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Land taxes in a Latin-American context *

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Abstract

Since Henry George (1839-1897) economists have been arguing that a tax on unimproved land is an ideal tax on efficiency grounds. Output taxes, on the other hand, have distortional effects on the economy. This paper shows that under asymmetric information output tax might be used along with land tax in order to implement the optimal taxation scheme in a Latin-American context, i.e., in an economy with imperfect land-rental market, non-agricultural land use and non-revenue objectives of land taxation. Also, we show that: (i) schemes based on land taxes alone might not be implementable; and (ii) tax evasion is more acute among large landholders.

JEL Classification: H21, H26, Q15

Key words: Optimal taxation, tax evasion, land use.

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1 Introduction

Taxes on land are among the oldest forms of taxation. Arnott and Stiglitz (1979), extending the so-called Henry George theorem, demonstrate that a tax on land rents is not only efficient, but the sole tax instrument necessary to finance a pure public good in large economies where the differential land rents are well defined and the distribution of economic activity over space is Pareto-optimal. On the other hand, land taxation is one of the few cases of lump-sum tax, being based on assets rather than on agricultural production. However, while land taxation enjoys striking advantages from a theoretical point of view, it is hardly ever used in developing countries. Trying to explain this fact, Skinner (1991) indicates two major drawbacks of land taxes relative to output taxes - they increase risk borne by farmers as well as entailing high administrative costs. Hoff (1991) points to the use of a mix of land and output taxes as an attempt to mitigate the adverse effect of a pure land-taxation regime in an economy with imperfect insurance markets.

In this paper, the mix of land and output taxes arises as the optimal taxation instrument in an economy where farmer's ability, and therefore land use, is private information. We show that the problem of informational costs raised by Skinner (1991) can be at least partially resolved in a typical Latin-American country by using an output tax as a part of the tax mechanism. This additional component takes advantage of the fact that the information about agricultural output is more reliable than self-reported indicators of productivity.

Many countries in Latin America have attempted to implement progressive land taxes in order to induce large landholders to use their land more intensively rather than to finance local government. Progressive tax rates are treated as a means of dissuading land speculation, inducing large landowners to sell out or use land more intensively. Implementation of this instrument has been disappointing - farmers often find ways around such taxes [Binswanger, Deininger and Feder (1995), Deininger and Feder (2000) and Bird and Slack (2002)]. In Brazil, for example, the enactment of the Land Statute in 1964 imposed non-revenue functions on land taxation, which became involved on the job of assisting public land-redistribution policies [Oliveira (1993)]. Since then, the land-tax mechanism has aimed at imposing penalties on unimproved land. Nevertheless, the original objectives of the tax were not achieved. Land taxes in Brazil have never constituted a good source of revenue and further, have hardly managed to achieve any of the desired changes in the rural environment.

Land ownership in Latin America can be used for agricultural production or for other non-agricultural purposes. Land is used not only as a productive asset but also as a source of other benefits - "*as a hedge against inflation, as* *an asset that can be liquidated to smooth consumption in the face of risk, as collateral for access to loans, as a tax shelter, or as a means of laundering illicit funds*" [De Janvry, Key and Sadoulet (1997)]. Therefore, we consider a continuum of types of producers who are differentiated by their proclivity for non-agricultural activity and agricultural productivity. Each type of producer constitutes private information and the area under cultivation is not observed.

Land-rental markets in Latin America are underdeveloped. Typically only 5 to 10% of the land is rented out, which is remarkably low when compared to United States or European patterns. A large body of literature has been theoretically addressing the reasons for the imperfections in the land-rental market. [1] Many reasons provide explanations for a reduction in the share of output appropriated by the tenants. Specifically, one factor that is likely to be important in most Latin American countries is the landlord's fear of loss of the land [Macours, De Janvry and Sadoulet (2001)]. In the absence of a land rental market, farmers with low agricultural productivity cannot use their titles to obtain non-agricultural gains and also lease out their land to other farmers for agricultural production. The only option for them is to retain a portion of their tracts in the form of idle land. Although farmers with low agricultural productivity choose farm size and cultivated area separately, those with high agricultural productivity and low non-agricultural yields choose only the farm size, since they are constrained by the no-rental-market constraint. The no-rental-market constraint determines two groups of farmers: those who face a one-dimensional problem of maximization and those who have two different choices (farm size and cultivated area).

Summarily, we study the problem of optimal taxation in an economy where (i) there are non-revenue objectives of land taxation; (ii) land provides both agricultural and non-agricultural payoffs; and (iii) the land rental market is imperfect. The main result is that relying solely on land tax is optimal only when the social-planner information about farmers is complete or when farmers do not hold unimproved land; otherwise, the optimal scheme is a linear combination of output and land taxes. Under additional assumptions regarding the relationship between the agricultural and non-agricultural productivities for different farmers, we show that a tax mechanism based solely on land taxes can be implemented for small farms because they have no idle land. This is consistent with the Brazilian experience with land taxation, where the difficulty of accurately appraising the extent of area utilized has determined a high rate of evasion and under-reporting, especially among large landholdings.

[1] The basic arguments are: risk sharing [Cheung (1969)], hidden actions and moral hazard [Stiglitz (1974); Ghatak and Pandey (2000); Eswaran and Kotwal (1985)], screening [Hallagan (1978) and Allen (1982)] and limited liability constraints [Shetty (1988); Laffont and Matoussi (1995)].

This paper attempts to fill a gap between optimal taxation models under asymmetric information, in the tradition initiated by Mirrlees (1971), and the land-taxation models mentioned above. The mechanics of our model is quite similar to that of Stern (1982). Asymmetric information about farmers introduces a role for output taxes, even considering their distortional effect. The mix of output and land taxes arises as a solution for the typical trade-off between rent extraction and distortion commonly observed in adverse-selection models.

The plan of the paper is as follows. Section 2 introduces and analyzes the basic model. In Section 3 we link tax evasion to farm size using an additional assumption about the ordering of farmer types and present, as an example, the Brazilian experience with land taxation. The discussion about the assumptions adopted, the generality of the results and implications for public policies are made in Section 4. Concluding comments are offered in Section 5.

2 The Model

The analysis is carried out in a partial-equilibrium environment. The price obtained for agricultural output is normalized at 1, an unlimited quantity of land is available at price p and each harvested hectare costs w. For the sake of simplification, we assume that labor and other inputs are used in fixed proportions with land at a cost w per hectare. Both production and land are perfectly homogeneous.

