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Modeling and forecasting short-term electric load demand: a two step methodology

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Modeling and Forecasting Short-Term Electric Load Demand: A Two-Step Methodology

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***Abstract*— The goal of this paper is to develop a forecasting model for the hourly electric load demand in the area covered by an utility company located in the southeast of Brazil. A different model is constructed for each hour of the day. Each model is based on a decomposition of the daily series of each hour in two components. The first component is purely deterministic and is related to trends, seasonality, and special days effect. The second one is stochastic and follows a linear autoregressive model. Nonlinear alternatives may be also considered in the second step. The multi-step forecasting performance of the proposed methodology is compared with a benchmark model and the results indicate that our proposal is a useful tool for electric load forecasting in tropical environments.**

***Index Terms*— Short-term load forecasting, time series, seasonality, linear models, SARIMA, decomposition.**

I. Introduction

ONE time series with major academic and practical interest is the hourly electric load demand series. From the academic point of view, the interest is remarkable because it has a number of interesting features, such as, trends, annual and daily seasonal patterns, influence of external variables, and possible nonlinearities. In addition, load series have been used along the years as a benchmark data set for different forecasting models and methods.

From the applied point of view, short-term load forecasting is a very important task for the electric utilities in order to manage the production, the transmission, and the distribution of electricity in a more efficient and secure way. As an example of the importance of accurate forecasts, it was estimated that an increase of only 1% in forecast error (in 1984) caused an increase of 10 million pounds in operating costs per year for one electric utility in the United Kingdom [1].

Over the years, different forecasting techniques have been developed to model electric load demand both in the classical time series literature (See [2] for a comprehensive review or [3], [4], [5], [6], [7] for recent applications) and in the machine intelligence community; see [8] and [9] for a recent survey or [10] for a successful application.

In this paper, we propose a methodology based solely on rigorous statistical arguments to model and forecast the hourly electric load demand of part of the southeast of Brazil. The area covered by the electric utility represents 25% of the province of Rio de Janeiro, totalizing 11,132 km^2 with a population of more than ten million people. The energy consumption corresponds to 75% of the total consumption in the Rio de Janeiro province. It is worth mentioning that this is one of the most important regions for tourism in Latin America. We adopt the same strategy as in [11], [12], [13], [14], and [15], treating each hour as a separate time series, such that 24 different models are estimated, one for each hour of the day. The model considered in the paper is based on a two-step decomposition of the load series. In the first step, a component based on Fourier series, dummy variables, and a linear trend, is estimated to describe the long-run trend, the annual seasonality, the effects of the days of the week, and any other special days effect such as public holidays. In the second step, different linear AR (autoregressive) models are estimated and lags are selected based on information criteria. The type of decomposition considered here is not new. Similar proposals have been discussed in the literature during the last two decades; see, for example, [16], [17], and [4]. However, we contribute to the literature in several different directions. First, to the best of our knowledge, the way in which we combine different aspects of classical techniques is new and relies only on rigorous classical statistical arguments. Recently, [14] proposed a similar approach, but their methodology is fully based on Bayesian statistics and it is computationally very demanding. Our methodology is simpler and can be used efficiently for real-time on-line load forecasting. Second, although very simple, the model proposed in this paper is robust and very flexible. For example, confidence intervals may be easily constructed without assuming any particular distribution for the errors of the model. In addition, it is important to stress that different models, such as neural networks or other nonlinear models,

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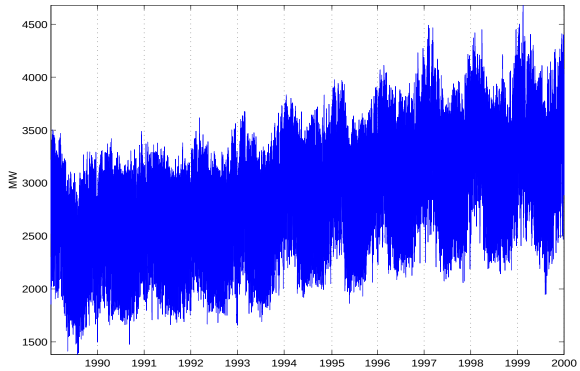

Fig. 1. Hourly loads from January 1, 1990 to December 31, 2000.

can be considered in the second step of the modeling strategy. However, we show in the paper that the nonlinear effects are caused by time-varying conditional variance and are not present in the conditional mean. Thus, the linear model is adequate to describe the dataset considered here. Exogenous variables, when available, may be also easily incorporated into the model. Finally, the model described in this paper is particularly useful for short-term load forecasting in tropical environments, where reliable temperature forecasts are not available.

The plan of the paper is as follows. Section II describes the dataset used in the paper. Section III presents the model and the modeling strategy. The benchmark model is discussed in Section IV. Section V-A shows the modeling results and Section V-B presents the forecasting results. Final remarks are made in Section VI.

II. The Data

In this paper we consider a dataset containing hourly loads from January 1, 1990 to December 31, 2000. The period from January 1, 1990 to December 31, 1998 is used for estimation purposes (in-sample) and the data concerning the years 1999 and 2000 are left for forecast evaluation (out-of-sample). The data were obtained from an utility company from Rio de Janeiro, Brazil and are shown in Figures 1 and 2. This is the same dataset considered in [15]. Figure 1 shows the hourly loads for the entire sample and Figure 2 shows the daily loads for each hour of the day during the in-sample period.

III. The Model

A. Mathematical Definition

Our approach to model the electric load time series is based on a two-step procedure for each hour of the day. The load is modeled as a sum of two components. The first component is deterministic, representing the trend, the annual cycle, and the effects of different types of days. The second component is described as a low-order linear autoregressive (AR) model. As discussed in the Introduction, two-level models have been considered extensively in the literature.

First, to remove the daily cycle we follow the ideas of [13] [1] by considering a separate model for each hour of the day, which avoids modeling complicated intra-day patterns in the hourly load, commonly called load profile, and enables each hour to have a distinct weekly pattern; see also [11], [18], [12], [19], [14], and [15]. This last feature is desirable, since it is expected that the day of the week will affect more work-time hours, when shops and industry may or not be open, compared to the first and last hours of the day, when most people are expected to be asleep. In [8], the authors report that difficulties in modeling the load profile are common to several load forecasting models.

The data seem to have a linear positive trend; see Figure 1. This is corroborated by the traditional Phillips-Perron unit-root test [20], where the null hypothesis of a stochastic trend (unit-root) is strongly rejected for all the 24 individual series. Furthermore, the positive trend in the load is correlated with economic and demographic factors. Hence, it is expected that the trend has a high positive correlation with the potential Gross Domestic Product (GDP), which in the case of Brazil is known to be almost linear; see [21] for a discussion. All that said, we model the trend as a deterministic linear function of time. Most papers in the load forecasting literature take first-order differences of the load series without previously testing for unit-roots; see [22] for example. This has a major drawback. When the trend is deterministic, taking first-differences introduces a non-invertible moving average component in the data generating process, which causes serious estimation problems. Furthermore, there is no linear autoregressive model that is able to correctly describe the dynamics of the data; see the discussion in Chapter 4 of [23]. Finally, it seems that there is a break in the trend after 1999. As this break belongs to the out-of-sample period, we ignore it during the specification and estimation of the proposed model. This is important in order to test the robustness of the proposed model.

As shown in Figures 1 and 2, the time-series displays a clear daily, weekly, and annual seasonality. Observing Figure 2 we can see that the annual seasonality is more apparent during the night. This is mainly due to the fact that during the night the effects of the days-of-the-week are less significant. The weekly seasonality – effects of the days of the week and special days, such as holidays – is modeled with dummy variables. Several authors claim

[1] The model described in [13] was the top first model in a load forecast competition organized by Puget Sound Power and Light Company, USA.

