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Multiplier analysis\*

Edward Joaquim Amadeo



PUC-Rio – Departamento de Economia

[www.econ.puc-rio.br](http://www.econ.puc-rio.br)

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## Abstract

The paper studies the multiplier mechanism from an analytical perspective. It emphasizes the notions of equilibrium and stability associated with the functioning of the multiplier. It studies the role of producers' expectations during the adjustment process, the relationship between the multiplier and income distribution, and the constraints to the workings of the mechanism.

## Resumo

O trabalho estuda o mecanismo do multiplicador a partir de uma perspectiva analítica. Enfatizamos as noções de equilíbrio e estabilidade associadas ao funcionamento do multiplicador. Estudamos o papel das expectativas dos produtores ao longo do processo de ajuste, a relação entre o multiplicador e a distribuição da renda, e os limites para o funcionamento do mecanismo.

## Multiplier analysis

What is the effect of a change in the level of investment? Wicksell (1935) was the first economist to explicitly pose this question in the context of his “pure credit economy”. Voluntary or anticipated saving is not a requirement if the banking system is willing to supply the necessary credit to finance an increase of investment demand. The effect of this increase of investment demand is an increase in the level of prices (if the level of output is fixed or given), or output if there is idle capacity and unemployed labor.

In his *Treatise on Money* (1930), Keynes analyses the same question. Just as in Wicksell’s model, in the *Treatise*, investment is independent from current saving. The effects of a change of investment are studied through the *Treatise’s* ‘fundamental equations’ according to which a difference between current (or voluntary) saving and investment will give rise to a change in the price level. It is a pure excess demand effect. Changes in the price level will lead to unforeseen (or windfall) profits or losses which, in turn, will affect producers’ next period decision to produce and employ. Windfall profits will have the effect of inducing producers to increase the level of output losses will have the opposite effect. The effect may not be as mechanical as described here if new informations (concerning, e. g., changes in economic policies) come into the picture.

Book IV of the *Treatise* studies the ‘credit cycle’, that is, the effects of changes in monetary or banking policies or the rate of interest which may have an effect on the decisions to save and invest, and therefore, on the price and output levels. Changes in both the price and output levels are seen as deviations from their long-period or equilibrium counterparts: they are short-period or disequilibrium levels of price and output which, so to speak, oscillate around the equilibrium as defined by the equality between voluntary saving and investment. However, Just as in Wicksell’s analysis, once the system deviates from the equilibrium position, very little is said in terms of the path towards a new equilibrium; indeed, the latter is not really determined.

Multiplier analysis is very much related to the adjustment process described above. The real *diferentia* is that it focuses predominantly on the notions of stability and equilibrium of the process. The most important contributors for the development of the multiplier analysis were Kahn (1931), Keynes (1936) and Kalecki (1971).

### The Multiplier as an exercise on statics

Let us consider the effects of a change in the level of investment which is known to all the relevant agents of the economy. Also let us temporarily assume that producers of consumption goods fully anticipate the effects of this change in investment on the demand for their products. An

increase in the level of investment demand implies a greater level of production of capital goods. The degrees of capacity utilization and employment in the capital goods sector increase, thus leading to higher profits and a greater wage bill. Part of the extra profits and wages earned will be spent in consumption goods: the rest will be saved. The share of profits and wages spent in consumption goods are determined respectively by the propensities to consume out of profits and wages. These, according to Keynes (1936, chs. 8 and 9), depend on objective factors (other than income) such as the money wage rate and agents' rates of time-discounting, and subjective factors such as precaution and avarice.

Thus, the main effect of an increase in investment is that it induces an increase in consumption, saving, and income. The final effect on the level of income will depend essentially on the propensity to consume of the economy. The greater the propensity to consume, the greater will be the increase in the demand for consumption goods resulting from an initial increase in the income generated in the capital goods sector. The intermediate effect on the demand for consumption goods will be given by  $\Delta C = c\Delta I$ , where  $C$  and  $I$  are respectively the levels of consumption and investment, and  $c$  is the weighted average of the propensities to consume out of profits and wages. The immediate effect on the level of income will be given by  $\Delta Y = \Delta I + c\Delta I$ . Note that a second round of the multiplier process will lead to an increase in the level of income given by  $\Delta Y = \Delta I + c\Delta I + c^2\Delta I$ . After an infinite number of rounds the effect will be given by  $Y = \Delta I + c\Delta I + c^2\Delta I + \dots = [1/(1 - c)]\Delta I$ . The term  $1/(1 - c)$  is called the investment multiplier. According to Keynes, the multiplier "tells us that, when there is an increment of aggregate investment, income will increase by an amount which is  $1/(1 - c)$  times the increment in investment" (Keynes, 1936, p 115).