We consider a heterogeneous population of farmers indexed by $\theta \in \Theta = \left[\underline{\theta}, \overline{\theta}\right]$, distributed according to the distribution function F and positive density f. The parameter θ reflects the technology available for each farmer, indicating the revenues obtained from agricultural and non-agricultural land use.

Land in our framework is not only an agricultural input but an asset that provides non-agricultural payoffs to its owners. This is a key feature of land markets in Latin America which is introduced in our model. As pointed out by Berry and Cline (1979), "in countries with poorly developed capital markets, especially those with chronic inflation, landowners may find it attractive to hold land for speculative gain - or merely to accomplish the store of value objective". Land ownership is also a source of market power (Conning, 2001) or political pressure in countries without secret ballots (Baland and Robinson, 2004). In summary, we assume one of the many reasons that generate a wedge between the land price and the discounted expected value of income streams from agricultural production.[2] Therefore, a farmer of type θ who holds T

[2] See Deininger and Feder (2000) for a survey.

hectares of land, grows A and pays a transfer t to the social planner, obtains profits given by:

$$\Pi = Q(A,\theta) + \phi(T,\theta) - wA - pT - t, \tag{1}$$

where Q and ϕ are respectively the agricultural and non-agricultural production functions with $Q_A > 0 > Q_{AA}$, $\phi_T > 0 > \phi_{TT}$, $Q_A(0,\cdot) = \phi_T(0,\cdot) = \infty$ and $Q_A(\infty,\cdot) = \phi_T(\infty,\cdot) = 0$, i.e., both functions are subject to decreasing marginal returns and Inada's conditions.[3] The function ϕ is a reduced-form representation of non-agricultural payoffs of landholding. We assume these additional gains are not influenced by the cultivated area A and are determined exclusively by the farm size T.

Peasant are ordered such that the parameter θ is positively related to the marginal non-agricultural benefits of landholding. The effect of θ on Q is considered later. Initially, it can be positive or negative. In order to simplify the analysis, we assume θ affects Q and ϕ linearly.[4] These preliminary functional-form hypotheses are summarized as:

Assumption A1 (ordering and linearity of ϕ): $\phi_{T\theta}(\cdot,\theta) > 0$ and $Q_{\theta\theta}(\cdot,\theta) = \phi_{\theta\theta}(\cdot,\theta) = 0$ for every $\theta \in \Theta$.

We assume there is no rental market and hence the choice of each producer must respect the condition $A \leq T$. Thus, based on a taxation scheme t, farmers are faced with the program:

$$\max_{A,T} \Pi \quad \text{s.t.} \quad A \leq T. \tag{P}$$

Individuals decide to participate whenever $\Pi \geq 0$.

Finally, we consider a social planner that mimic the effort of some Latin American governments, maximizing the tax revenue and hindering the non-agricultural motive of landholding.[5] Policy-makers from these countries have been attempting (without success) to use land taxes as an instrument of land redistribution. Thus, we study the problem of a social planner with utility

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[3] Notice that our analysis is carried out in a current approach, where A and T are modelled as flow variables. Since we are primarily interested in land use, this approach does not have major consequences and substantially simplifies the analysis.
[4] This condition avoids some assumptions regarding third derivatives which generally have little economic insight.
[5] See Binswanger, Deininger and Feder (1995), Deininger and Feder (2000) and Bird and Slack (2002).

function defined by[6]

$$U = t - \lambda\phi(T, \theta), \tag{2}$$

where $\lambda \in [0, 1]$ is the social cost attributed to the non-agricultural use of land.

2.1 The first-best case

Initially, we consider the design of a tax mechanism in an environment of complete information, i.e., we assume the social planner can exactly observe the type of agents and manages to establish tax-collection rules that internalize precisely the willingness of each producer to use land for agricultural production or not.

Under complete information, the social planner can determine the allocations for each producer via a punitive taxation scheme. The choice of the tax scheme is restricted only by the participation constraint (IR) and the absence of a land-rental market (RM). The first-best program in this case defines the optimal taxation mechanism for a producer of type θ:

$$\max_{t,A,T} U \tag{P.FB}$$

subject to

$$\Pi \geq 0, \tag{IR}$$

$$A \leq T. \tag{RM}$$

First, note that (IR) is binding.[7] Therefore, the transfers required by the optimal tax scheme represent all the producer's profits.

Substituting (IR), the solution of (P.FB) is given by the following system of equations:[8]

$$Q_A(A^*, \theta) = w + \mu^*, \tag{3a}$$

$$(1 - \lambda)\phi_T(T^*, \theta) = p - \mu^*, \tag{3b}$$

$$\mu^*(T^* - A^*) = 0, \tag{3c}$$

$$t^* = Q(A^*, \theta) + \phi(T^*, \theta) - wA^* - pT^*; \tag{3d}$$

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[6] Section 4 provides some discussion on the social planner's objective function. This particular functional form is useful to produce pure land taxes as the first-best solution. More general forms could determine that the pure land tax mechanisms, which are widespread in Latin America, were be no longer first-best optimal.

[7] Otherwise, the social planner could reach a higher level of U increasing t marginally, without violating (IR) or (RM).

[8] Hereafter, in order to facilitate notation, we drop the dependence on θ, i.e., we write $\mu^*(\theta)$ as μ^*, $A^*(\theta)$ as A^*, $T^*(\theta)$ as T^*, and so on.

where μ^* is the Lagrangian multiplier corresponding to the no-rental-market constraint (RM).

Define $\Theta_C = \{\theta \in \Theta : \mu^* > 0\}$ and its complement $\Theta_U = \Theta - \Theta_C$. While farmers in Θ_C (constrained farmers) harvest all land they have, those in Θ_U (unconstrained farmers) hold some amount of idle land.

When $\lambda = 0$, the social planner is not bothered by the non-agricultural use of land, and the choices of A and T are not affected by the tax mechanism. In this case, equations (3a) and (3b) become the first-order conditions of farmer's best choice of A and T. If $\lambda = 1$, equation (3b) determines that $\Theta_C = \Theta$, i.e., there is no idle land in the optimal taxation scheme. The shadow price of the rental-market constraint in the latter case becomes constant and equal to the price of land for all $\theta \in \Theta$. Therefore, λ actually represents the effort of the social planner to hinder non-agricultural land use.

As a consequence, there is a threshold $\bar{\lambda} < 1$ such that every economy with $\lambda < \bar{\lambda}$ has some pieces of idle land in the first-best solution. Since we are primarily focused in such economies, we will restrict the analysis to the cases where $\lambda < \bar{\lambda}$ as pointed by the following assumption.

