TEXTO PARA DISCUSSÃO

Nº 140

Testing Monetarist Models of Hyperinflation

Gustavo H. B. Franco

PUC-Rio – Departamento de Economia
www.econ.puc-rio.br

October 1986

1. Introduction

The idea that inflation is governed primarily by monetary forces has been perhaps too often applied to the hyperinflations. Most economists have a considerable amount of tolerance for non-monetary causes for inflation but in such extraordinary cases this tolerance, even among the less "orthodox", is rarely found. Conventional views on the hyperinflations are heavily influenced by the extreme views of Philip Cagan and Thomas Sargent[1]; yet these interpretations do not enjoy the same acceptance as far as "ordinary" inflations – these being OECD type inflations and even Latin American inflations prior to the oil shocks – are concerned. The experience of the 1970s and early 1980s did much to blur the frontier between what should be conceded to these extreme views and what should be explained by other influences. In general, the attitudes about the semi-hyperinflations of the 1970s have been careful and ambivalent; very few really attributed these inflations exclusively to money creation or seigniorage collection motives. This is not merely a product of economists celebrated pragmatism: the external shocks of the 1970s have been flagrant enough. We know that there is an extensive list of suggestive similarities between the 1970s and the 1920s, including huge external shocks, large transfers, impossible debts, flexible exchange rates and high inflations[2]. It makes little sense, at least at a first sight, to imagine that the so called hyperinflations would have been less affected by such non-monetary influences than the semi-hyperinflations of today. In this connection, a revision on the "extreme" Cagan/Sargent interpretation seems timely.

But more important than that is to observe that, surprisingly enough, the Cagan/Sargent model for the hyperinflations has really been tested. Cagan's work was basically addressed to established the empirical stability of the demand for money. It has been given support by the evidence to the extent to which it is compatible with the autoregressive structure of the price data and also with the strong positive correlation observed between money and prices. But as it is often the case with reduced forms equations the evidence could support many different models; for this case in particular it is hard to think of any one model that would not be compatible with such facts. The presence of inflationary inertia would follow from assumptions such as adaptive expectations or less than perfect adjustment of real balances to desired levels. Both hypothesis have become somewhat unpopular since they are difficult to reconcile with an equilibrium under rational expectations. In this respect, Sargent and Wallace suggested that if the money supply was determined by a random walk process then Cagan's adaptive expectations would be rational in the sense of producing inflation forecasts with the lowest mean square error[3]. The empirical results of such a model, however, were not

[1] P. Cagan (1956) and T. Sargent (1982).
[2] See, for example, M. de Ceocof (1983) and (1985), T. Balogh/A. Graham (1979), R. Aliber (1980), J. A. Frenkel (1978) and A. Fraga (1984).
[3] T. Sargent/N. Wallace (1973), p. 411, also D. A. Peel (1978), p. 34.

encouraging, and besides, it has been shown that the data does not support the random walk specification for the money supply process[4]. The coexistence of inflationary Inertia and rational expectations in monetarist models seems problematic for it could only obtain in the presence of some "rigidity", such as overlapping contracts or indexation arrangements, which these authors are generally very reluctant to admit.

Cagan's model predicts the strong correlation between money and prices that is indeed observed; yet as it is well known causality might run both ways and the correlation in itself would establish nothing. In this respect Granger causality tests have been performed by Frenkel and by Sargent and Wallace with results favouring strongly the hypothesis of prices Granger-causing money creation. Using wholesale prices data for Germany, Frenkel rejected the hypothesis that prices did not "cause" money and was unable to reject that money did not "cause" prices; using CPI figures Frenkel detected a two-way causality[5]. Similarly Sargent and Wallace rejected the hypothesis that prices did not Granger-cause money creation for Germany, Austria and Hungary (though not for Poland) and did not reject that money did not Granger-cause prices for all four countries[6]. Box Jenkins time series analysis were also employed by Frenkel and Evans for the German case, and the former rejected the hypothesis of independence between money and prices and the latter more specifically rejected the exogeneity of money[7]. The latter result has been challenged by Protopapadakis who also argued that the money supply process appears exogenous with respect to the revenue needs of the government[8]. Steven Webbs modelling of the money supply process in Germany concludes that in addition to the government needs, corporate needs for credit and expectations of inflation were also prime determinants of the money supply[9].