Note that the change in the level of saving ( $\Delta S$ ) is given by the propensity to save ( $s = 1 - c$ ) times the level of income, that is,  $\Delta S = s\Delta Y$ , which, according to the above analysis, is also equal to the initial change in the level of investment. Thus, through the multiplier mechanism, a change in the level of investment gives rise to an equal level of saving. The multiplier is essentially an equilibrating mechanism. It refers to the adjustment of the economy given an exogenous change, and it determines the equilibrium levels of income and saving associated to different levels of investment demand. It describes the changes in the level of consumption which eventually makes the latter compatible to each level of investment given the propensity to consume of the economy.

The essential difference between the multiplier mechanism and the description of credit cycles found in Keynes's *Treatise on money* as well as in the analysis of Wicksell and the Swedish economists (Ohlin and Lindhal, for example), is that it emphasizes the notion of equilibrium. It determines the new equilibrium configuration associated with any change in the level of investment demand rather than only its immediate effects. Because it is an equilibrating mechanism it must

also take into account the stability conditions of the process. In terms of the simple static version discussed above, the only stability condition is that the propensity to consume must be smaller than one. If it was greater than one the system would always explode either to a situation of full employment or zero-employment of the labor force and capacity utilization. As noted by Keynes, “if the [community] seek to consume the whole of any increment in income, there will be no point of stability and prices will rise without limit”. (Keynes, 1936, p. 117) However, since the propensity to consume is always positive, the multiplier is always greater than one which implies that fluctuations in investment will lead to fluctuations of income of greater magnitude. Thus, the workings of the multiplier mechanism itself may be regarded as a source of instability.

### The Multiplier as an exercise on dynamics

What makes the analysis of the above section static is the fact that it emphasizes the equilibrium configuration associated with a given (and known) level of investment, and a given propensity to consume. The decision to consume is rather passive and taking it into account does not really make the analysis dynamic. What is most important, however, is that the decisions to produce are not considered. Production takes time, and therefore decisions to produce involve expectations over a period of time. A dynamic approach to the analysis of the multiplier should emphasize the role of time and expectations associated with the decisions to produce.

What is the appropriate time-unit for the analysis of the multiplier process if decisions to produce are to be explicitly considered? Following Keynes, we shall take the short-period as the appropriate time-unit. The short-period is associated with ‘daily’ decisions, and daily here stands “for the shortest interval of time after which the firm is free to revise its decisions as to how much employment to offer”. (Keynes, 1936, p. 47) Producers make their decisions as to how much to produce based on their short-period expectations.

On the demand side the object of such expectations are either the expected sale-proceeds or the expected price, that is, the price which the producer expects to get for his product at the period of production. Let us take the expected price as the relevant variable, and assume that the producer knows the remuneration rates of the variable inputs and the shape of his cost curve. Given these informations we may assume that the producer goes through the following optimization exercise in order to determine the levels of output and employment:  $\max E[p]X - wN$  st.  $X = F(N, K)$  where  $E[p]$  is the expected price,  $X$  and  $N$  are the levels of output and employment respectively,  $w$  is the money-wage rate,  $K$  is the stock of capital (assumed to be given), and  $F$  is a production function. The level of employment associated with the expected price must satisfy the following condition:  $w/E[p] = F'(N^*)$ . The level of output is obviously  $X^* = F(N^*)$ .

Let us assume that the level of investment has been stable for a rather long period of time. Producers of consumption goods know not only the level of investment but also the demand for their products associated with this level. Therefore, they are able to form correct expectations concerning the demand for their products and their price. In short, in each and every period the expected price corresponds to the market price, i.e.,  $E[p] = p$ . We now let the level of investment increase, but assume that the producers of consumption goods either do not know that the change has taken place or the effect of the change on the demand for their products. If the latter is the case, assume that they underestimate the effect on demand. In either case the actual price will be greater than the expected price associated with the predetermined level of output ( $X^*$ ), that is,  $p > E[p]$  where  $p$  is the market price. In this example producers will experience a windfall profit given by  $Q = (p - E[p])X^*$ . The same exercise could be carried on taking stocks rather than the price as the adjustment variable (see Hicks, 1974, ch. 1).

