D11, D50, D51*

1. Introduction

The goal of this paper is to explore the robustness to wealth externalities of the general equilibrium model with finite numbers of goods and consumers. The study of general forms of externalities started immediately with the axiomatic approach adopted by Arrow, Debreu and McKenzie in the 1950s but results for very general forms of externalities have been limited to discussions of the existence of equilibrium and the validity of the theorems of welfare economics[1]. The only exception I am aware of is a paper by Bonnisseau and del Mercato [5] that proves the genericity of regular economies.

I show in the current paper that, while it may be difficult to come up with general results for the most general forms of externalities, the situation is radically different for the less general wealth externalities. This phenomenon is quite remarkable given the emphasis that has been placed in the past 15 years on the relevance of wealth externalities for economic theory. First, two recent expository papers by Fehr and Gächter [11] and by Sobel [20]

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[1] See in particular the references made in this paper to McKenzie, Arrow and Hahn, Shafer and Sonnenschein for existence [1, 13, 19] and to Osana, Kranich and Rader [12, 16, 17] on the welfare theorems for models with finite numbers of goods and consumers. Section 5.1 of the recent paper by Dufwenberg et al. [10] also contains a short update of the literature on externalities. For measure spaces of consumers, see the paper by Noguchi and Zame [15] and its references.

emphasize the appropriateness of "social preferences" or "other-regarding preferences" or "ORP" over classical selfish preferences and other forms of externalities. Second, the study by Dufwenberg et al. [10] of the theorems of welfare economics in the case of separable social preferences marks the beginning of a systematic study of the general equilibrium model with finite numbers of consumers and goods at the level of generality of Debreu's Theory of Value [6]. Finally, the paper by DeMarzo et al. [8] identifies rational bubbles to some of the equilibria of an overlapping-generations model with wealth externalities. That paper may stand apart because it involves an overlapping-generations model instead of a standard general equilibrium model. However, the many examples of simple models that bring invaluable insights into the properties of more complex models suggest that even bubbles (in overlapping-generations models) may benefit from a study of the simpler general equilibrium model with wealth externalities.

In this paper, I show that the approach through the equilibrium manifold and the natural projection that I first applied to the classical exchange model (i.e., without externalities) in [2] extends almost unhampered to wealth externalities under very general assumptions regarding consumers' individual demand functions. It is therefore possible to establish that the following properties are satisfied by the exchange model with wealth externalities provided that total resources are variable: existence of equilibrium for all economies; genericity of regular economies; at regular economies, finiteness and oddness of the number of equilibria; also at regular economies, local continuity of equilibrium selection maps; identification (or diffeomorphism) of the equilibrium manifold with a Euclidean space; pathconnectedness of the set of equilibrium allocations. These properties do not depend on the size and extent of externalities.

This paper is organized as follows. Section 2 is devoted to the main assumptions and definitions regarding the social exchange model with finite numbers of goods and consumers and also variable total resources, the main characteristic of the model being the dependence of every consumer's demand function on the wealth of the other consumers. Properties associated with the equilibrium manifold are proved in Section 3 while those related to the natural projection map from the equilibrium manifold into the parameter (or endowment) space are dealt with in Section 4. Concluding comments end this paper with Section 5.

2. The social exchange model

Prices and their two normalizations

There is a finite number ℓ of goods. The consumption space X of every consumer is the strictly positive orthant of the commodity space $\mathbb{R}^\ell$. Prices $p = (p_j) \in X$ are all strictly positive. The default assumption is to normalize prices by the numeraire convention $p_\ell = 1$. The set of numeraire normalized prices is denoted by $S = \mathbb{R}^{\ell-1}_{++} \times \{1\}$. The study of the behavior of a consumer's demand when some relative prices tend to zero is significantly easier with the alternative simplex normalization: the set of simplex normalized prices is the open simplex $S_\Sigma = \{p \in X \mid \sum_j p_j = 1\}$. The closure $\overline{S_\Sigma} = \{p \in \mathbb{R}^\ell_+ \mid \sum_j p_j = 1\}$, is compact.