Assumption A2: $\lambda < \bar{\lambda}$, where $\bar{\lambda}$ is implicitly defined by $\bar{\lambda} = 1 - \frac{p}{\phi_T\left(T^*\left(\bar{\theta}\right),\bar{\theta}\right)}$.

The following result shows that the optimal tax scheme under complete information can be implemented by pure land taxes.

Proposition 1 *The solution to the optimal-taxation problem (P.FB) can be decentralized through a menu of linear taxes of the form:*

$$t = \beta^* T + \gamma^*,$$

where $\beta^* = \lambda \phi_T\left(T^*, \theta\right) = \frac{\lambda}{1-\lambda}\left(p - \mu^*\right)$ and $\gamma^* = t^* - \beta^* T^*$.

PROOF. Consider $t = \beta^* T + \gamma^*$, where β^* and γ^* are defined as above. The first-order conditions for (P) can be written as:

$$Q_A\left(A, \theta\right) = w + \mu,$$

$$\phi_T\left(T, \theta\right) = p + \beta^* - \mu,$$
$$\mu\left(T - A\right) = 0.$$

Using the definition of β^* it is easy to check that the system above is equivalent to the system (3). The value of γ^* adjusts the level of the transfers in order to satisfy (IR) with equality.■

Proposition 1 shows that the social planner can implement the optimal tax scheme under complete information offering a pair (β^*, γ^*) to the producer of type θ. Solving (P), each farmer chooses the amounts of A and T determined by the solution of (P.FB). The land-tax rate is equal to the social cost of non-agricultural landholding which is increasing with respect to λ. If $\lambda = 0$, the government does not distort the choice of the farm size by not taxing the ownership of land ($\beta^* = 0$). If $\lambda \geq \bar{\lambda}$, on the other hand, there is no idle land after taxation ($\Theta_C = \Theta$).

Observe that in Θ_U we get a flat rate $\beta^* = \frac{\lambda}{1-\lambda} p$ which does not vary according to the type of producer. Thus, the model shows that in a context of complete information, one may use a single rate for all producers operating with unimproved land.

In short, if the social planner could precisely observe landowner-productivity parameters, there is a pure land-tax scheme capable of implementing the optimal taxation schedule. In this solution, if $\lambda > 0$, the social planner discourages the non-agricultural use of land, and with $\lambda \geq \bar{\lambda}$, there would be no idle land in equilibrium.

2.2 The second-best case

Consider now a more realistic case in which θ is private information. The choice of the optimal taxation scheme becomes a typical mechanism-design problem. The revelation principle ensures it is sufficient to concentrate on a direct mechanism that induces truthful revelation of the farmer's productivity parameter (see Mirrlees, 1971). Each mechanism is an allocation determined by a set of three functions (t, A, T) defined on Θ and should be interpreted as the principal collecting a tax t in exchange for a choice (A, T).[9]

Let $\Pi(\hat{\theta}|\theta)$ the payoff of a producer of type θ who declares to be type $\hat{\theta}$, i.e.,

$$\Pi(\hat{\theta}|\theta) = Q\left(\hat{A}, \theta\right) + \phi\left(\hat{T}, \theta\right) - w\hat{A} - p\hat{T} - \hat{t},$$

where $\hat{A} \equiv A\left(\hat{\theta}\right)$, $\hat{T} \equiv T\left(\hat{\theta}\right)$ and $\hat{t} \equiv t\left(\hat{\theta}\right)$. The truth-telling requirement establishes a restriction on the set of all feasible mechanisms which can be summarized in the following definition.

Definition *An allocation (t, A, T) is implementable (or incentive compatible)*

[9] The fact that $Q_A > 0$ implies that it is equivalent to designing a mechanism in terms of either (t, A, T) or (t, Q, T). This choice is a matter of convenience. For more details, see the proof of proposition 5 in appendix.

if and only if

$$\Pi(\theta|\theta) \geq \Pi(\hat{\theta}|\theta), \quad \forall\, \theta, \hat{\theta} \in \Theta. \tag{IC}$$

Every implementable allocation is such that the agent is interested in revealing his correct type rather than untruthfully declaring himself to be of some other type. The next result establishes a more tractable form for implementable allocations based on Guesnerie and Laffont (1984).

Proposition 2 *Let (t, A, T) be piecewise $\mathcal{C}^1$. Then (t, A, T) is implementable only if*

$$\frac{d}{d\theta}\Pi(\theta|\theta) = Q_\theta(A, \theta) + \phi_\theta(T, \theta), \tag{IC_1}$$

and

$$Q_{A\theta}(A, \theta)\dot{A} + \phi_{T\theta}(T, \theta)\dot{T} \geq 0, \ \textit{almost surely in } \Theta. \tag{IC_2}$$

This and all the proofs of the results that follow are in the appendix.

We know that the local first- and second-order conditions (IC_1) and (IC_2) of incentive compatibility are in general only necessary. Our strategy, therefore, is to define and characterize the solution of the relaxed mechanism design program for this problem, i.e., the one with these local conditions in the place of the incentive compatibility constraint. [10]

Thus, the problem of optimal mechanism design is represented by the following maximization program:

$$\max_{\{t,A,T\}_{\theta\in\Theta}} \int_\Theta [t - \lambda\phi(T, \theta)] f(\theta)\, d\theta \tag{$P.SB$}$$

subject to (IC),

$$\Pi(\theta|\theta) \geq 0, \tag{IR_θ}$$

and

$$A \leq T. \tag{RM_θ}$$

Analogously, we define $(P.SB_R)$ as the relaxed version of $(P.SB)$, where we replace (IC) by its first-order condition. We will follow the first-order approach to $(P.SB)$, examining later the conditions for which the solution of $(P.SB_R)$ and $(P.SB)$ coincide.

[10] We will show, in the proposition that follows, that these local conditions are sufficient for implementability under monotonicity of the mechanism and the single crossing property.

In order to solve the program above using the standard techniques, we need a condition of sorting regarding the effect of θ on Q and ϕ.

Assumption A3: $Q_\theta(\cdot,\theta)+\phi_\theta(\cdot,\theta)>0$ for every $\theta\in\Theta$.

Assumption (A3) establishes that, holding the farm size fixed for no idle land, farmers with higher values of θ can obtain a larger payoff. Under (A3), conditions (RM_θ) and (IC_1) imply (IR_θ) is binding only for $\theta=\underline{\theta}$. Otherwise, the social planner could improve by increasing t without violating the constraints.