In sum, the empirical stability of the demand for money relationship does not inform on the dynamic process by which the hyperinflations were generated; if anything, the evidence produced by the authors mentioned above casts doubts on the validity of the monetarist interpretation of the hyperinflations. This paper is specifically addressed to discuss the empirical relevance of the dynamic processes associated with the monetarist models of hyperinflations. In this connection this paper reviews the theories of inflation in which money plays the sole causal role in the process and proposes ways to verify empirically the predictions of these models.

There are two basic varieties of monetarist explanations for high inflations in general and hyperinflations in particular: one is what we can term the "optimal" inflation variety, according to which the government chooses the level of expenditure (and money growth) so as to maximize the

[4] B. Friedman (1976).
[5] J. A. Frenkel (1977), pp. 666-667.
[6] T. J. Sargent/N. Wallace (1973), p. 419.
[7] J. A. Frenkel (1977), p. 665 and P. Evans (1978), pp. 202-204.
[8] A. Protopapadakis (1983), pp. 86-91.
[9] S. Webb (1984) and (1985b).

proceeds of the "inflation tax"; the other assumes an exogenously given, and supposedly very high, budget deficit whose financing determines money growth and inflation. Both labor under the presumption that the collection of seigniorage is the one and only purpose of inflation; they can be seen as two alternative rules for the determination of the amount of seigniorage to be collected. The first basically establishes an optimality criteria and the very large literature produced in the regard – quite appropriately termed by Phelps a "Chicago discussion"[10] – has aimed at broadening the problem's setup and on giving due consideration to factors like the welfare costs of inflation, the role of economic growth, the influence of inflation over tax revenues and many more. The next section provides a somewhat extensive survey of the several alternative concepts of the "optimal" inflation obtained by these models. Special attention is devoted to the role of tax revenues in the determination of the "optimal" inflation for when the former are adversely affected by inflation the governments spending power becomes determined by its total revenues, i.e., taxation and seigniorage, and not only with the proceeds of the "inflation tax".

The purpose of section 3 is precisely the one of examining how these models perform as applied to the hyperinflations. Empirical computations are offered and their results suggest two sorts of difficulties for the models of "optimal" inflation. On one hand, the data reveals a remarkable instability of inflation rates during all four episodes, and on the other, the well-known paradox of actual inflations above revenues maximizing levels is once again observed. The latter is troublesome for these models can be seen as attempts to provide a theoretical rationale for the case against inflationary finance, which is ordinarily a natural companion for the usual monetarist policy prescriptions. But in assigning low values for the "optimal" inflations these models should be able to explain the so pervasively observed departures from this norm. The blame is sometimes laid on the politics of inflation, which is certainly an important element in the process and a relevant research program in itself[11]; in the context of this paper, however, it stands as a way to evade the issue of the economic determinants of the inflationary process.

The marked instability of inflation rates is difficult to reconcile with the models of "optimal" inflation; this leads us directly to the second variety of monetarist explanation for the hyperinflations that is basically related to Cagan's model. It is assumed that an exogenously given fiscal deficit is entirely financed by money creation, and that inflation is simply determined by "solving" the money demand equation for the equilibrium price level. The model is perfectly dichotomized in the sense of the monetary sector being entirely dissociated with real factors, or that it abstracts entirely from all possible non-monetary influences on inflation. The key aspect of this model is that the private sector

---

[10] E. Phelps (1973), p. 67.