Social demand functions

The price-income vector $b = (p, w_1, \dots, w_i, \dots, w_m)$ where $p \in S$ and $(w_1, \dots, w_m) \in \mathbb{R}^m_{++}$ respectively describes the price of every good and the wealth of every consumer. The set of price-income vectors $b = (p, w_1, \dots, w_i, \dots, w_m)$ is denoted by $B = S \times \mathbb{R}^m_{++}$.

A *classical demand function* is a map $f_i : S \times \mathbb{R}_{++} \to X$ that associates with the price vector $p \in S$ and the consumer i's wealth w_i the demand $f_i(p, w_i) \in X$. A *social demand function* is a map $f_i : B \to X$ that associates with the price-income vector $b = (p, w_1, \dots, w_i, \dots, w_m)$ the demand $f_i(b)$. A classical demand function f_i for consumer i is a social demand function that does not depend on the wealths of the other consumers $w_{-i} = (w_1, \dots, w_{i-1}, w_{i+1}, \dots, w_m) \in \mathbb{R}^{m-1}_{++}$.

Definition 1. *The following properties are defined for consumer i's (social) demand function f_i:*

(W) **Walras law:** *$p \cdot f_i(b) = w_i$ for any $b \in B$.*

(S) **Smoothness:** *f_i is smooth.*

(A) **Boundary behavior:** *Let the sequence $b^n = (p^n, w_1^n, \dots, w_m^n)$ (where prices p^n simplex normalized) tend to $b^0 = (p^0, w_1^0, \dots, w_m^0) \in \overline{S_\Sigma} \times \mathbb{R}^m_{++}$ where some coordinates of p^0 are equal to 0. Then,*

$$\limsup_{n \to +\infty} \|f_i(b^n)\| = +\infty.$$

The social exchange models

The *social exchange model* is defined by the m-tuple of social demand functions (f_i) where all demand functions satisfy (S) and (W) and one function at least (A). The *initial endowment* vector $\omega = (\omega_i) \in X^m$ represents consumers' endowments before exchange takes place. The set of these endowments is denoted by $\Omega = X^m$ and is known as the endowment or parameter space. Note that total resources $r = \sum_i \omega_i$ are variable.

The map $\varphi : S \times \Omega \to B$ associates with every pair $(p, \omega) \in S \times \Omega$ the price-income vector $b \in B$ where $b = (p, p \cdot \omega_1, \dots, p \cdot \omega_m)$.

The map $f : B \to S \times \Omega$ is defined by

$$f(b) = (p, f_1(b), \dots, f_m(b))$$

where $b = (p, w_1, \dots, w_m) \in B$.

The aggregate excess demand vector $z(p, \omega) \in \mathbb{R}^\ell$ associated with the pair $(p, \omega) \in S \times \Omega$ is by definition equal to

$$z(p, \omega) = \sum_{1 \le i \le m} f_i(\varphi(p, \omega)) - \sum_i \omega_i.$$

The *classical exchange model* is a special case of the social exchange model where every consumer's demand function does not depend on the wealth of the other consumers.

Definition 2. *i) The pair $(p, \omega) \in S \times \Omega$ is an equilibrium if $z(p, \omega) = 0$.*

ii) The equilibrium manifold E is the subset of $S \times \Omega$ defined by equation $z(p, \omega) = 0$.

iii) A no-trade equilibrium $(p, \omega) \in S \times \Omega$ satisfies the relation $\omega_i = f_i(\varphi(p, \omega))$ for $1 \leq i \leq m$.

iv) The natural projection $\pi : E \to \Omega$ is the restriction of the projection map $S \times \Omega \to \Omega$ to the equilibrium manifold E.

A no-trade equilibrium is evidently an equilibrium. The subset of the equilibrium manifold E consisting of the no-trade equilibria is denoted by T. The equality $T = f(B)$ is obvious. The set of no-trade equilibria T will be seen to play a pivotal role in the study of the exchange model in two areas at least: 1) Structure of the equilibrium manifold E; 2) Structure of the set of equilibrium allocations as the image $\pi(T)$ of the set of no-trade equilibria by the natural projection $\pi : E \to \Omega$.