Integrating (IC_1) by parts and substituting it into $(P.SB_R)$ we eliminate t to get:

$$\max_{\{A,T\}_{\theta\in\Theta}}\int_\Theta \Psi(A,T,\theta)\ f(\theta)\,d\theta$$

subject to (IR_θ) and (RM_θ), where

$$\begin{aligned}\Psi(A,T,\theta)\equiv\ & Q(A,\theta)-wA+(1-\lambda)\phi(T,\theta)-pT\\ &-R(\theta)\left(Q_\theta(A,\theta)+\phi_\theta(T,\theta)\right)\end{aligned}$$

and $R(\theta)\equiv\frac{1-F(\theta)}{f(\theta)}$.[11] We make the following assumption to assure that the use the first-order conditions for the maximization program above is sufficient.

Assumption A4: $Q_{AA}(\cdot,\theta)-R(\theta)Q_{AA\theta}(\cdot,\theta)<0$ and $(1-\lambda)\phi_{TT}(\cdot,\theta)-R(\theta)\phi_{TT\theta}(\cdot,\theta)<0$, for each $\theta\in\Theta$.

This implies that $\Psi(A,T,\theta)$ is strictly concave in (A,T) and, by Inada's condition, it has a unique maximum for each $\theta\in\Theta$. The first-order conditions of $(P.SB_R)$ are given by:

$$\Psi_A(A^{**},T^{**},\theta)=\mu^{**}, \tag{4a}$$
$$\Psi_T(A^{**},T^{**},\theta)=-\mu^{**}, \tag{4b}$$
$$\mu^{**}(A^{**}-T^{**})=0. \tag{4c}$$

Define a transfer profile for θ given by

$$\begin{aligned}t^{**}=\ & Q(A^{**},\theta)-wA^{**}+\phi(T^{**},\theta)-pT^{**}\\ &-\int_{\underline{\theta}}^{\theta}\left[Q_\theta\left(A^{**},\hat{\theta}\right)+\phi_\theta\left(T^{**},\hat{\theta}\right)\right]d\hat{\theta},\end{aligned}\tag{5}$$

[11] Notice that $\Psi_{AT}=0$.

obtained from the integration of (IC_1). If (IC) is verified for every pair (A^{**}, T^{**}) in Θ, the solution of $(P.SB)$ is completely defined by the system (4) and (5), under (A4). We assure this is the case by imposing the next two assumptions.

Assumption A5: (Strong monotonic hazard-rate condition) $\dot{R}(\theta) \leq -\lambda$ for all $\theta \in \Theta$.

Assumption A6: $Q_{A\theta}(\cdot, \theta) + (1 - \lambda) \phi_{T\theta}(\cdot, \theta) \geq 0$ for every $\theta \in \Theta$.

Assumptions (A5) and (A6) are used to avoid the need of the "ironing principle" in the solution. Condition (A5) is satisfied for distributions such as uniform.[12] Assumption (A6) is another sorting condition establishing that a marginal increase in θ determines an increase on the total revenue from landholding, even if $Q_{A\theta} < 0$. Notice that the sign of $Q_{A\theta}$ is not defined. The next result shows that (A1)-(A6) are sufficient to characterize (4) as the unique solution for $(P.SB)$.

Proposition 3 *Suppose that (A1)-(A6) are satisfied. Then the solution of $(P.SB)$ is determined by the system (4) along with the definition of t^{**}.*

The result above shows that the best direct revelation tax mechanism is given by the triple (t^{**}, A^{**}, T^{**}). Under this mechanism, each farmer is induced to announce that his type is the true θ receiving the allocation (A^{**}, T^{**}) and paying t^{**}. However, there are more reasonable mechanisms which yield the same results derived above. The implementation of the optimal solution via a menu of linear taxes is presented in the next proposition. Similarly to the first-best case, let $\tilde{\Theta}_C = \{\theta \in \Theta : \mu^{**} > 0\}$ and $\tilde{\Theta}_U = \Theta - \tilde{\Theta}_C$.

In order to implement the optimal solution via a menu of linear taxes, we need additional properties involving $Q_{AA\theta}$ and $\phi_{TT\theta}$. A sufficient condition is given by the following assumption.

Assumption A7: $Q_{AA\theta}(\cdot, \theta) \leq 0$ and $\phi_{TT\theta} \leq 0$ for every $\theta \in \Theta$.[13]

[12] Actually, in order to assure that system (4) determines the optimal solution for $(P.SB_R)$, we need only that $\dot{R}(\theta) \leq 0$. However, this stronger version is necessary to the implementation result.

[13] Notice that assumptions (A4) and (A7) determine that:

$$(1 - \lambda) \phi_{TT}(\cdot, \theta) - R(\theta) \phi_{TT\theta}(\cdot, \theta) < 0 \leq -\phi_{TT\theta}(\cdot, \theta),$$

which is perfectly consistent under (A2).

Now, our main result can be presented.

Proposition 4 *Under (A1)-(A7), the solution of (P.SB) can be decentralized by a menu of linear taxes through the following: (i) the social planner offers a menu of linear taxes based on the observable variables T (farm size) and Q (agricultural output) and the announcement of the farmer's productivity parameter $\hat{\theta}$ given by*

$$t(Q, T; \hat{\theta}) = \alpha(\hat{\theta})Q + \beta(\hat{\theta})T + \gamma(\hat{\theta});$$

(ii) based on the tax schedule $t(Q, T; \hat{\theta})$, farmers of each type choose Q and T and reveal their types (truthfully); (iii) the social planner collects the tax $t\left(T, Q; \hat{\theta}\right)$. The tax schedule is such that $\alpha(\hat{\theta}) = 0$ for every $\hat{\theta} \in \Theta_C$.

The total tax is constituted by a three-part tariff: an output tax, a land tax and a fixed part. Observe that the scheme put forward as a solution for the model with complete information cannot be implemented under asymmetric information, i.e., it is not possible to implement the optimal tax mechanism relying only on land taxes. From Proposition 4, it is necessary to use output taxes along with land taxes in order to obtain the optimal allocation to farmers with idle land. Notice that the fixed part does not alter the choices of the cultivated area A and farm size T and it is used only to extract rent from the farmers.

3 Farm Size and Tax Evasion

This section examines some implications of the previous analysis for land taxation in Latin American countries, exploring the contents of Proposition 4. Despite the absence of a land rental market, there is a consensus regarding the existence of a dualism in agrarian organization in Latin America. Small farmers with high yields per hectare coexist with less productive large landholders. We adopt the next assumption to concentrate our analysis on this Latin American situation.

Assumption A8: $Q_{A\theta}(\cdot, \theta) < 0$ for every $\theta \in \Theta$.