[11] A number of such contributions can be found in F. Hirsch/J. H Goldthorpe (1978) and L. N. Lindberg/C. S. Maier (1985).

is incapable of defending from the government endeavours to collect seigniorage, either because of adaptive expectations or in view of less than instantaneous adjustments of money balances. In these conditions the government could always collect seigniorage above the "optimal" or maximum steady state level by promoting inflations larger than it was expected by the public. If the exogenously given budget deficit is above that maximum seigniorage collection the government could be forced into promoting systematic "surprises" which could only take place under an "explosive" inflation.

Section 4 discusses two versions of Cagan's model: a basic one in which the budget deficit is exogenously given and another in which taxes are adversely affected by inflation and government expenditure is exogenous. The dynamics of both versions is the basically the same, though the extended version provides a richer and more realistic description. Both versions predict a hyperinflation in certain conditions, namely when the government is forced to collect seigniorage (or seigniorage plus taxes) above the maximum steady State levels; this is subject to empirical verification in section 5.

Section 4 also discusses a "null hypothesis" against which Cagan's model is compared. It is basically argued that for inflationary shocks unrelated to the monetary sector the actual increases in prices are observed before the corresponding move of the money supply; as the central bank accommodates the shock the money supply grows *pari passu* with the adjustment of money balances to the desired levels so that there is no collection of seigniorage above the steady state levels. That means that if the hyperinflations were predominantly produced by non-monetary shocks then we should observe that, in contrast to what is predicted by Cagan's model, the amounts of seigniorage (or seigniorage plus taxes) actually collected would be lower and at most only equal to the steady state levels. It is further argued that the level of government expenditure (or the exogenous budget deficit) may adjust as to match the revenues effectively collected by taxation and by the amount of seigniorage produced in accommodating the current level of inflation. While in Cagan's model the money supply is whatever necessary to get the seigniorage to finance what the government spends, in the "null hypothesis" the government would do the opposite, namely it spends what it gets.

These hypotheses are easily confronted in section 5 by the comparison between the levels of seigniorage and of total revenues (seigniorage and taxes) actually collected (and also government expenditure) and their corresponding "optimal" and steady state levels. Next, our "null hypothesis" is subject to a closer look; we verify whether there was monetary expansion in excess of the government budgetary needs, and also if the levels of expenditure act according to the rule of maintaining these economies permanently on the total revenues (seigniorage) curve. The results were remarkably favourable to the "null hypothesis" as opposed to Cagan's model, which call into serious question the notions that a disproportionate budget deficit (or level of government expenditure) was the driving force of the hyperinflations. It is strongly suggested, therefore, that the role of money and the budget in the hyperinflation episodes was mostly accommodative and that the driving forces of the inflationary processes seemed to come from the "outside".

2. The Optimal Inflation

The simplest possible setting in which the discussion of the optimal inflation usually takes place considers a closed economy in which there are no financial assets other than the national money and there is no tax revenue (or taxes in real terms or as a share of GNP are invariant with respect to inflation). Government expenditure is assumed "desirable", and since it would be entirely financed by money creation the government would then maximize the real value of the additions to the money stock subject to the private sector's demand for real balances. For a Cagan-type money demand schedule[12] and considering a steady State configuration[13], the problem can be written as:

$$\max G = \frac{\Delta M}{P} = \frac{\Delta M}{M}\frac{M}{P} = \Pi\left(\frac{M}{P}\right)^{d} \tag{1}$$

$$s.t. \quad \left(\frac{M}{P}\right)^{d} = \alpha e^{-\beta\Pi}$$

from which we obtain the familiar solution that the "optimal" revenue-maximizing inflation rate is given by the reciprocal of the interest semi-elasticity of the demand for money[14]:

$$\Pi^{*} = \frac{1}{\beta} \tag{2}$$

The first extensions proposed to this simple rule were associated with the consideration of economic growth. Friedman argued that in an economy experiencing autonomous growth the revenue maximizing inflation could be substantially lower than in a stationary economy depending on the income elasticity of the demand for money; the optimal inflation under autonomous growth would be