The notation φ is also used for the restriction to the equilibrium manifold E of the map $\varphi : S \times \Omega \to B$ where $\varphi(p, \omega_1, \ldots, \omega_m) = (p, p \cdot \omega_1, \ldots, p \cdot \omega_m)$.

3. The equilibrium manifold

Local structure

The name of equilibrium manifold given to the subset E of $S \times \Omega$ suggests that this set is actually a smooth submanifold of $S \times \Omega$. This is confirmed by:

Proposition 3. *The equilibrium manifold E is a smooth submanifold of $S \times \Omega$ of dimension ℓm.*

Proof. The proof of Proposition 4.9 in [4] for the case of classical demand functions works also here. □

For $x = (x^1, \ldots, x^{\ell-1}, x^\ell) \in \mathbb{R}^\ell$, denote by $\bar{x} = (x^1, \ldots, x^{\ell-1}) \in \mathbb{R}^{\ell-1}$ the projection of x in $\mathbb{R}^{\ell-1}$ defined by its first $\ell - 1$ coordinates. With $(p, \omega) \in S \times (\mathbb{R}^\ell)^m$, let $b = (p, p \cdot \omega_1, \ldots, p \cdot \omega_m) \in S \times \mathbb{R}^m$ and $\bar{\omega}_{-m} = (\bar{\omega}_1, \ldots, \bar{\omega}_{m-1}) \in (\mathbb{R}^{\ell-1})^{m-1}$. Define the map $\theta : S \times (\mathbb{R}^\ell)^m \to (S \times \mathbb{R}^m) \times (\mathbb{R}^{\ell-1})^{m-1}$ by $\theta(p, \omega) = \big((p, p \cdot \omega_1, \ldots, p \cdot \omega_m), \bar{\omega}_{-m})\big)$.

Proposition 4. *Let $(p^*, \omega^*) \in E$. There exist open neighborhoods U and V of $(p^*, \omega^*) \in E$ and $\theta(p^*, \omega^*) \in B \times (\mathbb{R}^{\ell-1}_{++})^{m-1}$ respectively that are diffeomorphic by the map θ restricted to the open set U.*

Proof. Let $\rho : B \times (\mathbb{R}^{\ell-1})^{m-1} \to S \times (\mathbb{R}^\ell)^m$ be the map $\rho\big((p, w_1, \ldots, w_m), \bar{\omega}_{-m}\big) = (p', \omega'_1, \ldots, \omega'_{m-1}, \omega'_m)$ where $p' = p$, $\omega'_i = (\bar{\omega}_i, w_i - \bar{p} \cdot \bar{\omega}_i)$ for $1 \leq i \leq m - 1$ and $\omega'_m = \sum_{1 \leq i \leq m} f_i(b) - \sum_{1 \leq i \leq m-1} \omega'_i$.

It follows from $\rho\big(\theta(p^*, \omega^*)\big) = (p^*, \omega^*) \in E$ and the continuity of the map ρ that there exists an open neighborhood U' in $S \times (\mathbb{R}^\ell_{++})^m$ of (p^*, ω^*) such that $V = \rho^{-1}(U')$ is contained in $B \times (\mathbb{R}^{\ell-1}_{++})^{m-1}$. Let $U = U' \cap E$. It follows from the formulas defining the maps θ and ρ that the restrictions of $\rho \circ \theta$ and $\theta \circ \rho$ to U and V respectively are the identity maps of U and V. These two open sets are therefore diffeomorphic since the maps θ and ρ are smooth. □

The open neighborhood U of $(p^*, \omega^*) \in E$ is known as a chart of the smooth manifold E at the point (p^*, ω^*). A local coordinate system for the equilibrium manifold E (i.e., a set of coordinates for the open set U) is therefore defined by $\big((\bar{p}, w_1, \ldots, w_m), \bar{\omega}_{-m}\big) \in (\mathbb{R}^{\ell-1}_{++} \times \mathbb{R}^m_{++}) \times (\mathbb{R}^{\ell-1}_{++})^{m-1}$. These coordinate system is obtained by composing the restriction of the map θ to U with the projection of $S = \mathbb{R}^{\ell-1}_{++} \times \{1\}$ onto $\mathbb{R}^{\ell-1}_{++}$.