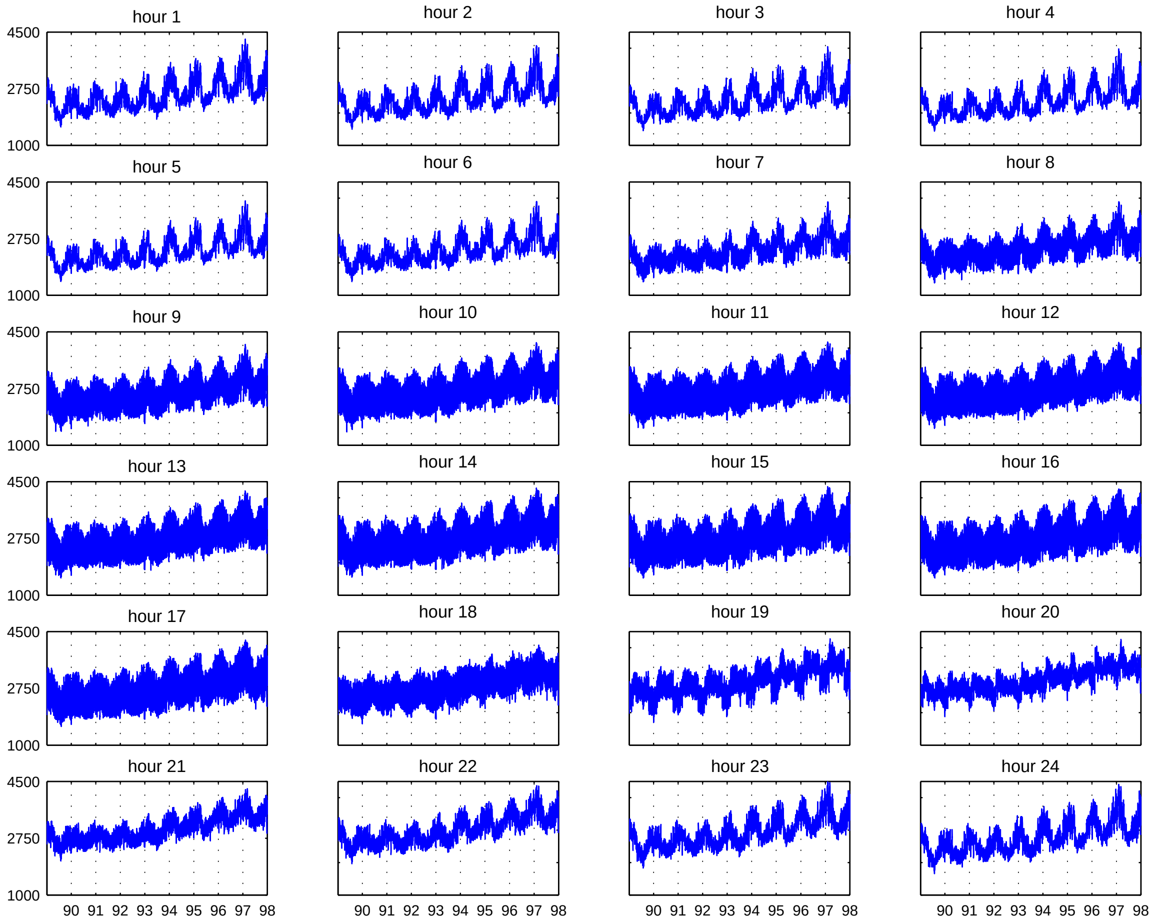

Fig. 2. Load of each hour from January 1, 1990 to December 31, 1998.

that Tuesdays, Wednesdays, Thursdays and Fridays can be modeled as a single type of day. Since we have a large amount of data we prefer to model each day as a dummy variable. We also consider dummies for holidays, part-time holidays, and special days. Table I gives summary of the variables used. Cottet and Smith [14] have adopted a similar approach.

The annual cycle is modeled as a sum of sines and cosines, as in a Fourier decomposition. The motivation for that can be easily seen by the graphical inspection of Figure 2. The number of trigonometric functions is determined by the Bayesian Information Criteria (SBIC) proposed by Schwarz [24]. Schneider, Takenawa, and Schiffman [25] and El-Keib, Ma, and Ma [26] have considered the same strategy. However, they have applied the Fourier decomposition to a single hourly series instead of 24 different daily series. Furthermore, they have determined the number of terms in the decomposition in a different way than the one considered here. More recently, Cotted and Smith [14] have used trigonometric functions to model the seasonality in the load series. The authors have also considered a distinct model for each hour of the day, but they have kept fixed the number of sines and cosines. In addition, their approach was based on Bayesian statistics.

We do not include external variables, such as those related to temperature. This is a point to draw attention to, as some temperature measures (maxima, averages, and others) could improve substantially the prediction if used, particularly in the summer, when the air conditioning appliances constitute great part of the load. The reasons for not using weather variables are threefold. First, as mentioned in the Introduction, the area covered by the electric utility considered in this paper represents 25% of the province of Rio de Janeiro, totalizing 11,132 km^2[2], which includes sub-regions with temperatures that range from 10 degrees Celsius during the winter to 24 degrees during the summer; as well as other sub-regions with temperatures that varies from 23 (winter) to 42 (summer) degrees Celsius. For example, in a given

[2]The total area of the Rio de Janeiro province is 43,910 km^2.

day and at the same time, it is common to observe two or more sub-regions with temperature differences around 10 degrees Celsius. However, the available hourly temperature measures are collected at few points in the city of Rio de Janeiro (not the province) and do not give a complete picture of the temperature profile of the covered area. Second, the available data have a large number of deficient observations, including outliers and missing values, which distort the results and do not bring any relevant contribution to the forecasting performance of the model [3]. Finally, it is well known, that the forecasting of hourly temperatures in tropical environments is not precise, specially for a few days ahead. All that said, we decide not to include the temperature in our model. Nevertheless, it should be mentioned that whenever available, including weather variables in the model is straightforward.

The model proposed in this paper is called the Two-Level Seasonal Autoregressive (TLSAR) model and is defined as:

Definition 1: The time series $L_{h,d}$ representing the load of the hour h, $h = 1, \ldots, 24$ and day d, $d = 1, \ldots, D$, D is the total number of observations, follow a Two-Level Seasonal AutoRegressive (TLSAR) model if

$$L_{h,d} = L^P_{h,d} + L^S_{h,d}, \tag{1}$$

where

$$L^P_{h,d} = \alpha_0 + \rho d + \sum_{r=1}^{H} \alpha_r \cos(\omega r d) + \beta_r \sin(\omega r d) + \sum_{i=1}^{K} \mu_i \delta_i \tag{2}$$

is the “potential load”,

$$L^S_{h,d} = \boldsymbol{\phi}' \mathbf{z}_{h,d} + \varepsilon_{h,d} \tag{3}$$

is the “irregular load”, $\alpha_r \cos(\omega r d) + \beta_r \sin(\omega r d)$ is known as the r^{th}harmonic, $\omega = 2\pi/365$, δ_i, $i = 1, \ldots, K$ are dummy variables identifying the days of the week, public holidays, special days, etc. α_0, ρ, α_r, β_r, $r = 1, \ldots, H$, and μ_i, $i = 1, \ldots, K$ are unknown parameters. The vector $\mathbf{z}_{h,d}$ is formed by constant and a subset of p lags of $L^S_{h,d}$, $\boldsymbol{\phi} \in \mathbb{R}^{p+1}$ is a vector of unknown parameters and $\varepsilon_{h,d}$ is an error term.

We make the following assumption about the error term.

Assumption 1: The sequence of random variables $\{\varepsilon_{h,d}\}$, $h = 1, \ldots, 24$, is drawn from a continuous (with respect to Lebesgue measure on the real line), positive everywhere density, and bounded in a neighborhood of 0. Furthermore, $\mathsf{E}(\varepsilon_{h,d}|\mathcal{F}_{d-1}) = 0$ and $\mathsf{E}(\varepsilon^2_{h,d}|\mathcal{F}_{d-1}) = \sigma^2_{h,d} < \infty, \forall\, t$, where $\mathcal{F}_{d-1}$ is the full information set at day $d-1$.

[3]In [27], the authors also considered Brazilian data and temperature was not included in the model.