---

[12] In fact, nothing but habit indicates that this should be the appropriate functional form to be considered. By and large the empirical work on money demand estimation for hyperinflations and for high inflations more generally follows Cagan in using the exponential specification with good results. Some experimentation with other functional forms have been performed with the hyperinflation data, such as in R. Jacobs (1977a), R. Barro (1972), J. A. Frenkel (1977) and (1979). These studies produced good results but nothing clearly superior to what vas obtained by the simple Cagan model.

[13] This meaning that the money market is equilibrium, or that $\frac{M^S}{P} = (\frac{M}{p})^d$, and that inflation equals the rate of growth of the money supply.

[14] Incidentally, at this point the elasticity of the demand for real balances equals one: as in the famous Cournot example of the owner of a mineral spring, the government would have the monopoly of a resource with zero cost of production and, in a stationary economy, would operate at the point where demand elasticity equals one, cf. M. Friedman (1971), pp. 854-55.

given by[15]:

$$\Pi^* = \frac{1}{\beta} - \eta\hat{y} \qquad (3)$$

where η is the income elasticity of the demand for money and $\hat{y}$ stands for the economy's rate of growth. Similarly, Mundell considered the possibility that all government expenditure represented additions to the stock of capital in a Harrod-type model; in this case growth would be endogenized and the optimal inflation would be given by[16] :

$$\Pi^* = \frac{1}{\beta}\left(1 - \frac{\emptyset}{v}\right) \qquad (4)$$

where $\emptyset$ stands for the output/capital ratio and v for the velocity of circulation. Both authors attitude towards the potential contribution of inflationary finance to economic growth was very pessimistic: Mundell in this respect showed that for reasonable parameter values his model would produce very modest increases in economic growth for very large additions to the inflation rate[17].

Both Friedman and Mundell warned that the case against inflationary finance would be further strengthened if the welfare costs of inflation were explicitly considered. In this connection it had been shown that like any other tax inflation generated welfare costs corresponding to the area under the appropriately defined demand for money[18]. But costs and benefits were not teamed together into a computation of a broadly defined "optimal" inflation until somewhat later with the formulation of neo-classical models of optimal growth in which the welfare costs of inflation were explicitly considered. Cathcart, for example, obtains the optimal inflation by maximizing the present value of the difference between the revenue of the inflation tax and the welfare loss caused by inflation, from which the following expression is derived[19]:

$$\Pi^* = \frac{1}{2\beta} - \hat{y} - \delta\left(1 - \frac{1}{\alpha\beta}\right) \qquad (5)$$

where δ corresponds to the real rate of return on capital, or the intertemporal discount rate. It should

[15] M. Friedman (1971), p. 850. Friedman considers also the rate of growth of population, which we ignored.
[16] R. Mundell (1971), p. 41.
[17] R. Mundell (1971), pp. 40-41. Other authors considering the issue, such as A. L. Marty (1973) and E. Tover (1971), reached similar conclusions.
[18] M. J. Bailey (1956).
[19] C. D. Cathcart (1974), p. 180.

be noted that this model assumed a unit income elasticity of money demand, and derived expression (5) under adaptative expectations; under perfect forecast Cathcart shows that the term in δ vanishes.

A somewhat different setup is considered by Aghevli who maximizes the discounted flow of consumption in a model where inflationary finance is used to finance part of the investment expenditure and real money balances belong in the utility function as a consumer good. Aghevli does not derive an explicit expression for the optimal inflation, but performs detailed calculations for several arbitrarily selected values for the parameters of the model. It is significant that his results invariably shows optimal inflation rates much lower than the benchmark of equation (2)[20].