The set of no-trade equilibria

We will see shortly that the structure of the set of no-trade equilibria T is essential in the study of the equilibrium manifold E.

Proposition 5. *i) The map $f : B \to E$ is a smooth embedding with image $f(B) = T$.*
ii) The set of no-trade equilibria T is a smooth submanifold of E diffeomorphic to B.

Proof. Same as for Proposition 5.2 in [4]. □

Corollary 6. *The set of equilibrium allocations $\pi(T)$ is pathconnected.*

Proof. The image $\pi(T)$ of the pathconnected set T by the continuous map $\pi : E \to \Omega$ is pathconnected. □

Remark 1. The pathconnectedness of the set of equilibrium allocations $\pi(T)$ with variable total resources is the strongest global topological property that can be proved short of much stronger assumptions regarding the individual social demand functions $f_i : B \to X$. Because the two theorems of welfare economics do not hold true here, the set of equilibrium allocations cannot be parameterized by consumers' utility levels and total resources.

The global structure of the equilibrium manifold

Without sign restrictions on wealth and consumption, the diffeomorphism $\theta : U \to V$ in the proof of Proposition 4 can easily be extended to the full equilibrium manifold E to define a global diffeomorphism. See for example Corollary 5.9 in [4] for the classical case.

Sign restrictions on wealth and consumption make more sense here than in the classical case because of the difficulty or even impossibility of giving some economic consistence to externalities associated with negative wealth. This justifies working out a proof of the diffeomorphism property despite the difficulties created by those sign restrictions. In the classical case with sign restrictions, the diffeomorphism of the equilibrium manifold with a Euclidean space results from Theorem 3 of [2]. See also [18] for a different proof. The proof given here is new and takes advantage of recent mathematical results on fibrations. Written for the social exchange model, it also works for the classical case. This proof starts by two lemmas.

Lemma 7. *The map $\varphi : E \to B$ is a surjective submersion.*

Proof. Surjectivity follows from Proposition 5, (ii).

By definition, the map φ is a submersion at (p, ω) if its derivative (or tangent map) $D\varphi(p, \omega)$ at (p, ω) is onto (i.e., a surjection). A projection map is a special case of a submersion. (The converse is also true locally.) With local coordinates for E and B defined by $\big((\bar{p}, w_1, \dots, w_m), \bar{\omega}_{-m}\big) \in U$ and $(\bar{p}, w_1, \dots, w_m)$ respectively, it results from

$$\varphi\big((\bar{p}, w_1, \dots, w_m), \bar{\omega}_{-m}\big) \to (\bar{p}, w_1, \dots, w_m)$$

that the map φ (restricted to U) is a projection and, therefore, a submersion. □

Lemma 8. *Every fiber $\varphi^{-1}(b)$ with $b \in B$ is diffeomorphic to $\mathbb{R}^{(\ell-1)(m-1)}$.*

Proof. Let $b = (p, w_1, \ldots, w_m) \in B$. The fiber $\varphi^{-1}(b)$ consists of the points $(p, \omega) \in S \times (\mathbb{R}^\ell)^m$ with $\omega = (\omega_1, \ldots, \omega_m) \in (\mathbb{R}^\ell)^m$ such that $p \cdot \omega_i = w_i$ for $1 \leq i \leq m$, $\sum_i \omega_i = \sum_i f_i(b)$ and $\omega_i^j > 0$ for $1 \leq i \leq m$ and $1 \leq j \leq \ell$. This set of linear equalities and strict inequalities defines a convex set. That set is non empty since it contains the point $f(b)$ and its relative interior has dimension $(\ell - 1)(m - 1)$ as follows from the local chart U in Proposition 4. The fiber $\varphi^{-1}(b)$ is therefore diffeomorphic to $\mathbb{R}^{(\ell-1)(m-1)}$ as a non-empty open convex subset of $\mathbb{R}^{(\ell-1)(m-1)}$. □

Proposition 9. *The equilibrium manifold E is diffeomorphic to $\mathbb{R}^{\ell m}$.*

Proof. It follows from Meigniez [14], Corollary 31 that the surjective submersion $\varphi : E \to B$ with fibers diffeomorphic to a Euclidean space is a locally trivial fiber map. The base space B being contractible as diffeomorphic to a Euclidean space, the fibration defined by the map $\varphi : E \to B$ is therefore trivial. The equilibrium manifold E is diffeomorphic to the Cartesian product $B \times \mathbb{R}^{(\ell-1)(m-1)}$. □

As in the classical case, the equilibrium manifold E is therefore a disjoint union of convex fibers, each fiber containing a unique no-trade equilibrium.