Under (A6) and (A8), the types of farmers are such that a higher θ refers to an increase in the non-agricultural land benefits and a decrease in the agricultural productivity of land. Farmers with high θ obtain profits mostly from non-agricultural land use, experiencing lower agricultural productivity. As a consequence, those farmers are expected to have larger tracts with a smaller cropped area. In other words, assumptions (A1) and (A8) are sufficient to generate an inverse relationship between farm size and agricultural productivity.[14]

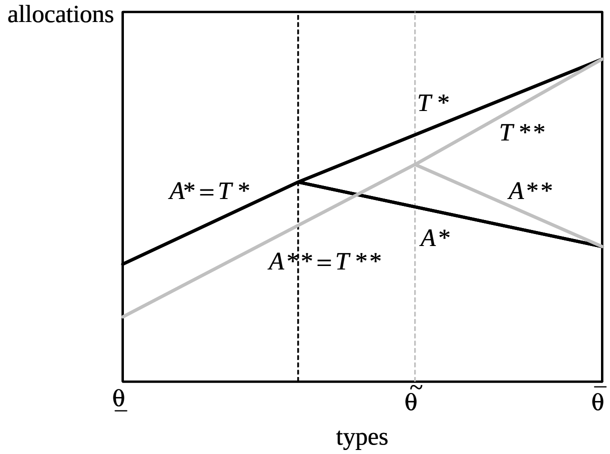

Fig. 1. Optimal allocations in the first-best (black) and second-best (gray) cases

Lemma 5 *Assume (A1)-(A8). There exists at most one critical type* $\tilde{\theta} \in \Theta$ *such that* $\tilde{\Theta}_C = [\underline{\theta}, \tilde{\theta}]$ *and* $\tilde{\Theta}_U = (\tilde{\theta}, \bar{\theta}]$*. Also,* $\dot{T}^{**} = \dot{A}^{**} > 0$ *for all farmers in* $\tilde{\Theta}_C$*,* $\dot{T}^{**} > 0$ *and* $\dot{A}^{**} < 0$ *for all farmers in* $\tilde{\Theta}_U$*.*

Lemma 5 has two important consequences. First, all types can be completely described by the farm size - formally, T^{**} is an one-to-one function. Second, only small farmers (those with $T^{**}(\theta) < T^{**}\left(\tilde{\theta}\right)$) are restricted by the rental-market constraint. It is also straightforward to obtain a similar result from (A8) to the complete information case. Figure 1 compares the first-best and second-best allocations.

Assumption (A8), through Lemma 5, provides an easier interpretation for the contents of Proposition 4. The output tax rate should be zero for small farmers who are restricted by the absence of a land rental market, operating without idle land (see Figure 2). Even though the tax on output causes a distortion in resource allocation, its use is justified by its ability to compose an implementable (or self-revealing) taxation mechanism, especially for large landholders. Therefore, the tax evasion tends to be more acute for these large farmers in countries with pure land tax instruments. The next proposition summarizes these findings.

[14] This inverse relationship between farm size and productivity can be theoretically explained either by interaction of different market imperfections (Feder, 1985; Eswaran and Kotwal, 1986) or by a self-selection argument (Assunção and Ghatak, 2003). The empirical evidence is vast, including; for example, Berry and Cline (1979), Rosenzweig and Binswanger (1993), Benjamin (1995), Barret (1996) and Lamb (2003).

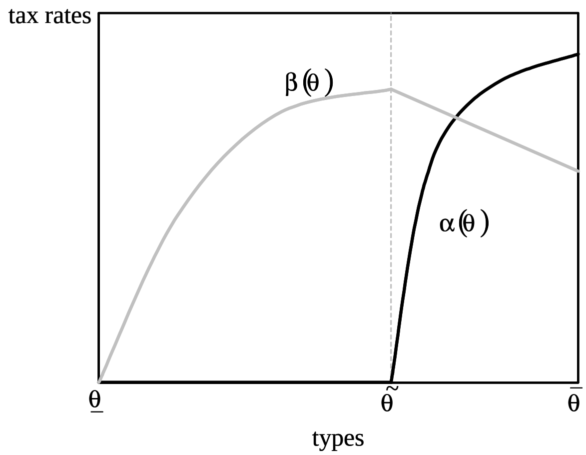

Fig. 2. Optimal rates: taxes on output (black) and land (gray).

Proposition 6 *Under (A1)-(A8), the optimal land-taxation problem is such that: (i) there is evasion when a pure land tax regime is used and* $\Theta_U \neq \varnothing$*; (ii) evasion is more accentuated among large landholders (those with* $T^{**}(\theta) > T^{**}\left(\tilde{\theta}\right)$*).*

An example: the Brazilian experience

Land tax (Rural Property Tax - ITR) in Brazil, since its creation through the Land Statute of 1964, has been used to support public policies for land redistribution. Nevertheless, there is a high level of evasion and default that hinders its efficiency as an instrument of landholding policy. Two large-scale reforms of ITR were carried out, in 1979 and 1996, but the results have not sufficed to overcome the associated problems. According to the current scheme, the rate would vary from 0.03% to 20% of the land value. The rates differ only by degree of utilization and total area, being sharply progressive rates in relation to the farm size and regressive with respect to the percentage of cropped area, so that productive properties are benefited.

In an economy such as Brazil's, where producers operate with idle land and the government is often in the dark as to the true productivity parameters of the agricultural and non-agricultural activities available to various landholders, Proposition 4 shows that there are no tax rates able to make ITR an optimal taxation scheme. The use of an output tax becomes necessary so that producers with better access to non-agricultural activities will pay their share of the tax burden.

A natural way to implement this result, using the pre-existent and well-defined structure of the output tax (which is called ICMS in Brazil), is to consider the tax rate a function of the farm size and the total output tax collected per hectare. The degree of use, which is a non-observable variable, should be replaced by the amount of ICMS collected per hectare. This implementation has a great potential to improve not only the application of ITR but also the collection of ICMS. The scheme proposed implies an additional cost to the evasion and default of ICMS given by the increase in the ITR. Therefore, this reformulation constitutes a new step towards more reliable tax institutions. Note that only the information about the ICMS is required in this scheme. Hence, there is no need for large modifications in the existent institutions and mechanisms.

This inability to implement ITR as a solution to the problem with asymmetric information lends theoretical support to what the government found by comparing declared against actual data. According to the Presidential Press Office, the percentage of value declared in relation to real value in the 1980s varied from 20% for properties of less than 10 hectares down to as low as 1.2% for large properties of over 10 thousand hectares. The area reported as usable was also far short of the true figure, with large landowners declaring around 50% and small ones 94% of the real measure. This casual evidence is compatible not only with the inadequacy of a pure land-tax regime but also with the fact that small producers, in general with no idle land, are more likely to declare their land use correctly. The Brazilian case shows not only that a pure scheme based on land taxes is not implementable but also that evasion is more accentuated among large landholders.

