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INTER-TEMPORAL DISCOUNTING AND UNIFORM IMPATIENCE

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ABSTRACT. The uniform impatience hypothesis, a joint requirement on endowments and preferences, was imposed in the literature to prove equilibrium existence in infinite horizon sequential economies. In this note, we characterize this assumption in terms of asymptotic properties on inter-temporal discount factors.

KEYWORDS: Uniform impatience, inter-temporal discounting.

JEL classification: D50, D52.

The uniform impatience assumption—see Magill and Quinzii (1994, Hypotheses A2 and A4), Hernandez and Santos (1996, Assumption C.3) or Magill and Quinzii (1996, Assumptions B2 and B4)—is a usual requirement for existence of equilibrium in economies with infinite-lived debt-constrained agents. In this note, we characterize uniform impatience in terms of inter-temporal discount factors. As a consequence, we show that the uniform impatience assumption does not hold for agents with hyperbolic inter-temporal discounting (see Laibson (1998)).

We follow the notation of Magill and Quinzzi (1994). Consider a framework where an infinite-lived price-taker agent demands L different commodities at any node ξ of an infinite countable event-tree D. This agent may trade financial assets to implement inter-temporal transfers. She receives at any $\xi \in D$ a physical endowment $w(\xi) \in \mathbb{R}^L_+$ and she makes contingent consumption plans, $x(\xi)$, to maximize her preferences, which are represented by a function $U : \mathbb{R}^{L\times D}_+ \to \mathbb{R}_+ \cup \{+\infty\}$. Aggregated physical endowments in the economy at node ξ are given by $W_\xi \in \mathbb{R}^L_{++}$. The date associate with a node $\xi \in D$ is denoted by $t(\xi)$.

ASSUMPTION A. Let $U(x) := \sum_{\xi\in D} \beta_{t(\xi)}\rho(\xi)u(x(\xi))$, where $\rho(\xi_0) = 1$ and, for each $\xi \in D$, $\rho(\xi) = \sum_{\mu\in\xi^+} \rho(\mu)$. For any $\xi \in D$, $\beta_{t(\xi)}$ is a strictly positive number and the function $u : \mathbb{R}^L_+ \to \mathbb{R}_+$ is concave, continuous and strictly increasing. Also, $\sum_{\xi\in D} \beta_{t(\xi)}\rho(\xi)u(W_\xi)$ is finite.

UNIFORM IMPATIENCE. *There are $\pi \in [0,1)$ and $(v(\mu); \mu \in D) \in \mathbb{R}^{D\times L}_+$ such that, given a consumption plan $(x(\mu); \mu \in D)$, with $0 \le x(\mu) \le W_\mu$, we have that,*

$$u\left(\xi, x(\xi) + v(\xi)\right) + \sum_{\mu>\xi} u(\mu,\, \pi'\, x(\mu)) > \sum_{\mu\geq\xi} u(\mu, x(\mu)), \qquad \forall \xi \in D,\ \forall \pi' \geq \pi.$$

Moreover, there is $\delta > 0$ such that, $w(\xi) \ge \delta v(\xi)$, $\forall \xi \in D$.

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The requirements of impatience above depend on both preferences and physical endowments. As particular cases we obtain the assumptions imposed by Hernandez and Santos (1996) and Magill and Quinzii (1994, 1996). Indeed, in Hernandez and Santos (1996), for any $\mu \in D$, $v(\mu) = W_\mu$. Also, since in Magill and Quinzii (1994, 1996) initial endowments are uniformly bounded away from zero by an interior bundle $\underline{w} \in \mathbb{R}^L_+$, they suppose that $v(\mu) = (1, 0, \ldots, 0)$, $\forall \mu \in D$.

Our main result is,

Proposition 1. *Suppose that Assumption A holds,* $(W_\xi; \xi \in D)$ *is a bounded plan and that there is* $\underline{w} \in \mathbb{R}^L_+ \setminus \{0\}$ *such that,* $w(\xi) \geq \underline{w}$, $\forall \xi \in D$. *For each* $t \geq 0$, *let* $s_t = \frac{1}{\beta_t} \sum_{r=t+1}^{+\infty} \beta_r$.