Two points are important to stress. First, Assumption 1 is weak. As in [28] and [6], we do not assume that errors are Gaussian. The distribution of the errors may be skewed and leptokurtic. However, different from these authors, we allow for possible conditional heteroskedasticity as $\mathsf{E}(\varepsilon^2_{h,d}|\mathcal{F}_{d-1})$ is not assumed to be constant; see [29] for a discussion. Second, although not considered in this paper, the model for the “irregular load” may include, if available, other exogenous variables, such as, for example, hourly temperatures. In addition, the specification do not need to be linear. In this paper, we consider a simple linear autoregressive model because more complicated neural network model do not improve significantly the forecasting performance.

B. Modeling Strategy

As previously mentioned, the specification and estimation of the TLSAR model is divided in two steps. Summarizing, the estimation procedure is carried out as follows:

- For each hour, estimate α_0, ρ, α_r, β_r, and μ_i, $r = 1, \ldots, H$, and $i = 1, \ldots, K$ in (2) by ordinary least squares (OLS). The number of harmonics (H) is determined by minimizing the Schwarz Bayesian Information Criteria (SBIC) [24]. The number of dummies (K), representing the different types of days is kept fixed and equal to 15, as described in Table I.
- After estimating the “potential load”, we compute

TABLE I
TYPES OF DAYS USED IN THE TLSAR MODEL.

Code	Description
1	Sunday
2	Monday
3	Tuesday
4	Wednesday
5	Thursday
6	Friday
7	Saturday
8	Holiday (official or religious)
9	Working day after a holiday
10	Working day before a holiday
11	Working day between a holiday and a weekend
12	Saturday after a holiday
13	Work only during the mornings
14	Work only during the afternoons
15	Special holidays

the residuals $\widehat{L}^S_{h,d} = L_{h,d} - \widehat{L}^P_{h,d}$, where

$$\widehat{L}^P_{h,d} = \widehat{\alpha}_0 + \widehat{\rho} d + \sum_{r=1}^{H} \widehat{\alpha}_r \cos(\omega r d) + \widehat{\beta}_r \sin(\omega r d) + \sum_{i=1}^{K} \widehat{\mu}_i \delta_i.$$

- Again, using the SBIC, we select the best combination of lags in $\mathbf{z}_{h,d}$ in (3) among the first seven lags of the series. The autoregressive model is estimated by OLS and standard errors that are robust to heteroskedasticity are computed using White´s estimator [30]. Apart from the intercept, statistically insignificant lags are excluded from the model [4].

After a model is estimated, it is submitted to number of misspecification tests. First, we test for no remaining serial correlation in the residuals using the Ljung-Box test [31]. We also test for possible nonlinearities in the conditional mean, using the neural network test proposed in [32] with the heteroskedasticity correction discussed in [33]. This test is important to justify the linear specification of the "irregular component". The Engle´s [34] Lagrange Multiplier test for conditional heteroskedasticy is also considered. Conditional heteroskedasticity, if present, must be taken into account when computing confidence intervals. Finally, in order to verify if the trend is correctly modeled, we apply the Phillips-Perron unit root test to the estimated residuals. Any evidence of nonstationarity is an indication that the trend has been incorrectly modeled. Although misspecification testing is a standard procedure in time series econometrics, it has been neglected in most of the applications in short-term load forecasting.

In order to check the forecasting performance of the TLSAR model in forecasting, we consider a benchmark model as describe in Section IV.

IV. The Benchmark Model

The benchmark model considered in this paper is a Seasonal Integrated Autoregressive Moving Average (SARIMA) model with dummy variables to correct the effects of holidays and special days. Several authors use similar models as benchmarks; see, for example, [35] and [4] among others.

Definition 2: The time series $L_{h,d}$ representing the load of the hour h, $h = 1, \ldots, 24$ and day d, $d = 1, \ldots, D$, D is the total number of observations, follow a Dummy-Adjusted SARIMA (DASARIMA) model if

$$(1 - \phi_1 B)\ \overline{\Delta_7 \Delta_1 L_{h,d}} = (1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3)(1 + \beta B^7)\varepsilon_{h,d}, \quad (4)$$

[4]We consider the standard 5% significance level.

where

$$\overline{\Delta_7 \Delta_1 L_{h,d}} = \Delta_7 \Delta_1 L_{h,d} - \sum_{i=1}^{M} \alpha_i \delta_{i,d} - \sum_{i=1}^{M} \lambda_i \delta_{i,d-1} - \sum_{i=1}^{M} \gamma_i \delta_{i,d-7} \quad (5)$$

and $\Delta_j = (1 - B^j)$, $j = 1, 7$, B is the lag operator [5], θ_k, $k = 1, 2, 3$, β, α_i, λ_i, γ_i, $i = 1, \ldots, M$, are parameters, δ_i , $i = 1, \ldots, M$ are dummy variables identifying public holidays, special days, etc. $\varepsilon_{h,d}$ is a zero mean error term with finite second-moment.

The selection of lags is based on the analysis of the autocorrelation (ACF) and partial autocorrelation (PACF) functions and one the use of the SBIC. The ACF and PACF for each $\overline{\Delta_7 \Delta_1 L_{h,d}} =$, $h = 1, \ldots, 24$, $d = 1, \ldots, D$, are used to roughly identify the order the ARMA component, which are further refined with the SBIC. After estimating the models, the Ljung-Box autocorrelation test is used to verify model adequacy. This approach is the standard Box and Jenkins methodology in time series analysis; see [36] for a detailed discussion.

In Table II we illustrate the types of days included in the DASARIMA model. The classification of days is different from the one considered in the TLSAR methodology because the first and seventh-order differences in (5) remove the days-of-the-week effect. Consequently, we have to consider only the anomalous days, such as special holidays.

It is important to mention that the DASARIMA considers that the trend in the load series is stochastic instead of deterministic. This is a major difference between the DASARIMA and TLSAR models and may be of extreme importance as there is an apparent break in the trend in the out-of-sample period.

[5]The lag operator is defined as $B^j y_t = y_{t-j}$.

TABLE II
Types of days used in the DASARIMA model.

Code	Description
1	Weekdays (Sun, Mon, Tue, Wed, Thu, Fri, and Sat)
2	Holiday (official or religious)
3	Working day after a holiday
4	Working day before a holiday
5	Working day between a holiday and a weekend
6	Saturday after a holiday
7	Work only during the mornings
8	Work only during the afternoons
9	Special holidays

V. The Experiment

The experiment considered in this paper consists in computing from 1- to 7-days ahead, multi-step forecasts of the hourly electric load using both the TLSAR and DASARIMA models. Section V-A shows the specification and estimation results. Forecasting results and comparisons are described in Section V-B. All models are estimated in a computer with a Pentium V 2.2 GHz processor with 1 Gb of Ram memory and running Matlab. The computational time to specify and estimated all the 24 models are negligible, not being over 60 seconds.

A. *Specification and Estimation*

Table III shows, for each hour of the day, the estimated number of harmonics and the estimated parameters of the autoregressive model with their White´s standard-errors robust to heteroskedasticity. All autoregressive coefficients are significant at a 5% level. The table also shows the p-value of the Ljung-Box test for no error serial autocorrelation of order 7 [31]. It is clear that the errors are not serially correlated, which indicates correct specification of the lags. Although not shown in the Table, the Phillips-Perron test strongly rejects the null of nonstationarity (unit-roots) for all the series, indicating that the linear detrending has successfully removed the trend from all the 24 series [6]. The p-values of the neural network linearity test proposed in [32] with the heteroskedasticity correction discussed in [33] are also reported in Table III. At a 5% level, the null of linearity is rejected, although not strongly, only for hours 10, 13, and 14. When a 1% level is considered, there is no evidence of nonlinearity for any series, apart from hour 14. For those series, a neural network model is estimated with Bayesian regularization in conjunction with the Levenberg-Marquadt algorithm; see [37] and [38]. However, the forecasting results are inferior from the ones from the linear model, and are thus omitted for the sake of conciseness. A similar result has been reported in the literature by Darbellay and Slama [35]. The authors have found that the short-term evolution of the Czech electric load is primarily a linear problem. On the other hand, when conditional heteroskedasticity is tested using Engle´s ARCH LM test [34], the null hypothesis of homoskedasticity is strongly rejected for all series, indicating the presence of time-varying conditional variances. In terms of estimation and point forecasts, this is not a drawback. However, in order to compute confidence intervals for the future loads it is important to take the conditional heteroskedasticity into account.