4 Discussion

We have used a model to study an optimal land-taxation problem in the context of a typical Latin American country. In this section, we describe the role of each key ingredient in our main results.

Some authors recognize that a tax on land has both fiscal and non-fiscal effects. Although it can be an important source to finance local governments, it must also be considered from a more general policy perspective. Land taxes can be an alternative to traditional confiscatory land-reform programs. Land taxes might provide an incentive for greater density and better use of land. Based on this literature, our analysis has also considered non-revenue objectives for land taxation. [15]

In a more general scenario, we could consider the social planner's utility function as $U(t, A, T) = t - \psi(A, T)$, where ψ is strictly decreasing in A and

[15] E.g. Binswanger, Deininger and Feder (1995), Deininger and Feder (2000), Bird and Slack (2002).

increasing in T. In other words, the social planner could promote agricultural production and avoid large landholdings.[16] In this case, it is straightforward to check that the use of output tax is always required in a linear implementation of the optimal taxation scheme, even in the complete information case. Therefore, relying only on land tax would not be optimal even if the productivity parameter were publicly observed.

On the other hand, the utility function we have used provides a meaningful benchmark. It allows us to analyze the need of output taxes as part of the optimal tax mechanism. A comparison between Proposition 1 and Proposition 4 reveals that asymmetric information may lead the need of output taxes along with land taxes in order to control both cultivated area and farm size.

Another key feature of our analysis is the non-agricultural component of land demand. As mentioned before, many reasons have contributed to distort the allocation of land in favor of large landholders, especially in Latin American countries. In terms of the model, this is represented by the function $\phi(T,\theta)$, which is essential for our analysis. If $\phi(T,\theta)=0$ for all T and θ, no idle land would exist in equilibrium. In this case, all farmers exploit the full agricultural capacity of their holdings, keeping $A=T$. Therefore, $\Theta_U=\tilde{\Theta}_U=\varnothing$ and the optimal tax mechanism can be implemented using only the tax on land in both cases - complete or incomplete information. The output tax is required only when there is a separation between the choice of cultivated area and farm size.

Finally, we have assumed that the land rental market is absent. Alternatively, we could consider an imperfect land rental market. In this case, we believe that the qualitative results still hold, with consequences only to the notation.

5 Conclusion

This paper presents a model of asymmetric information in which a mix of output and land taxes arises as a means of implementing an optimal taxation mechanism. The model is stripped down to highlight characteristics commonly observed in Latin American countries. The role of each one in our main propositions is discussed in the last section.

The mechanics of our results is the following. The absence of a land rental market restrains the cultivated area to be not larger than the farm size. As a consequence, depending on the proclivity of agricultural and non-agricultural

[16] This functional form includes the cases in which the social planner combats idle land, measured as T/A or $T-A$.

land use, farmers have one or two key decisions to make. Whenever the land-rental market constraint is binding, the cultivated area is equal to the farm size and there is only one relevant choice. On the other hand, if the land rental market constraint is not binding (the case with idle land), farmers choose both farm size and area under cultivation. While we need only one tax instrument in the first case, the second case requires two. Therefore, output taxes have a key role in the taxation of those farmers operating with idle land, even considering their distortional effect.

The lack of a land rental market along with the inverse relationship between farm size and productivity explains not only the evasion of land taxes but also why this practice is more evident among large landholders. Small farmers do not operate with idle land due to their higher agricultural productivity. Hence, a land-tax scheme is more likely to be effective for them.

The present model can be useful for issues of land policy in Latin American countries. Appropriate land taxes might correct land prices in economies where they are above the discounted present value of agricultural inflows, inducing land redistribution from large landowners to more productive small peasants [Deininger and Feder (2000)]. We have argued that the implementation of such taxes comprises both land and output taxes.

Appendix

PROOF. [**Proposition 2**] Let (t, A, T) be an implementable allocation piecewise $\mathcal{C}^1$. Thus, for every $\theta \in \Theta$,

$$\Pi(\theta|\theta) \geq \Pi\left(\hat{\theta}|\theta\right) = \Pi\left(\hat{\theta}|\hat{\theta}\right) + \left[Q\left(\hat{A},\theta\right) - Q\left(\hat{A},\hat{\theta}\right)\right] + \left[\phi\left(\hat{T},\theta\right) - \phi\left(\hat{T},\hat{\theta}\right)\right],$$

where $\hat{A} \equiv A\left(\hat{\theta}\right)$ and $\hat{T} \equiv T\left(\hat{\theta}\right)$. Hence,

$$\Pi(\theta|\theta) - \Pi\left(\hat{\theta}|\hat{\theta}\right) \geq \left[Q\left(\hat{A},\theta\right) - Q\left(\hat{A},\hat{\theta}\right)\right] + \left[\phi\left(\hat{T},\theta\right) - \phi\left(\hat{T},\hat{\theta}\right)\right],$$

and, switching θ and $\hat{\theta}$, dividing by $\left(\theta - \hat{\theta}\right)$ and taking $\hat{\theta} \to \theta$, the function Π is proved to be piecewise $\mathcal{C}^1$ and

$$\frac{d}{d\theta}\Pi(\theta|\theta) = Q_\theta(A,\theta) + \phi_\theta(T,\theta) \quad a.s. \text{ for all } \theta \in \Theta.$$

Repeating this computation considering $\left(\theta - \hat{\theta}\right)^2$, we get

$$Q_{A\theta}(A,\theta)\dot{A} + \phi_{T\theta}(T,\theta)\dot{T} \geq 0 \quad a.s. \text{ for all } \theta \in \Theta. \blacksquare$$

PROOF. [**Proposition 3**] Notice that we need only to check the incentive compatibility constraint for the mechanism determined by (4). From Proposition 2 and its proof, it is sufficient to test:

$$0 \leq \Pi(x|x) - \Pi(y|x) = \int_y^x \int_y^\theta \left[Q_{A\theta}(\widetilde{A}^{**}, \theta)\dot{\widetilde{A}}^{**} + \phi_{T\theta}(\widetilde{T}^{**}, \theta)\dot{\widetilde{T}}^{**} \right] d\widetilde{\theta} d\theta$$

for every $x, y \in \Theta$, where the equality results from (IC_1) applied to the mechanism (t^{**}, A^{**}, T^{**}), after some manipulation, and $\widetilde{A}^{**} = A^{**}(\widetilde{\theta})$, etc. Therefore, it is sufficient to check that the sign of the expression in the bracket above is always non-negative. For this we separate the analysis in two cases:

For all $\theta \in \Theta_C$, we can differentiate (4) to obtain

$$\dot{A}^{**} = \dot{T}^{**} = -\frac{\Psi_{A\theta} + \Psi_{T\theta}}{\Psi_{AA} + \Psi_{TT}} > 0 \tag{6}$$

under (A1), (A4), (A5) and (A6). In this case, the non-negativeness of the expression in the bracket reduces to

$$\left[Q_{A\theta}(\widetilde{A}^{**}, \theta) + \phi_{T\theta}(\widetilde{T}^{**}, \theta) \right] \dot{\widetilde{T}}^{**} \geq 0,$$

which is clearly true under (A1) and (A6).