Another group of extensions to the benchmark of equation (2) is related to the consideration of rational expectations. The models of “optimal” inflation had been criticized on grounds that the solutions proposed were unstable as the government could always increase the proceeds of the inflation tax in the short-run by means of unexpected inflation[21]. But it was objected that as the public learns the governments motivations it would adjust its holdings of real balances accordingly, so that in a rational expectations equilibrium, or in a steady state, no systematic pattern of surprises could subsist[22]. The consideration of the issue of inflationary finance under rational expectations in a dynamic context would introduce a problem of time consistency which would be the dynamic counterpart for the continuous incentive for the government to surprise the public; at any point in time the government could raise any amount as seigniorage, so that no equilibrium would actually be defined[23]. The difficulty would be removed by Barro by considering explicitly in the governments objective function the costs of inflation, which would basically determine infinite inflations. Such objective function could be written in a simplified form as[24].

$$L = e^{\Pi_t} - \psi_t R_t, \;\; \psi_t > 0 \tag{6}$$

where L should be interpreted as a loss function. R_t stands for the revenue from the inflation tax and the parameter ψ, which accounts for the government’s valuation of inflationary finance is assumed to follow a random walk: $\psi_t = \psi_{t-1} + \epsilon_t$. Barro derives an expression for the optimal inflation under a purely discretionary monetary regime that, considering the simplifications we introduced, results to be:

---

[20] B. B. Aghevli (1977), p. 1304. This was also observed by L. Auernheimer (1974), p. 601.
[21] The point was made in L. Sjaastad (1976) and H. G. Johnson (1977).
[22] R. J. Barro (1983), p. 2. The point was also made in connection of Sjaastad 1976 work by D. Laidler and A. L. Marty who were discussants of the paper, cf. L. Sjaastad (1976) pp. 86-87.
[23] G. Calvo (1978).
[24] We made all his parameters equal to one simplify matters, and ignored his assumption that the parameter α, in our notation, would follow a random walk. See R. J. Barro (1983), pp. 5-8.

$$\Pi^* = \left[\frac{1}{1+\beta}\right][\log(\alpha\psi_{t-1})] + \epsilon_t \quad (7)$$

The optimal inflation in this case would depend on the characteristics of the stochastic process governing ψ_t, namely the government's varying willingness to undertake inflationary finance, and also on the parameter associated with the costs of inflation that we assumed to be equal to one. The implied departures from the $1/\beta$ benchmark are thus determined by the form of the government's utility function.

One fundamental element for the determination of the optimal inflation is the tax system. The basic point is that if the real value of tax revenues is sensitive to inflation it follows that attempts to increase the proceeds of the inflation tax could be defeated by reductions in the real yields of ordinary taxation. Presumably governments should be concerned about their *total* revenues, namely seigniorage plus ordinary tax revenues, for this would account for their ultimate ability to undertake expenditure. It might very well be that governments attribute different weights to revenues from taxation and from inflation; in fact, the case for inflationary finance is usually made on grounds that "weak" government would regard inflation as an apparently painless, and relatively easy to implement, *substitute* for additional taxation. In this connection, it seems plausible to assume that governments undertaking inflationary finance would see as an undesirable development that revenues from pre-existing taxes are increased in real terms by inflation; after all, inflationary finance would be an alternative to increased taxation. Correspondingly, if the real value of the revenues from existing taxes is reduced by inflation, the very purpose of increasing the governments command over real resources would be defeated. It is therefore assumed that governments undertaking inflationary finance would be interested in maximizing their total spending power provided that inflation would not increase the real yields of existing taxes.