[6]Detailed results can be obtained from the authors.

Figure 3 shows the sum of the estimated harmonics for each hour of the day. As can be seen, for most the hours only two harmonics are sufficient to model the annual pattern. Apart from hours 7, 8,14–15, 18–21, the annual pattern is rather clear. First, there is a "summer regime" that begins more or less in November and goes approximately until March. The "winter regime" starts in April and ends in July as the temperatures usually start to raise in August. However, the extremely high temperatures (over 30 degrees Celsius) are more common from November to March. For that reason, there is a "spring regime" starting approximately in August and ending in October. This pattern is clearer during the night, mostly because of the use of air-conditioning. Hours 18–20 are quite different because of several factors: Public lightning, daylight saving period, holidays, etc.

Table IV shows the estimated results for the DASARIMA model.

B. *Forecasting*

This section reports the forecasting results for both the TLSAR and DASARIMA models. One of the most used measure of forecasting accuracy in the load forecasting literature is the Mean Absolute Percentage Error (MAPE) (see [39] and [35]), which measures the proportionality between the error and the observed load. An important point deserves attention. Several authors (see, for example, [39]) achieve MAPEs as low as 2% when predicting the total daily load, but results of different models cannot be compared on different datasets because of the differences among load curves in different countries. For example, a load profile of a country with tropical weather, such as Brazil, is distinct from one like USA or United Kingdom. Hence, if different datasets are used, the same model(s) must be used, and the comparison should be made among data sets and not models. If the researcher wants to compare the performance of different models, the same data with the same forecasting period must be used.

As to the present dataset, Tables V–VII show the MAPEs for one- to seven-days-ahead for the years of 1999 and 2000, both for the TLSAR model and the DASARIMA benchmark specification. The bold figures indicate which model attains the lowest MAPE. By inspection of the tables, it is clear that the TLSAR model outperforms the benchmark for all hours when one-step-ahead forecasts are considered. The benchmark is better than the TLSAR model only during the night and when more than one-step-ahead forecasts are evaluated. The superiority of the proposed model over the DASARIMA specification is huge when the middle hours are analyzed. For example, consider hour 13 for 1999 (Table

TABLE III

ESTIMATED PARAMETERS FOR THE TLSAR MODEL.

Hour	Number of Harmonics	$\widehat{\phi_0}$	$\widehat{\phi_1}$	$\widehat{\phi_2}$	$\widehat{\phi_3}$	$\widehat{\phi_4}$	$\widehat{\phi_5}$	$\widehat{\phi_6}$	$\widehat{\phi_7}$	Ljung-Box	NN
1	2	−0.052 (2.443)	0.855 (0.023)	−0.137 (0.026)	–	0.046 (0.018)	–	–	0.057 (0.015)	0.982	0.318
2	2	−0.082 (2.321)	0.875 (0.023)	−0.148 (0.027)	–	0.044 (0.018)	–	–	0.058 (0.015)	0.936	0.226
3	2	−0.067 (2.174)	0.912 (0.023)	−0.181 (0.026)	–	0.051 (0.018)	–	–	0.052 (0.015)	0.943	0.456
4	2	−0.081 (2.080)	0.924 (0.022)	−0.191 (0.026)	–	0.056 (0.018)	–	–	0.049 (0.015)	0.922	0.322
5	2	−0.083 (1.991)	0.933 (0.022)	−0.199 (0.025)	–	0.060 (0.017)	–	–	0.047 (0.014)	0.702	0.306
6	2	−0.103 (1.886)	0.916 (0.022)	−0.175 (0.025)	–	0.054 (0.018)	–	–	0.053 (0.014)	0.791	0.145
7	5	−0.164 (1.773)	0.814 (0.022)	−0.090 (0.023)	–	0.047 (0.018)	–	–	0.068 (0.015)	0.995	0.068
8	3	−0.136 (1.770)	0.721 (0.018)	–	–	–	–	–	0.086 (0.014)	0.335	0.659
9	2	−0.140 (1.972)	0.660 (0.022)	–	–	–	–	–	0.115 (0.015)	0.972	0.424
10	2	−0.164 (2.092)	0.610 (0.024)	–	–	–	–	–	0.143 (0.016)	0.645	0.038
11	2	−0.211 (2.169)	0.577 (0.026)	–	–	–	–	–	0.156 (0.016)	0.988	0.094
12	2	−0.219 (2.135)	0.583 (0.026)	–	–	–	–	–	0.159 (0.016)	0.958	0.113
13	2	−0.224 (2.165)	0.592 (0.025)	–	–	–	–	–	0.154 (0.016)	0.911	0.027
14	3	−0.201 (2.302)	0.582 (0.034)	−0.004 (0.026)	0.042 (0.019)	–	–	–	0.138 (0.015)	0.940	0.001
15	3	−0.219 (2.402)	0.588 (0.033)	−0.007 (0.026)	0.045 (0.019)	–	–	–	0.130 (0.014)	0.884	0.133
16	2	−0.229 (2.363)	0.611 (0.023)	–	–	–	–	–	0.135 (0.015)	0.683	0.090
17	2	−0.267 (2.367)	0.555 (0.026)	–	–	0.047 (0.022)	–	–	0.136 (0.017)	0.972	0.146
18	1	−0.283 (2.167)	0.519 (0.023)	–	–	0.084 (0.024)	–	–	0.127 (0.017)	0.886	0.282
19	5	−0.318 (1.887)	0.597 (0.021)	–	–	0.072 (0.021)	–	0.043 (0.020)	0.092 (0.018)	0.410	0.393
20	6	−0.291 (1.672)	0.641 (0.017)	–	–	0.063 (0.016)	–	–	0.110 (0.014)	0.661	0.053
21	3	−0.215 (1.654)	0.706 (0.027)	−0.043 (0.026)	0.049 (0.018)	–	–	0.056 (0.018)	0.077 (0.018)	0.862	0.457
22	2	−0.210 (1.891)	0.784 (0.023)	−0.080 (0.023)	–	0.045 (0.017)	–	0.031 (0.020)	0.058 (0.019)	0.787	0.101
23	2	−0.185 (2.201)	0.825 (0.022)	−0.121 (0.023)	–	0.054 (0.017)	–	–	0.058 (0.015)	0.938	0.382
24	2	−0.163 (2.374)	0.858 (0.021)	−0.144 (0.024)	–	0.046 (0.017)	–	–	0.059 (0.015)	0.815	0.388

V). The MAPEs of the TLSAR model range from 2.86% to 3.56%, while the MAPEs of the DASARIMA model go from 5.04% to 15.72%. Considering the same hour for year 2000 (Table VI) the results do not differ much. The MAPEs of the TLSAR specification are between 3.8% and 4.6% while the ones from the benchmark are from 5.6% to 17.12%. This superiority of the TSLAR model is also confirmed by the average figures shown at the bottom of the tables. One interesting point is that during the peak hours (19-21) the TSLAR model attains its lowest MAPEs. Comparing the results between the years, we do not see a difference in the comparative performance between the models. However, the results in 2000 are slightly worse than the ones obtained by the same model in 1999, mainly because the linear trend is not re-estimated but seems to suffer a break in the former year, as explained before. Even so, the results are good and are qualitatively equal to the year of 1999. This shows that the TSLAR model is quite robust. It is important to note that when we speak of h-steps-ahead, we consider the sectional data and hence refer to days. As the primary data are hourly, one must interpret as 24h-steps-ahead, so that $1, 2, \ldots, 7$ daily steps ahead actually correspond to $24, 48, \ldots, 168$ hourly steps ahead. In practice, it would be interesting to use the model proposed here and the benchmark for the hours and time horizons in which each one fares better, or even in a combined way. However, forecast combination is beyond the scope of this paper. Confidence intervals may be computed taking into account the conditional heteroskedasticity. One way of proceeding is estimating a GARCH (Generalized Autoregressive Conditional Heteroskedastic) model [29]. Another option is to use a block bootstrap [40] or the stationary bootstrap [41] to resample the residuals.