The process initiated with a change in investment demand could go on for a long period. Producers would continue to get their expectations wrong, profits or losses would appear, new decisions would be taken and so on. Will producers ever get their prices (and production decisions) right? If we assume that the level of investment will not be affected by changes in short-period expectations, and depending on the way producers form their expectations, they will eventually converge to an equilibrium position. If, for example, producers form their short-period expectations in an adaptive fashion, for certain values of the parameters of the expectation function, the system will converge to a position of rest. For other values of the parameters the system will not converge. This only implies that the way producers form their expectations may affect the stability of the multiplier process and the trajectory of the relevant variables.

Does the way producers form their expectations affect the equilibrium configuration? The answer here is no, if the level of investment is assumed to be given and the process is assumed to be stable (which, again, depends on the parameters of the expectation function), the equilibrium configuration will be exactly the same as the one associated with a process in which producers form their expectations in a rational fashion. By rational we mean that expectations are recurrently correct, Keynes was aware of this result: in his lectures notes written in 1937 he argued that his principle of effective demand is substantially the same independently of the way expectations are formed (see Keynes, 1973, pp. 180-1).

### The Multiplier and the notion of 'Shifting Equilibrium'

So far we have examined the multiplier mechanism assuming that either the level or the expected level of investment is given. In both the static and dynamic analyses the multiplier tells us

the levels of income and saving compatible with a given level or expected level of investment. The advantage of these approaches to the multiplier is that they emphasize the notion of equilibrium, that is, they provide a definite result to the effect of a change in investment.

However, once the notion of equilibrium has become clear, we should turn our attention to the interactive relation between the level of investment and the workings of the multiplier. The level of investment is quite a volatile variable. Long-period expectations (which play a central role in the determination of the level of investment) change for various reasons. They change due to changes in the political or international environments; due to changes in economic policies; or due to objective problems of individual industries which tend to affect the expected performance of other industries of the economy. To different states of long-period expectations there corresponds different levels of investment and, therefore, different “levels of long-period employment” (Keynes, 1936, p. 48). The extent to which short-period expectations are fulfilled may also affect the level of investment. If the actual demand is persistently greater than the expected demand, producers will tend to revise their long-period expectations and investment decisions.

We may associate the notion of “shifting equilibrium” with the evolution of the economic system as determined by different states of long-period expectations and, therefore, characterized by a sequence of equilibrium configurations of income and saving. By shifting equilibrium Keynes meant “the theory of a system in which changing views about the future are capable of influencing the present situation” (1936, p. 293).

#### Price and quantity adjustments

The standard view of the evolution of Keynes’s theory of employment is that in the *Treatise on Money* only the price level adjusts to changes in investment whereas in the *General Theory* only output and employment take part in the adjustment. As mentioned already, in Book IV of the *Treatise* both the levels of output and price adjust; also in the *General Theory* both do the adjustment. The central difference between the two analysis is that in the former there is not any discussion of the new equilibrium associated with the new level of investment; in the latter, due to the introduction of the multiplier mechanism, equilibrium is greatly emphasized.

Let us concentrate on the *General Theory*. In the short-period when expectations associated with the level of demand can be falsified, either the price level or the level of stocks may adjust. If the supply of goods is fixed in the short-period, prices will adjust to any discrepancy between supply and demand. If firms keep stocks, part of the adjustment may take place through changes in stocks. Both types of adjustments are considered in the *General Theory* (Keynes, 1936, p.123-4).

In the multiplier period, that is, after the multiplier mechanism has come to an end, both the

levels of output and price can adjust. It all depends on the shape of the cost curves. Kahn is quite clear about this point: “The rise in prices... is a natural concomitant of increased output, to a degree indicated by the slope of the supply curve” (1931, p. 7). If constant marginal returns are assumed only the levels of output and employment will adjust: if, on the contrary, the economy is operating at full capacity, only the price level will adjust. In the intermediary situation where decreasing marginal returns prevails both the levels of price, output and employment will change given a change in investment.

### Distribution and the Multiplier

The relationship between the distribution of income (or the real wage) and the multiplier depends on the assumptions about the exogeneity or endogeneity of the real wage. In the *General Theory*, Keynes assumed perfect competition cum profit maximization and decreasing marginal returns which, for a given money-wage rate, implies that the real wage is endogenously determined. It also implies that the greater the levels of employment and output, the smaller the real wage. This result has an important implication for the workings of the Keynesian multiplier. If we assume – as Keynes and Kalecki usually do – that the propensity to consume out of wages is greater than the propensity to consume out of other types of incomes (profits, interests etc.), as the level of income increases and the real wage falls, the value of the multiplier decreases. Keynes pointed out to this result in the *General Theory*: “the increase of employment will tend, owing to the effect of diminishing returns, ... to increase the proportion of aggregate income which accrues to the entrepreneurs, whose ... propensity to consume is probably less than the average for the community as a whole” (1936, p. 121).