On theoretical or *a priori* grounds it has been suggested that the behaviour of the real value of tax revenues when the government is following inflationary finance is undetermined, the net effects of inflation over taxes depending on the specific tax system at hand[25]. The matter appears to be largely empirical and in this connection a fair amount of work has been generated at the Fiscal Department of the IMF providing evidence on very strong negative effects of inflation over the real value of tax revenues in countries experiencing high inflations[26]. From a theoretical point of view, the point of interest is that if the government maximizes total revenues, that is seigniorage plus ordinary tax revenues, and if taxes are adversely affected by inflation, then the optimal inflation would be considerably smaller than the benchmark of equation (2). In order to see that let us write total

[25] L. H. Summers (1981), p. 191 and M. Friedman (1974), pp. 14-15.
[26] V. Tanzi (1977) and (1978), B. B. Aghevli/M. S. Khan (1978). See also J. H. G. Oliveira (1967).

government revenues as:

$$G = T(\Pi) + \Pi\left(\frac{M}{P}\right)^{d}, \quad T'(\Pi) < 0 \tag{8}$$

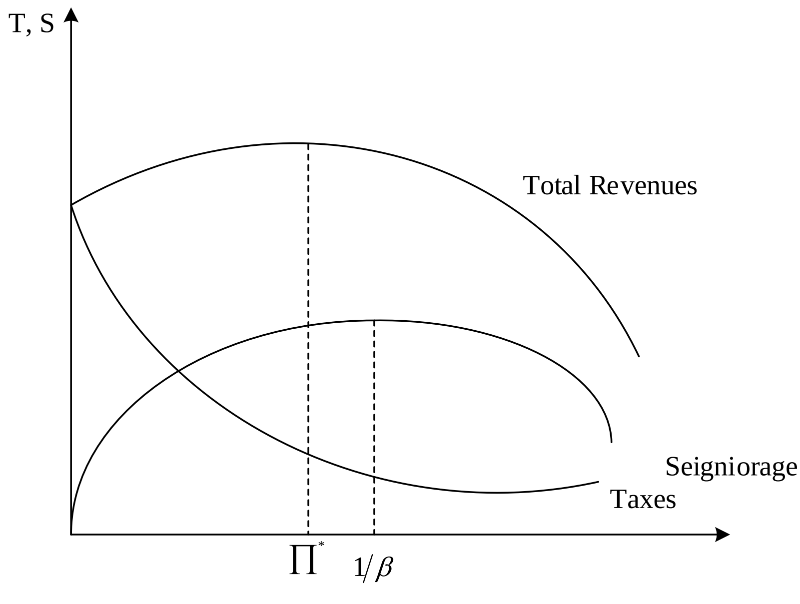
where $T(\Pi)$ stands for the real value of tax revenues, assumed to decrease with inflation, and the second term correspond to the proceeds of the inflation tax. Total revenues will be maximized at a point where:

$$T'(\Pi) = (1 - \beta\Pi)\alpha e^{-\beta\Pi} \tag{9}$$

i.e., where the marginal loss (gain) in real tax revenues equals the marginal revenue gain from the inflation tax. Equation (9) is not solved explicitly, but since $T'^{1/\beta} < 0$ it is clear that total revenues could be increased by a reduction of inflation, i.e., $1/\beta$ is thus greater than the optimal inflation when real tax revenues are negatively affected by inflation. It is interesting to observe that in this case, in contrast to some of the concepts of "optimal inflation" we derived earlier, the government not concerned with welfare costs or growth but just with spending power. This can be very easily seen in diagram below originally due to V. Tanzi[27]:

Graph 1

Tax Revenues, Seigniorage and Total Revenues



[27] V. Tanzi (1978), p. 429.

In Graph 1 we plot the behaviour of the proceeds of the inflation tax, the behaviour of ordinary tax revenues and the vertical sum of these two curves namely total revenues, all measured in real terms or as percentages of GNP. It is readily seen that the optimal inflation Π^*, or the rate of inflation that maximizes total revenues, is significantly lower than $1/\beta$ and the more so the faster tax revenues decrease as inflation accelerates.