Remark 2. The properties of this section depend crucially on the specific forms of externalities as depending on consumers' wealth only. These properties do not extend to more general forms of consumption externalities.

4. The natural projection

Proposition 10. *The natural projection $\pi : E \to \Omega$ is:*

i) smooth;
ii) proper;
iii) its topological degree is equal to $+1$ for suitable orientations of E and Ω;
iv) its modulo 2 degree is equal to 1.

Proof. i) Smoothness. The natural projection $\pi : E \to \Omega$ can be viewed as a map from $\mathbb{R}^{\ell m}$ into itself that is smooth because that map is the composition of a projection map that is smooth with the embedding of the equilibrium manifold E in $S \times \Omega$, a map that is also smooth because E is a smooth submanifold of $S \times \Omega$.

ii) Properness. Let K be a compact subset of Ω. In order to prove that the set $\pi^{-1}(K)$ is compact, it suffices to show that every sequence $x^n = (p^n, \omega^n)$ in $\pi^{-1}(K)$ has a convergent subsequence. Here, the simplex normalization is used for prices. The sequence (ω^n) belonging to the compact set K, there is no loss of generality in considering a subsequence still denoted by (ω^n) that converges to a limit $\omega^* \in K$.

The closed simplex $\overline{S}_\Sigma$ is also compact. We can therefore assume without loss of generality that the sequence p^n has a convergent subsequence with limit some $p^* \in \overline{S}_\Sigma$. If p^* belongs to the interior S_Σ, i.e., to the strictly positive (price) simplex, the pair (p^*, ω^*) is an equilibrium by the continuity of the equations defining an equilibrium and also belongs to the preimage $\pi^{-1}(K)$.

Assume now that the limit p^* belongs to the boundary $\partial \bar{S}_\Sigma = \bar{S}_{Sigma} \setminus S_\Sigma$ and let us get a contradiction.

It follows from the compactness of K that its image by the projection map $\omega = (\omega_1, \dots, \omega_m) \to \omega_i$ is a compact subset of $X = \mathbb{R}^{\ell}_{++}$. There exist A_i and B_i in X such $A_i \le \omega_i \le B_i$ for all $\omega = (\omega_i) \in K$ and $1 \le i \le m$.

All the coordinates of the demand $f_i(b^n)$ where $b^n = (p^n, w_1^n, \dots, w_m^n)$ are strictly positive. The equilibrium equation $\sum_i f_i(b^n) = \sum_i \omega_i^n$ therefore implies the inequality $f_i(b^n) \le \sum_i B_i$ for i with $1 \le i \le m$.

It follows from $A_i \le \omega)i^n$ that $p^n \cdot A_i \le p^n \cdot \omega_i^n$. At the limit, this gives the inequality $p^* \cdot A_i \le w_i^*$. All the coordinates of A_i are strictly positive and one coordinate of p^* at least is different from zero and, therefore, strictly positive. This implies $p^* \cdot A_i > 0$ and, therefore, w_i^* is strictly positive for $1 \le i \le m$.