For $\theta \in \Theta_U$, we get

$$\dot{A}^{**} = -\frac{\Psi_{A\theta}}{\Psi_{AA}} \tag{7}$$

and

$$\dot{T}^{**} = -\frac{\Psi_{T\theta}}{\Psi_{TT}} > 0, \tag{8}$$

under (A1), (A4) and (A5). The sign of $\dot{A}^{**}$ depends on $Q_{A\theta}$. Using Ψ, the required non-negative condition is equivalent to

$$\frac{\left(Q_{A\theta}(\widetilde{A}^{**}, \theta)\right)^2 \left(1 - \dot{R}\right)}{\Psi_{AA}(\widetilde{A}^{**}, \widetilde{T}^{**}, \theta)} + \frac{\left(\phi_{T\theta}(\widetilde{T}^{**}, \theta)\right)^2 \left(1 - \lambda - \dot{R}\right)}{\Psi_{TT}(\widetilde{A}^{**}, \widetilde{T}^{**}, \theta)} \leq 0,$$

which is true under (A4) and (A5).■

PROOF. [**Proposition 4**] Let $Q = Q(A, \theta)$ denote the agricultural production. Since $Q_A(A, \theta) > 0$, we can use the implicit function theorem to define a function $A = A(Q, \theta)$. Therefore, we will define the tax schedule in terms of (A, T) and the announcement $\widehat{\theta}$ without loss of generality. This proof owes much to Laffont and Tirole (1993). Consider a tax scheme $t(A, T; \widehat{\theta})$ as in the statement of the proposition (only substituting Q for A) in which, for every

$\widehat{\theta} \in \tilde{\Theta}_U$,

$$\alpha(\widehat{\theta}) = Q_A(A^{**}(\widehat{\theta}), \widehat{\theta}) - w,$$
$$\beta(\widehat{\theta}) = \phi_T(T^{**}(\widehat{\theta}), \widehat{\theta}) - p,$$
$$\gamma(\widehat{\theta}) = t^{**}(\widehat{\theta}) - \alpha(\widehat{\theta})A^{**}(\widehat{\theta}) - \beta(\widehat{\theta})T^{**}(\widehat{\theta}),$$

and, for $\widehat{\theta} \in \tilde{\Theta}_C$,

$$\alpha(\widehat{\theta}) = 0,$$
$$\beta(\widehat{\theta}) = Q_A(A^{**}(\widehat{\theta}), \widehat{\theta}) + \phi_T(T^{**}(\widehat{\theta}), \widehat{\theta}) - p - w,$$
$$\gamma(\widehat{\theta}) = t^{**}(\widehat{\theta}) - \beta(\widehat{\theta})T^{**}(\widehat{\theta}),$$

where (A^{**}, T^{**}, t^{**}) is the second best solution.

Given this linear tax scheme, a type θ farmer chooses agricultural production and farm size in order to solve:

$$\max_{A,T,\hat{\theta}} \; Q(A,\theta) + \phi(T,\theta) - wA - pT - t(A,T;\widehat{\theta}) \quad \text{s.t. } A \leq T. \tag{9}$$

The first-order condition gives:

$$Q_A(A,\theta) - w - \widehat{\alpha} - \mu = 0 \tag{10a}$$
$$\phi_T(T,\theta) - p - \widehat{\beta} + \mu = 0 \tag{10b}$$
$$-\widehat{\dot{\alpha}}\left(A - \widehat{A}\right) + \widehat{\alpha}\widehat{\dot{A}} - \widehat{\dot{\beta}}\left(T - \widehat{T}\right) + \widehat{\beta}\widehat{\dot{T}} - \widehat{\dot{\gamma}} = 0 \tag{10c}$$

where μ is the Lagrange multiplier of (RM_θ). In order to facilitate notation, we use $\hat{\alpha} \equiv \alpha\left(\hat{\theta}\right)$, $\hat{\beta} \equiv \beta\left(\hat{\theta}\right)$, $\hat{\gamma} \equiv \gamma\left(\hat{\theta}\right)$, $\hat{A} \equiv A^{**}\left(\hat{\theta}\right)$ and $\hat{T} \equiv T^{**}\left(\hat{\theta}\right)$. Using Proposition 3 and the definition of α, β and γ, we can show that the (10c) is equivalent to $-\widehat{\dot{\alpha}}\left(A - \widehat{A}\right) - \widehat{\dot{\beta}}\left(T - \widehat{T}\right) = 0$. From this, it is easy to see that for each θ (either in $\tilde{\Theta}_U$ or $\tilde{\Theta}_C$), $A = A^{**}(\widehat{\theta})$, $T = T^{**}(\widehat{\theta})$ and $\widehat{\theta} = \theta$ satisfy the first-order condition of $(P.SB)$. The following lemma shows that this allocation is indeed the optimal for the program above and concludes the proof.■

Lemma *(i)* $sign(\widehat{\dot{\alpha}}) = -sign(\widehat{\dot{A}}) = -sign\left(Q_{A\theta}\right)$, $\widehat{\dot{\beta}} < 0$ *and* $\widehat{\dot{T}} > 0$.

(ii) For given θ and $\widehat{\theta}$, there exists only one (A,T) satisfying (10a) and (10b) of the first-order condition above. Moreover, this (A,T) is the optimal for the program (9) when we consider $\widehat{\theta}$ fixed.

(iii) For (A,T) given of (ii), $\widehat{\theta} > \theta$ if and only if $-\widehat{\dot{\alpha}}\left(A - \widehat{A}\right) - \widehat{\dot{\beta}}\left(T - \widehat{T}\right) < 0$. In particular, $\widehat{\theta} = \theta$ is the optimal for the program (9).