Kalecki (1971) assumed constant marginal returns and gave up profit maximization. Instead he assumed that firms determine their prices through a mark-up over variable costs which, in a closed economy, also determines the real wage. Therefore, according to Kalecki, the real wage is exogenously determined, and does not change as the levels of output and employment change. This means that the multiplier does not change either as the level of output changes; it depends on the propensity to consume out of wages and profits and the level of the mark-up, both assumed to be constant over the cycle.

### Constraints to the workings of the Multiplier

There are two central constraints to the workings of the capacity in the consumption goods sector and/or the existence of unemployed workers in the economy. The second is associated with

the availability of finance. Changes in output and employment as a result of an increase in investment will take place only if there is idle capacity in the consumption goods industries and unemployed workers. If one of the two requirements is not satisfied the supply of consumption goods becomes inelastic, and only prices will adjust to an increase in the demand for goods. If this is the case, the increase in saving which necessarily follows from an increase in investment will result from the reduction of purchasing power of those with incomes denominated in nominal terms. Assume that money-wages are fixed in the short-period. If only prices adjust to the increase in aggregate demand, real wages as well as the share of wages in income will fall. The price level (rather than the levels of output and employment) will play the role of adjustment variable in this 'quantity-constrained' multiplier process.

Saving is not required to finance an increase in the level of investment demand. Instead of using retained profits or any other type of saved income, firms may prefer to demand credit from the banking system. If the banks are not operating at their reserve requirements they may provide credit to those firms which may need to pay for the purchase of new machines, and to those firms which are in the process of starting the production of new machines.

If banking credit is not available firms may choose to issue bonds. However, if there is not any change in the supply of money and the state of liquidity preference of the community remains stable, an increase in the supply of bonds will drive their prices down and the rate of interest up with deleterious effects on the level of investment. Firms may also choose to use retained profits to finance investment; if they are able to do that, this option is clearly independent of the availability of credit.

Thus, the finance for investment expenditures comes from either of the following sources: saving, credit, or the issuing of new bonds. What is important to note, however, is that saving is not the only source of finance. Indeed, investment may also be financed by credit money if the banking system is willing to accommodate changes in the demand for finance. Furthermore, if an addition to credit money is the source of finance, the effect on the level of price is not necessarily inflationary. As noted already, the effect on output and prices depends on the existence of idle capacity and unemployed workers or, what is essentially the same, the shape of the supply curves. There is a long way to go between an increase in credit money and an increase in the price level.

What has been said above should not be confused with the standard view that *ex-post* saving (resulting from the workings of the multiplier) finances a previous increase of investment expenditures. What the multiplier process and *ex-post* saving do is to reequilibrate the flows of aggregate expenditure and income. When investment demand increases, expenditure becomes greater than income. As the multiplier process starts working and income and saving begins to change, the equilibration of the flows of expenditure and income is on its way. At the end of the

multiplier process, the *ex-post* level of saving is equal to the initial level of investment. The new level of income is now compatible with the level of investment expenditure, and the value of the investment multiplier. In this sense, as noted by Chick (1983, ch. 14), *ex-post* saving can only fund the new investment if part of the saved income is used to buy bonds; it cannot be said to finance the new investment.

## References

- Chick, V. (1983). *Macroeconomics after Keynes*, Cambridge, MIT Press.
- Hicks, Sir John (1974). *The Crisis in Keynesian Economics*, Oxford, Blackwell.
- Kahn, R. (1931). "The Relation of Home Investment to Unemployment", *Economic Journal*, 41, june; reprinted in Kahn, R. (1972). *Selected Essays on Employment and Growth*, Cambridge, Cambridge University Press.
- Kalecki, M. (1971). *Selected Essays on the Dynamic of the Capitalist Economy*, Cambridge, Cambridge University Press.
- Keynes, J. M. (1930). *A Treatise on Money*, vol. 1, London, Macmillan.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*, London, Macmillan.
- Keynes, J. M. (1973). *Collected Writings*, vol. XIV, D. E. Moggridge and E. Johnson (eds., Macmillan for the Royal Economic Society.
- Wicksell, K. (1935). *Lectures in Political Economy*, vol. II, London, Roulledge & Kegan Paul.