Pick i arbitrary. Property (A) now implies $\limsup_{n\to\infty} \|f_i(b^n)\| = +\infty$, a contradiction with the fact $f_i(b^n)$ is bounded from above.

iii) and iv): Degrees. The topological and modulo 2 degree are homotopy invariants of continuous (and therefore smooth) proper maps. Let $f_i' : S \times \mathbb{R}_{++} \to X$ be a (classical) demand function that satisfy (S), (W) and (A). Extend f_i' to B by setting $f_i'(b) = f_i'(p, w_i)$ with $b = (p, w_1, \dots, w_i, \dots, w_m)$. One checks readily that the function $f_i(t, \cdot) = (1-t)f_i + tf_i'$ with $0 \le t \le 1$ satisfies (S), (W) and (A). Let E_t be the equilibrium manifold associated with the m-tuple of demand functions $(f_i(t, \cdot))$. Let $\pi_t : E_t \to \Omega$ the corresponding natural projection and $\epsilon_t : B \times \mathbb{R}^{(\ell-1)(m-1)} \to \Omega$ the composition of the diffeomorphism of $B \times \mathbb{R}^{(\ell-1)(m-1)}$ with E_t of Proposition 9 with the natural projection π_t. By varying t from 0 to 1, the same argument as in [3] proves that the maps ϵ_t define a proper homotopy between ϵ_0 and ϵ_1. These two maps have therefore the same topological and modulo 2 degrees. It suffices to observe that $\pi' : E_1 \to \Omega$ is the natural projection for classical demand functions. It then suffices to apply Proposition 7.12 and 7.14 of [4]. □

Properties (i) and (ii) of Proposition 10 imply that the natural projection $\pi : E \to \Omega$ is a "ramified" covering of Ω. Before translating the property of being a ramified covering in a more accessible language, a few definitions are in order.

A *regular equilibrium* $x = (p, \omega) \in E$ is a regular point of the natural projection $\pi : E \to \Omega$. This is equivalent to the derivative (or tangent map) $D\pi(x) : \mathbb{R}^{\ell m} \to \mathbb{R}^{\ell m}$ being invertible. The set of regular equilibria is a subset of the equilibrium manifold E denoted by $\mathfrak{R}$. That set is an open subset of the equilibrium manifold E. A *critical equilibrium* is an equilibrium that is not regular. The set of critical equilibria is the complement $\mathfrak{C} = E \setminus \mathfrak{R}$ of the set of regular equilibria in the equilibrium manifold. The set of critical equilibria $\mathfrak{C}$ is closed in the equilibrium manifold E.

The element $\omega \in \Omega$ is a *singular economy* if it is a singular value of π, i.e., if there exists a critical point (i.e., a critical equilibrium) $x \in \mathfrak{R}$ with $\omega = \pi(x)$. The set of singular values of π is denoted by Σ. It is a subset of Ω. The definitions imply $\Sigma = \pi(\mathfrak{C})$.

The element $\omega \in \Omega$ is a *regular economy* (i.e., a regular value of π) if it does not belong to Σ, the set of singular economies. The set of regular economies is denoted by $\mathcal{R}$ and this set is the complement in Ω of the set of singular economies: $\mathcal{R} = \Omega \setminus \Sigma$.

Proposition 11. *i) The set of singular values Σ of the map $\pi : E \to \Omega$ is closed with measure zero in Ω.*

ii) The set of regular values $\mathcal{R} = \Omega \setminus \Sigma$ is open with full measure in Ω.

iii) The set $\pi^{-1}(\omega)$ is finite for $\omega \in \mathcal{R}$ and its elements are locally smooth functions of $\omega \in \mathcal{R}$.

iv) The number of equilibria is constant over each connected component of the set of regular values $\mathcal{R}$.
v) The number of elements of $\pi^{-1}(\omega)$ is odd for $\omega \in \mathcal{R}$.
vi) Equilibrium exists for all $\omega \in \Omega$.

Proof. It suffices to reproduce the proofs in Chapter 7 of [4]. □

Remark 3. Properties (i), (ii), (iii), (iv) and (vi) of Proposition 11 were proved for the classical case by Debreu [7]. Property (v) is due to Dierker [9].

5. Concluding comments

The social exchange model has therefore the same properties as the classical exchange model provided total resources are variable when every consumer's individual demand function satisfy (S) and (W) and one consumer's demand function also satisfies (A). In the classical exchange model, these properties extend to the important case of fixed total resources if consumers' preferences can be represented by strictly quasi-concave utility functions, one consumer's utility function having everywhere non-zero Gaussian curvature. The problem of extending those properties to the case of fixed total resources for the classical and social exchange models considered in the current paper is open.

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