Table VIII shows the MAPEs for each type of day. As can be seen, the forecast performance of the TSLAR model do not differ among different standard weekdays (from Sunday to Saturday). However, as expected, during anomalous days, such as Christmas, New Years-Eve, Carnival, etc., the MAPEs are higher. Table IX shows the results according to the month in order to check

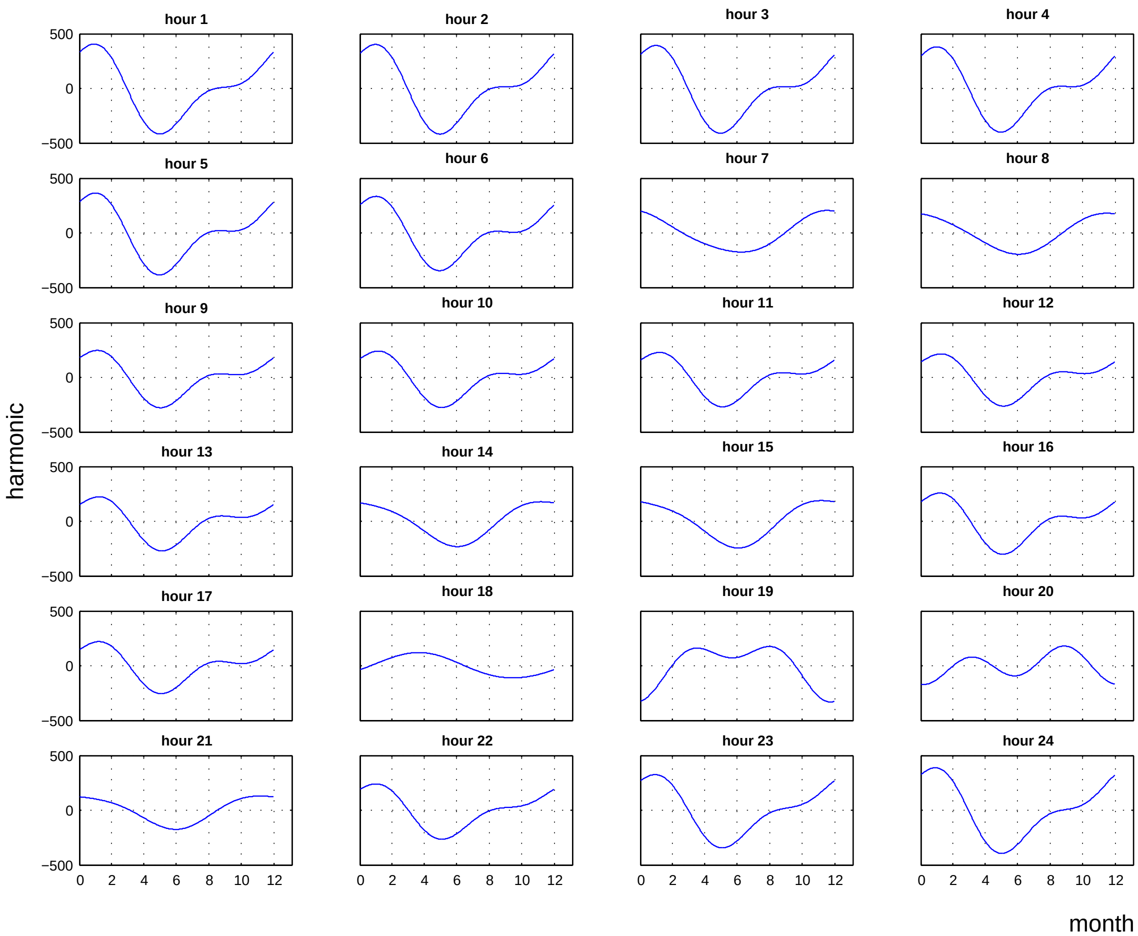

Fig. 3. Estimated harmonic shape for each hour.

if there are huge differences in performance depending on the time of the year. It is evident that the forecasts are worse during the warmer months, like January and February. Furthermore, the best performance is attained during the colder months (May–July).

Finally, comparing our results with the ones obtained by Soares and Souza [15], who estimate a generalized long memory model for the same dataset using a slightly different time span, our results compare favorably.

VI. Conclusions

In this paper we considered a two-level model for the hourly electric load demand from the area covered by a specific utility in the southeast of Brazil. This model applies to sectional data, that is, the load for each hour of the day is treated separately as a series. This model can be applied to other utilities presenting similar seasonal patterns, such as many in Brazil and other countries in Latin America and Africa. As previously discussed, weather variables were not considered in the model for three reasons: First, the area covered by the electric utility considered in this paper represents 25% of the province of Rio de Janeiro, which includes different sub-regions with completely distinct temperatures and the available hourly temperature measures do not cover all that diversity in temperature ranges. Second, the available dataset has a vast number of outliers and missing data. Finally, it is well known that hourly temperature forecasts in tropical environments are not reliable, specially for a few days ahead. A forecasting exercise against a specific class of Seasonal ARIMA models (the benchmark) is highly favorable to our proposal. This exercise included the entire years of 1999 and 2000, forecasting one to seven-days-ahead $(24, 48, \dots, 168$ hours ahead), using models estimated up to the end of 1998.

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TABLE IV
SIGNIFICANT PARAMETERS FOR THE DASARIMA MODEL.

hour	$\widehat{\mu}$	$\widehat{\phi_1}$	$\widehat{\theta_1}$	$\widehat{\theta_2}$	$\widehat{\theta_3}$	$\widehat{\beta}$
1	0.018 (0.315)	0.299 (0.080)	−0.318 (0.079)	−0.164 (0.018)	−0.076 (0.023)	−0.817 (0.011)
2	−0.008 (0.280)	0.307 (0.081)	−0.324 (0.080)	−0.165 (0.018)	−0.087 (0.024)	−0.823 (0.010)
3	−0.012 (0.271)	0.305 (0.082)	−0.284 (0.081)	−0.173 (0.018)	−0.098 (0.024)	−0.825 (0.010)
4	−0.008 (0.269)	0.282 (0.087)	−0.252 (0.086)	−0.174 (0.018)	−0.112 (0.025)	−0.819 (0.010)
5	−0.012 (0.275)	0.262 (0.089)	−0.219 (0.088)	−0.177 (0.018)	−0.114 (0.025)	−0.813 (0.011)
6	−0.007 (0.311)	0.261 (0.098)	−0.230 (0.097)	−0.149 (0.018)	−0.111 (0.025)	−0.787 (0.011)
7	0.013 (0.426)	–	−0.043 (0.147)	−0.094 (0.019)	−0.105 (0.024)	−0.726 (0.012)
8	0.034 (0.689)	–	−0.076 (0.286)	−0.041 (0.028)	−0.049 (0.022)	−0.623 (0.014)
9	0.080 (0.924)	–	−0.114 (0.363)	–	−0.033 (0.018)	−0.576 (0.014)
10	0.074 (1.097)	–	−0.143 (0.382)	–	–	−0.530 (0.015)
11	0.070 (1.098)	–	−0.157 (0.503)	–	–	−0.542 (0.015)
12	0.057 (1.073)	–	−0.133 (0.460)	–	–	−0.544 (0.015)
13	0.059 (0.989)	–	−0.135 (0.384)	−0.042 (0.054)	–	−0.567 (0.014)
14	0.053 (1.035)	–	−0.129 (0.292)	−0.064 (0.041)	–	−0.563 (0.015)
15	0.045 (1.064)	–	−0.122 (0.299)	−0.063 (0.041)	−0.035 (0.027)	−0.570 (0.014)
16	0.034 (1.019)	–	−0.125 (0.274)	−0.056 (0.039)	−0.053 (0.025)	−0.576 (0.014)
17	0.057 (0.889)	–	−0.179 (0.242)	−0.041 (0.047)	−0.055 (0.020)	−0.607 (0.014)
18	0.069 (0.719)	−0.548 (0.165)	0.301 (0.164)	−0.165 (0.046)	−0.080 (0.017)	−0.629 (0.014)
19	0.031 (0.617)	−0.476 (0.218)	0.264 (0.217)	−0.158 (0.050)	−0.070 (0.019)	−0.633 (0.014)
20	0.032 (0.480)	–	−0.167 (0.242)	−0.052 (0.044)	−0.065 (0.024)	−0.669 (0.013)
21	0.017 (0.423)	–	−0.131 (0.137)	−0.108 (0.026)	−0.062 (0.024)	−0.693 (0.013)
22	0.001 (0.387)	0.223 (0.104)	−0.300 (0.103)	−0.116 (0.020)	−0.076 (0.024)	−0.733 (0.012)
23	0.003 (0.342)	0.236 (0.084)	−0.292 (0.083)	−0.158 (0.019)	−0.091 (0.023)	−0.775 (0.011)
24	0.025 (0.369)	0.280 (0.081)	−0.316 (0.080)	−0.169 (0.019)	−0.080 (0.023)	−0.777 (0.011)