PROOF. (i) Assumption (A7) determines that $Q_{A\theta}$ does not change its sign more than once. Let us start considering the case where $Q_{A\theta} < 0$. First suppose that $\widehat{\theta} \in \tilde{\Theta}_U$. The first-order condition of the second-best problem is

$$\widehat{Q}_A - w - \widehat{R}\widehat{Q}_{A\theta} = 0$$
$$(1-\lambda)\widehat{\phi}_T - p - \widehat{R}\widehat{\phi}_{T\theta} = 0.$$

Taking the implicit derivative with respect to $\widehat{\theta}$ we get

$$\widehat{\dot{A}} = \frac{(\widehat{\dot{R}} - 1)\widehat{Q}_{A\theta}}{\widehat{Q}_{AA} - \widehat{R}\widehat{Q}_{AA\theta}} < 0$$
$$\widehat{\dot{T}} = \frac{[\widehat{\dot{R}} - (1-\lambda)]\widehat{\phi}_{T\theta}}{(1-\lambda)\widehat{\phi}_{TT} - \widehat{R}\widehat{\phi}_{TT\theta}} > 0$$

by assumptions A4 to A7. Thus, using the definition of $\widehat{\alpha}$ and $\widehat{\beta}$,

$$\widehat{\dot{\alpha}} = \widehat{Q}_{AA}\widehat{\dot{A}} + \widehat{Q}_{A\theta} = \widehat{R}\widehat{Q}_{AA\theta}\widehat{\dot{A}} + \widehat{\dot{R}}\widehat{Q}_{A\theta} > 0$$
$$\widehat{\dot{\beta}} = \widehat{\phi}_{TT}\widehat{\dot{T}} + \widehat{\phi}_{T\theta} = \frac{\widehat{R}\widehat{\phi}_{TT\theta}\widehat{\dot{T}} + \widehat{\dot{R}}\widehat{\phi}_{T\theta}}{1-\lambda} < 0.$$

Notice that in the case where $Q_{A\theta} > 0$ we get $\widehat{\dot{A}} > 0$ and $\widehat{\dot{\alpha}} < 0$ and, therefore, $\widehat{\dot{\alpha}}\ \widehat{\dot{A}} < 0$ whatever is the sign of $Q_{A\theta}$.

If $\widehat{\theta} \in \Omega_C$, we have $\widehat{\dot{\alpha}} = 0$ and, analogously, the first-order condition of the second-best problem gives

$$\widehat{Q}_A + (1-\lambda)\widehat{\phi}_T - w - p - \widehat{R}(\widehat{Q}_{A\theta} + \widehat{\phi}_{T\theta}) = 0.$$

Taking the implicit derivative with respect to $\widehat{\theta}$ we get

$$\widehat{\dot{A}} = \widehat{\dot{T}} = \frac{(\widehat{\dot{R}} - 1)(\widehat{Q}_{A\theta} + \widehat{\phi}_{T\theta})}{\widehat{Q}_{AA} + (1-\lambda)\widehat{\phi}_{TT} - \widehat{R}(\widehat{Q}_{AA\theta} + \widehat{\phi}_{TT\theta})} \geq 0$$

by assumptions A4-A7. Thus, using the definition of $\widehat{\beta}$,

$$\begin{aligned}\dot{\widehat{\beta}} &= (\widehat{Q}_{AA} + \widehat{\phi}_{TT})\dot{\widehat{T}} + \widehat{Q}_{A\theta} + \widehat{\phi}_{T\theta} \\ &= \dot{\widehat{R}}\left(\widehat{Q}_{A\theta} + \widehat{\phi}_{T\theta}\right) + \widehat{R}(\widehat{Q}_{AA\theta} + \widehat{\phi}_{TT\theta})\dot{\widehat{T}} + \lambda\widehat{\phi}_{TT}\dot{\widehat{T}} < 0.\end{aligned}$$

(ii) This is a direct consequence of the strict concavity (A4) and Inada's condition.

(iii) We will show that the announcement of $\hat{\theta} = \theta$ is optimal for the program without the condition $A \leq T$. Then, we conclude using part (ii) that this fact implies in unique choices of (A, T) which are equivalent to those of $(P.SB)$, with $A \leq T$.

Fix $\widehat{\theta}$ and consider $Q_{A\theta} < 0$. Let (A, T, θ) be the solution of the equations (10a) and (10b) above for $\theta < \widehat{\theta}$, considering $\mu = 0$ (relaxed program). By the implicit function theorem, there are functions $T = \Upsilon(A)$ and $\theta = \Theta(A)$ that solves (10a) and (10b) for every $A \geq 0$ and $\theta < \widehat{\theta}$ such that

$$\begin{aligned}\dot{\Theta} &= -\frac{Q_{AA}}{Q_{A\theta}} < 0 \\ \dot{\Upsilon} &= -\frac{\phi_{T\theta}}{\phi_{TT}}\dot{\Theta} < 0\end{aligned}$$

by assumption (A1). Then, by these signs and the ones given in (i), we have

$$-\dot{\widehat{\alpha}}\left(A - \widehat{A}\right) - \dot{\widehat{\beta}}\left(T - \widehat{T}\right) < 0$$

if and only if $\theta < \widehat{\theta}$, for (A, T, θ) solution of (10a) and (10b) (because for $\theta = \widehat{\theta}$, this is exactly zero, i.e., equation (10c)). Analogously, the same is true for $Q_{A\theta} > 0$. Since part (ii) determines that the system (10a) and (10b) has only one solution for each θ and $\hat{\theta}$, with $A \leq T$, we are done.■

PROOF. [**Lemma 5**] For constrained types $\theta \in \tilde{\Theta}_C$, equation (6) establishes that $\dot{A}^{**} = \dot{T}^{**} > 0$. For unconstrained types $\theta \in \tilde{\Theta}_U$, it is straightforward to check that (7) implies in $\dot{A}^{**} < 0$ under (A1), (A4) and (A8) and equation (8) determines $\dot{T}^{**} > 0$.

To complete the proof, it is sufficient to verify that the multiplier of the constraint $A \leq T$ is strictly decreasing in θ. Differentiating (4a) for all $\theta \in \Theta_C$ and rearranging, we get

$$\dot{\mu}^{**} = \left(1 - \dot{R}\right) Q_{A\theta} + (Q_{AA} - RQ_{AA\theta})\, \dot{A}^{**} < 0,$$

under assumptions (A1), (A4), (A5) and (A8). Therefore, there exists at most one critical type $\tilde{\theta}$ such that $\tilde{\Theta}_U = (\tilde{\theta}, \bar{\theta}]$.■

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