Then, the function U *satisfies uniform impatience if and only if* $(s_t)_{t\geq 0}$ *is bounded.*

Proof. Assume that $(W_\xi)_{\xi \in D}$ is a bounded plan. That is, there is $\overline{W} \in \mathbb{R}^L_+$ such that, $W_\xi \leq \overline{W}$, $\forall \xi \in D$. If $(s_t)_{t\geq 0}$ is bounded, then there exists $\overline{s} > 0$ such that, $s_t \leq \overline{s}$, for each $t \geq 0$. Also, since $\mathbb{F} := \{x \in \mathbb{R}^L_+ : x \leq \overline{W}\}$ is compact, the continuity of u assures that there is $\pi \in (0,1)$ such that $u(x) - u(\pi' x) \leq \frac{u(\overline{W}+\underline{w}) - u(\overline{W})}{2\overline{s}}$, $\forall x \in \mathbb{F}$, $\forall \pi' \geq \pi$. Thus, uniform impatience follows by choosing $\delta = 1$ and $v(\xi) = \underline{w}$, $\forall \xi \in D$. Indeed, given a plan $(x(\mu); \mu \in D) \in \mathbb{R}^{L \times D}_+$ such that, $x(\mu) \leq W_\mu$ $\forall \mu \in D$, the concavity of u assures that, for any $\xi \in D$ and $\pi' \geq \pi$,

$$\begin{aligned}\sum_{\mu > \xi} \beta_{t(\mu)} \rho(\mu) u(x(\mu)) - \sum_{\mu > \xi} \beta_{t(\mu)} \rho(\mu) u(\pi' x(\mu)) &\leq \frac{\beta_{t(\xi)} s_t}{2\overline{s}} \rho(\xi) \left(u(\overline{W} + \underline{w}) - u(\overline{W}) \right) \\ &< \beta_{t(\xi)} \rho(\xi) u(x(\xi) + v(\xi)) - \beta_{t(\xi)} \rho(\xi) u(x(\xi)).\end{aligned}$$

Reciprocally, suppose that uniform impatience property holds. Then, given $(x(\mu); \mu \in D) \in \mathbb{R}^{L \times D}_+$ such that, $x(\mu) \leq W_\mu$, for all $\mu \in D$, there are $(\pi, \delta) \in [0,1) \times \mathbb{R}_{++}$ and $(v(\mu); \mu \in D)$ $\mathbb{R}^{D \times L}_+$ satisfying, for any $\xi \in D$, $w^h(\xi) \geq \delta v(\xi)$, such that

$$\frac{1}{\beta_{t(\xi)} \rho(\xi)} \left[\sum_{\mu > \xi} \beta_{t(\mu)} \rho(\mu) u(x(\mu)) - \sum_{\mu > \xi} \beta_{t(\mu)} \rho(\mu) u(\pi x(\mu)) \right] < u(x(\xi) + v(\xi)) - u(x(\xi)), \quad \forall \xi \in D.$$

It follows that, for any node ξ,

$$\frac{1}{\beta_{t(\xi)} \rho(\xi)} \left[\sum_{\mu > \xi} \beta_{t(\mu)} \rho(\mu) u(\underline{w}) - \sum_{\mu > \xi} \beta_{t(\mu)} \rho(\mu) u(\pi \underline{w}) \right] < u\left(\left(1 + \frac{1}{\delta} \right) \overline{W} \right).$$

Therefore, we conclude that, for any $\xi \in D$,

$$\frac{1}{\beta_{t(\xi)}} \left(u(\underline{w}) - u(\pi \underline{w}) \right) \sum_{t = t(\xi)+1}^{+\infty} \beta_t < u\left(\left(1 + \frac{1}{\delta} \right) \overline{W} \right),$$

which implies that the sequence $(s_t)_{t \geq 0}$ is bounded. □

Suppose that Assumption A holds and initial endowments are bounded away from zero. If intertemporal discount factors are constant, i.e. $\exists c \in \mathbb{R}_{++} : \frac{\beta_{t(\xi)+1}}{\beta_{t(\xi)}} = c$, $\forall \xi \in D$, then $c < 1$ and $s_t = \frac{c}{1-c}$, for each $t \geq 0$. Therefore, in this case, if aggregated physical endowments are uniformly bounded along the event-tree then U satisfies uniform impatience condition.

However, even with bounded plans of endowments, uniform impatience is a restrictive condition when inter-temporal discount factors are time varying. For instance, if we consider *hyperbolic inter-temporal discount factors*, that is, $\beta_t = (1+at)^{-\frac{b}{a}}$, where $b > a > 0$, then the function U, as defined in the statement of Proposition 1, satisfies Assumption A and the sequence s_t goes to infinity as t increases. Therefore, in this case, uniform impatience does not hold.

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