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TABLE V

FORECASTING COMPARISON FOR EACH HOUR OF THE DAY FOR 1999. MEAN ABSOLUTE PERCENTAGE ERROR FROM THE TLSAR AND SARIMA MODELS

Hour	TLSAR Forecasting horizon							SARIMA Forecasting horizon						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	**3.76**	5.12	5.67	**5.82**	5.95	6.07	6.18	3.93	**4.93**	**5.63**	5.84	**5.83**	**5.63**	**4.72**
2	**3.68**	5.17	5.74	5.93	6.06	6.19	6.33	3.85	**4.79**	**5.37**	**5.50**	**5.45**	**5.39**	**4.66**
3	**3.60**	5.06	5.67	5.89	5.96	6.10	6.25	3.77	**4.65**	**5.30**	**5.43**	**5.28**	**5.21**	**4.49**
4	**3.63**	4.93	5.54	5.76	5.87	6.01	6.15	3.72	**4.65**	**5.31**	**5.46**	**5.38**	**5.23**	**4.43**
5	**3.38**	4.55	5.14	5.37	5.47	5.62	5.73	3.47	**4.24**	**5.02**	**5.21**	**5.18**	**4.89**	**4.08**
6	**3.08**	**4.15**	**4.63**	**4.84**	**4.93**	**5.07**	5.17	3.34	4.38	5.34	5.93	5.97	5.42	**4.36**
7	**2.83**	**3.60**	**3.94**	**4.06**	**4.13**	**4.20**	**4.27**	3.69	6.26	8.26	9.13	9.20	8.42	6.32
8	**2.69**	**3.28**	**3.53**	**3.61**	**3.67**	**3.74**	**3.80**	4.09	8.43	11.04	11.8	11.99	11.38	8.38
9	**2.74**	**3.29**	**3.47**	**3.51**	**3.56**	**3.59**	**3.60**	4.63	10.33	13.18	13.76	13.98	13.62	10.05
10	**2.76**	**3.28**	**3.46**	**3.46**	**3.49**	**3.50**	**3.51**	4.91	11.38	14.42	14.89	15.07	14.84	11.05
11	**2.76**	**3.28**	**3.45**	**3.44**	**3.44**	**3.46**	**3.47**	4.95	11.81	15.19	15.48	15.67	15.57	11.40
12	**2.71**	**3.21**	**3.32**	**3.33**	**3.33**	**3.33**	**3.34**	5.02	12.06	15.61	15.81	15.93	15.96	11.58
13	**2.86**	**3.38**	**3.52**	**3.55**	**3.55**	**3.56**	**3.56**	5.04	11.69	15.32	15.64	15.72	15.50	11.19
14	**3.11**	**3.71**	**3.88**	**3.92**	**3.90**	**3.90**	**3.89**	5.10	12.01	15.91	16.14	16.20	16.04	11.45
15	**3.23**	**3.84**	**4.02**	**4.07**	**4.08**	**4.07**	**4.07**	5.16	12.45	16.78	17.08	17.16	16.89	11.79
16	**3.18**	**3.70**	**3.97**	**3.99**	**4.01**	**4.01**	**4.02**	5.11	12.40	16.92	17.33	17.34	16.93	11.70
17	**3.04**	**3.50**	**3.72**	**3.76**	**3.78**	**3.79**	**3.80**	4.82	11.57	16.16	16.57	16.64	16.11	10.97
18	**2.81**	**3.25**	**3.43**	**3.51**	**3.53**	**3.54**	**3.55**	4.27	8.97	11.95	12.46	12.56	12.19	8.65
19	**2.73**	**3.27**	**3.41**	**3.46**	**3.51**	**3.54**	**3.56**	3.54	6.04	7.55	8.02	8.01	7.89	6.12
20	**2.33**	**2.76**	**2.80**	**2.82**	**2.87**	**2.89**	**2.93**	3.07	4.90	5.63	5.98	5.94	5.77	4.88
21	**2.31**	**2.78**	**2.89**	**2.94**	**2.92**	**2.95**	**3.02**	2.85	4.47	5.44	5.84	5.82	5.44	4.42
22	**2.56**	**3.18**	**3.45**	**3.46**	**3.44**	**3.47**	**3.53**	3.08	4.85	6.21	6.73	6.70	6.09	4.76
23	**3.50**	**4.35**	**4.68**	**4.69**	**4.67**	**4.73**	**4.82**	4.19	5.66	6.96	7.43	7.33	6.85	5.47
24	**4.71**	**5.85**	**6.28**	**6.36**	**6.35**	**6.43**	6.56	5.42	6.60	7.57	7.93	7.95	7.54	**6.43**
Minimum	2.31	2.76	2.8	2.82	2.87	2.89	2.93	2.85	4.24	5.02	5.21	5.18	4.89	4.08
Average	3.08	3.85	4.15	4.23	4.27	4.32	4.38	4.21	7.9	10.09	10.48	10.51	10.20	7.64
Maximum	4.71	5.85	6.28	6.36	6.35	6.43	6.56	5.42	12.45	16.92	17.33	17.34	16.93	11.79

TABLE VI

FORECASTING COMPARISON FOR EACH HOUR OF THE DAY FOR 2000. MEAN ABSOLUTE PERCENTAGE ERROR FROM THE TLSAR AND SARIMA MODELS

Hour	TLSAR Forecasting horizon							SARIMA Forecasting horizon						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	**4.34**	6.58	7.51	7.95	8.22	8.30	8.33	4.53	**5.77**	**6.55**	**6.71**	**6.73**	**6.65**	**5.70**
2	**4.45**	6.70	7.70	8.20	8.50	8.60	8.62	4.53	**5.73**	**6.42**	**6.55**	**6.54**	**6.50**	**5.65**
3	**4.30**	6.56	7.67	8.20	8.50	8.60	8.63	4.44	**5.58**	**6.27**	**6.36**	**6.35**	**6.33**	**5.48**
4	**4.18**	6.34	7.49	7.98	8.30	8.40	8.45	4.35	**5.30**	**6.05**	**6.15**	**6.15**	**6.06**	**5.24**
5	**3.99**	6.18	7.24	7.72	8.03	8.14	8.18	4.24	**5.07**	**5.82**	**5.92**	**5.97**	**5.87**	**5.11**
6	**3.74**	5.66	6.61	7.06	7.31	7.41	7.48	4.14	**5.10**	**6.21**	**6.48**	**6.46**	**6.16**	**5.14**
7	**3.55**	**5.06**	**5.74**	**6.13**	**6.35**	**6.45**	**6.51**	4.47	6.93	8.68	9.12	9.16	8.72	6.57
8	**3.29**	**4.55**	**5.15**	**5.42**	**5.58**	**5.64**	**5.69**	4.70	9.08	11.41	12.08	12.13	11.58	8.61
9	**3.23**	**4.36**	**4.82**	**4.99**	**5.11**	**5.15**	**5.17**	4.92	10.69	13.48	14.28	14.33	13.86	10.25
10	**3.21**	**4.20**	**4.56**	**4.65**	**4.70**	**4.73**	**4.75**	5.24	12.01	15.02	15.80	15.85	15.52	11.48
11	**3.20**	**4.12**	**4.44**	**4.52**	**4.57**	**4.59**	**4.60**	5.39	12.69	16.11	16.81	16.79	16.53	11.99
12	**3.27**	**4.06**	**4.41**	**4.47**	**4.51**	**4.52**	**4.53**	5.60	12.78	16.47	17.14	17.12	16.84	12.10
13	**3.28**	**4.17**	**4.48**	**4.54**	**4.57**	**4.59**	**4.59**	5.40	12.56	16.27	16.94	16.93	16.49	11.71
14	**3.52**	**4.50**	**4.83**	**4.90**	**4.93**	**4.93**	**4.94**	5.55	12.85	16.80	17.65	17.60	17.09	12.11
15	**3.61**	**4.68**	**5.08**	**5.15**	**5.17**	**5.18**	**5.18**	5.74	13.29	17.70	18.40	18.45	17.89	12.41
16	**3.61**	**4.72**	**5.11**	**5.22**	**5.26**	**5.26**	**5.26**	5.77	13.17	17.77	18.47	18.57	17.90	12.29
17	**3.50**	**4.42**	**4.71**	**4.75**	**4.79**	**4.78**	**4.79**	5.39	12.19	16.65	17.26	17.39	16.63	11.25
18	**3.29**	**4.04**	**4.32**	**4.42**	**4.47**	**4.48**	**4.48**	4.79	9.24	12.18	12.66	12.68	12.09	8.67
19	**3.15**	**3.77**	**4.12**	**4.26**	**4.31**	**4.32**	**4.33**	4.03	6.43	7.81	8.13	8.14	7.80	6.18
20	**2.85**	**3.48**	**3.75**	**3.82**	**3.91**	**3.93**	**3.95**	3.47	5.15	5.74	5.95	6.05	5.80	4.99
21	**2.72**	**3.58**	**3.93**	**4.09**	**4.18**	**4.22**	**4.23**	3.17	4.89	5.67	5.84	5.95	5.69	4.56
22	**3.04**	**4.22**	**4.76**	**5.00**	**5.13**	**5.18**	**5.19**	3.36	5.30	6.61	6.92	6.90	6.56	4.92
23	**3.68**	**5.26**	**5.88**	**6.21**	**6.39**	**6.43**	6.43	3.99	5.64	6.95	7.28	7.24	6.79	**5.30**
24	**4.26**	6.08	6.96	7.38	7.62	7.67	7.70	4.50	**5.79**	**6.78**	**7.09**	**6.98**	**6.70**	**5.59**
Minimum	2.72	3.48	3.75	3.82	3.91	3.93	3.95	3.17	4.89	5.67	5.84	5.95	5.69	4.56
Average	3.55	4.89	5.47	5.71	5.85	5.90	5.92	4.65	8.47	10.64	11.08	11.10	10.75	8.05
Maximum	4.45	6.70	7.70	8.20	8.50	8.60	8.63	5.77	13.29	17.77	18.47	18.57	17.90	12.41

TABLE VII

FORECASTING COMPARISON FOR EACH HOUR OF THE DAY FOR 1999 AND 2000. MEAN ABSOLUTE PERCENTAGE ERROR FROM THE TLSAR AND DASARIMA MODELS

Hour	*TLSAR* Forecasting horizon							*DASARIMA* Forecasting horizon						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	**4.05**	5.85	6.59	6.89	7.09	7.19	7.26	4.23	**5.35**	**6.09**	**6.27**	**6.28**	**6.14**	**5.21**
2	**4.07**	5.93	6.72	7.07	7.28	7.40	7.47	4.19	**5.26**	**5.90**	**6.03**	**5.99**	**5.95**	**5.16**
3	**3.95**	5.81	6.67	7.04	7.23	7.35	7.44	4.10	**5.12**	**5.79**	**5.90**	**5.82**	**5.77**	**4.99**
4	**3.91**	5.64	6.51	6.87	7.09	7.21	7.30	4.04	**4.97**	**5.68**	**5.80**	**5.77**	**5.65**	**4.83**
5	**3.69**	5.37	6.19	6.54	6.75	6.88	6.96	3.85	**4.66**	**5.42**	**5.56**	**5.57**	**5.38**	**4.59**
6	**3.41**	4.91	**5.62**	**5.95**	**6.12**	6.24	6.33	3.74	**4.74**	5.77	6.21	6.21	**5.79**	**4.75**
7	**3.19**	**4.33**	**4.84**	**5.10**	**5.24**	**5.33**	**5.39**	4.08	6.59	8.47	9.12	9.18	8.57	6.45
8	**2.99**	**3.92**	**4.34**	**4.51**	**4.62**	**4.69**	**4.75**	4.39	8.76	11.23	11.94	12.06	11.48	8.50
9	**2.99**	**3.82**	**4.14**	**4.25**	**4.33**	**4.37**	**4.39**	4.78	10.51	13.33	14.02	14.16	13.74	10.15
10	**2.99**	**3.74**	**4.01**	**4.06**	**4.10**	**4.12**	**4.13**	5.08	11.70	14.72	15.35	15.46	15.18	11.27
11	**2.98**	**3.70**	**3.94**	**3.98**	**4.01**	**4.03**	**4.03**	5.17	12.25	15.65	16.15	16.23	16.05	11.70
12	**2.99**	**3.64**	**3.86**	**3.90**	**3.92**	**3.93**	**3.94**	5.31	12.42	16.04	16.47	16.53	16.40	11.84
13	**3.07**	**3.78**	**4.00**	**4.05**	**4.06**	**4.07**	**4.08**	5.22	12.12	15.79	16.29	16.33	16.00	11.45
14	**3.32**	**4.11**	**4.36**	**4.41**	**4.42**	**4.42**	**4.42**	5.33	12.43	16.36	16.90	16.90	16.56	11.78
15	**3.42**	**4.26**	**4.55**	**4.62**	**4.63**	**4.62**	**4.63**	5.45	12.87	17.24	17.74	17.80	17.39	12.10
16	**3.39**	**4.21**	**4.54**	**4.61**	**4.63**	**4.64**	**4.64**	5.44	12.78	17.35	17.90	17.95	17.42	11.99
17	**3.27**	**3.96**	**4.21**	**4.26**	**4.29**	**4.29**	**4.30**	5.10	11.88	16.40	16.92	17.01	16.37	11.11
18	**3.05**	**3.64**	**3.88**	**3.96**	**4.00**	**4.01**	**4.02**	4.53	9.11	12.07	12.56	12.62	12.14	8.66
19	**2.94**	**3.52**	**3.77**	**3.86**	**3.91**	**3.93**	**3.95**	3.79	6.23	7.68	8.07	8.07	7.84	6.15
20	**2.59**	**3.12**	**3.28**	**3.32**	**3.39**	**3.41**	**3.44**	3.27	5.03	5.69	5.96	5.99	5.78	4.93
21	**2.51**	**3.18**	**3.41**	**3.51**	**3.55**	**3.59**	**3.62**	3.01	4.68	5.56	5.84	5.89	5.56	4.49
22	**2.80**	**3.70**	**4.10**	**4.23**	**4.28**	**4.33**	**4.36**	3.22	5.08	6.41	6.82	6.80	6.33	4.84
23	**3.59**	**4.80**	**5.28**	**5.46**	**5.54**	**5.58**	5.63	4.09	5.65	6.95	7.36	7.29	6.82	**5.39**
24	**4.48**	**5.97**	**6.62**	**6.87**	**6.98**	**7.05**	7.13	4.96	6.19	7.17	7.51	7.46	7.12	**6.01**
Minimum	2.51	3.12	3.28	3.32	3.39	3.41	3.44	3.01	4.66	5.42	5.56	5.57	5.38	4.49
Average	3.32	4.37	4.81	4.97	5.06	5.11	5.15	4.43	8.18	10.37	10.78	10.81	10.48	7.85
Maximum	4.48	5.97	6.72	7.07	7.28	7.40	7.47	5.45	12.87	17.35	17.90	17.95	17.42	12.10
1999														
Minimum	2.31	2.76	2.8	2.82	2.87	2.89	2.93	2.85	4.24	5.02	5.21	5.18	4.89	4.08
Average	3.08	3.85	4.15	4.23	4.27	4.32	4.38	4.21	7.9	10.09	10.48	10.51	10.20	7.64
Maximum	4.71	5.85	6.28	6.36	6.35	6.43	6.56	5.42	12.45	16.92	17.33	17.34	16.93	11.79
2000														
Minimum	2.72	3.48	3.75	3.82	3.91	3.93	3.95	3.17	4.89	5.67	5.84	5.95	5.69	4.56
Average	3.55	4.89	5.47	5.71	5.85	5.90	5.92	4.65	8.47	10.64	11.08	11.10	10.75	8.05
Maximum	4.45	6.70	7.70	8.20	8.50	8.60	8.63	5.77	13.29	17.77	18.47	18.57	17.90	12.41

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TABLE VIII

FORECASTING COMPARISON FOR EACH TYPE OF DAY.

	1999 and 2000						
	Forecasting horizon						
Day	1	2	3	4	5	6	7
Sunday	3.27	4.02	4.60	4.81	4.77	4.77	4.77
Monday	3.93	4.71	4.71	4.94	5.12	5.18	5.20
Tuesday	2.64	4.55	4.84	4.84	4.96	5.09	5.13
Wednesday	2.71	3.63	4.55	4.64	4.65	4.75	4.83
Thursday	3.11	4.13	4.62	5.01	5.08	5.05	5.16
Friday	2.84	4.05	4.51	4.71	4.93	4.94	4.96
Saturday	3.03	4.07	4.59	4.69	4.78	4.83	4.85
Others	5.21	6.03	6.23	6.25	6.35	6.45	6.46
minimum	2.64	3.63	4.51	4.64	4.65	4.75	4.77
average	3.34	4.40	4.83	4.99	5.08	5.13	5.17
maximum	5.21	6.03	6.23	6.25	6.35	6.45	6.46

	1999						
	Forecasting horizon						
Day	1	2	3	4	5	6	7
Sunday	3.10	3.53	4.02	4.02	3.94	3.92	3.96
Monday	3.26	3.88	3.85	4.06	4.08	4.15	4.17
Tuesday	2.29	3.75	3.78	3.79	3.94	4.02	4.07
Wednesday	2.55	3.13	3.87	3.89	3.85	3.95	4.03
Thursday	3.16	3.95	4.24	4.55	4.61	4.58	4.70
Friday	2.80	3.75	4.05	4.18	4.33	4.40	4.42
Saturday	2.85	3.62	3.86	3.93	3.91	3.93	3.99
Others	4.56	4.71	4.89	4.80	4.84	5.03	5.14
minimum	2.29	3.13	3.78	3.79	3.85	3.92	3.96
average	3.15	3.96	4.24	4.33	4.38	4.42	4.47
maximum	4.56	4.71	4.89	4.80	4.84	5.03	5.14

	2000						
	Forecasting horizon						
Day	1	2	3	4	5	6	7
Sunday	3.46	4.52	5.21	5.64	5.64	5.65	5.62
Monday	4.60	5.54	5.56	5.82	6.16	6.20	6.24
Tuesday	2.96	5.30	5.84	5.83	5.91	6.10	6.13
Wednesday	2.87	4.13	5.23	5.40	5.46	5.54	5.63
Thursday	3.06	4.33	5.00	5.49	5.57	5.55	5.64
Friday	2.88	4.33	4.95	5.23	5.51	5.47	5.49
Saturday	3.19	4.47	5.24	5.38	5.57	5.63	5.62
Others	6.02	8.56	8.26	8.29	8.30	8.39	8.47
minimum	2.87	4.13	4.95	5.23	5.46	5.47	5.49
average	3.53	4.83	5.41	5.63	5.77	5.82	5.85
maximum	6.02	8.56	8.26	8.29	8.30	8.39	8.47

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TABLE IX

FORECASTING COMPARISON FOR EACH MONTH.

	1999 and 2000						
	Forecasting horizon						
Month	1	2	3	4	5	6	7
Jan	3.58	4.69	5.12	5.28	5.28	5.30	5.34
Feb	3.20	4.34	4.76	5.03	5.17	5.14	5.08
Mar	3.60	4.80	5.48	5.71	5.86	5.92	5.93
Apr	3.06	4.23	4.64	4.66	4.62	4.70	4.74
May	2.72	3.61	4.09	4.25	4.37	4.48	4.55
Jun	2.29	2.80	2.95	3.06	3.09	3.05	3.04
Jul	2.80	3.63	3.82	3.84	3.90	3.91	3.93
Aug	3.31	4.53	5.21	5.53	5.73	5.89	6.02
Sep	3.49	4.66	5.25	5.47	5.61	5.69	5.76
Oct	3.97	5.11	5.57	5.82	5.97	6.10	6.19
Nov	3.82	4.83	5.11	5.28	5.46	5.57	5.66
Dec	3.90	5.20	5.65	5.64	5.60	5.50	5.47
Minimum	2.29	2.80	2.95	3.06	3.09	3.05	3.04
Average	3.31	4.37	4.80	4.96	5.05	5.10	5.14
Maximum	3.97	5.20	5.65	5.82	5.97	6.10	6.19

	1999						
	Forecasting horizon						
Month	1	2	3	4	5	6	7
Jan	3.20	3.83	3.91	3.90	3.88	3.96	4.10
Feb	2.76	3.25	3.32	3.62	3.80	3.81	3.85
Mar	3.89	4.92	5.51	5.61	5.72	5.80	5.79
Apr	2.78	3.74	4.02	4.00	3.88	3.96	4.04
May	2.55	3.07	3.45	3.49	3.52	3.66	3.77
Jun	2.57	2.89	2.91	2.93	2.97	2.96	2.96
Jul	2.49	3.10	3.03	2.90	2.87	2.89	2.90
Aug t	2.86	3.58	3.75	3.79	3.82	3.86	3.88
Sep	3.11	4.16	4.68	4.77	4.72	4.79	4.87
Oct	3.73	4.74	5.21	5.52	5.71	5.86	5.99
Nov	3.87	4.97	5.45	5.73	5.94	6.11	6.29
Dec	3.16	3.90	4.38	4.38	4.28	4.11	4.06
Minimum	2.49	2.89	2.91	2.90	2.87	2.89	2.90
Average	3.08	3.84	4.13	4.22	4.26	4.31	4.37
Maximum	3.89	4.97	5.51	5.73	5.94	6.11	6.29

	2000						
	Forecasting horizon						
Month	1	2	3	4	5	6	7
Jan	3.96	5.55	6.32	6.66	6.68	6.65	6.58
Feb	3.62	5.40	6.15	6.40	6.50	6.42	6.27
Mar	3.32	4.68	5.45	5.80	6.00	6.03	6.07
Apr	3.34	4.71	5.25	5.32	5.37	5.45	5.45
May	2.89	4.15	4.72	5.02	5.22	5.30	5.33
Jun	2.02	2.72	2.99	3.19	3.20	3.15	3.12
Jul	3.12	4.15	4.60	4.78	4.93	4.93	4.96
Aug	3.77	5.48	6.67	7.27	7.64	7.92	8.17
Sep	3.88	5.16	5.83	6.17	6.49	6.58	6.64
Oct	4.21	5.48	5.94	6.12	6.23	6.33	6.40
Nov	3.77	4.70	4.76	4.82	4.98	5.02	5.04
Dec	4.64	6.49	6.92	6.90	6.92	6.89	6.89
Minimum	2.02	2.72	2.99	3.19	3.20	3.15	3.12
Average	3.55	4.89	5.47	5.70	5.85	5.89	5.91
Maximum	4.64	6.49	6.92	7.27	7.64	7.92	8.17

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