3. Hyperinflations and the Theories of Optimal Inflation

This section is addressed to verify whether the observed rates of inflation should show any correspondence with the concepts of "optimal" inflation we discussed in the last section. The most basic of these concepts is given by equation (2), or it assumes that governments should set inflation at the seigniorage maximizing level $1/\beta$; we should start by examíníng the exístíng estimates of the parameter β. Since Cagan's seminal work on the estimation of money demand equations during the hyperinflations there has been more than one dozen reviews, refinements and replications of Cagan's results using the most varying methodologies[28], Table 1 reports a somewhat arbitrary sample of estimates of β obtained in several of these revisions.

Table 1

Alternative Estimates of the Interest Semi-Elasticity of the Demand for Money

Country	(1) Cagan	(2) Khan	(3) Jacobs	(4) Khan	(5) Frenkel	(6) Jacobs	(7) Jacobs	(8) Sargent	Median	Average
Austria	8.55	5.41	7.67	4.50	-	7.71	2.78	0.31*	4.96	4.88
Hungary	8.70	1.09	-	1.90	-	3.88	2.37	1.84	2.13	3.23
Germany	5.46	1.12	3.70	4.34	3.51	4.34	3.03	5.97*	4.02	3.34
Poland	2.30	1.23	-	2.53	-	3.65	2.62	2.53	2.53	2.48

Source and observations: (1) P. Cagan (1956), pp. 43, 45. (2) M. S. Khan (1975), p. 359. (3) R. L. Jacobs (1977b), p. 118. (4) M. S. Khan (1977), p. 823. (5) J. A. Frenkel (1979), p. 86. (6) R. L. Jacobs (1975), p. 343. (7) R. L. Jacobs (1977a), p.292. (8) T. J. Sargent (1977) p. 447. *Not significantly different from zero.

The estimates for each country reported in Table 1 are sensitive to the choice of methodology, but except for a few odd estimates – such as for example Sargent's for Austria (which is not significantly different from zero), Cagan's for Hungary and Khan's (1975) for Germany – there seems to be a fair amount of consistency. Medians and averages certainly provide *ad-hoc* indicators of the general tone of the results but it should be kept in mind that each number has been obtained in a different context so that these medians are not really meaningful in any theoretical sense. Any one choice would be arbitrary and would reflect a judgement as regards methodology. Simplicity appears

[28] A critical survey could be found in S. B. Webb (1983).

to be a safe criteria and this regard perhaps the simplest possible choice would be the estimates obtained considering Cagan's model corrected for serial correlation. This would leave aside Jacobs (1977a) and Sargent (1977) who consider different models. Cagan's own estimates as well as Jacobs (1975) both suffer from serial correlation that seems very strong especially as regards Germany and Hungary[29]. The remaining estimates were obtained by procedures that were attentive to this problem: Khan (1975) simply corrected Cagan's original estimates by considering that residuals followed a first order autoregressive process, obtaining the estimates of column (2) in Table 2. Jacobs (1977b) reestimated the equations for Austria and Germany correcting for specification errors, from which he obtained a substantially higher value for the estimate for Germany. Khan (1977) produced new estimates considering the possibility of a variable speed of adjustment for money balances and Frenkel (1979) reconsidered Cagan's model using the forward premium on foreign exchange to measure expected inflation.

It is significant that by considering only the latter estimates our choices would not be much different from what is expressed by the medians and averages: a value around 5.0 for Austria, between 3.5 and 4.0 for Germany, about 1.5 for Hungary and a little less than two for Poland. Although essentially an *ad-hoc* procedure it does not seem that we do violence to these authors efforts by considering the medians for purposes of computation of the concepts of "optimal" inflation. In fact, differences between medians and averages, and between these and the estimates proposed by Khan (1977), Frenkel (1979) and Jacobs (1977b) are small enough to have nearly negligible effects in our results. It is quite significant that the ranking of the estimates is exactly the one should expect *a priori*. Austria and Germany, the more open and financially developed economies, would show the largest elasticities[30], while the lowest ones would be observed in the more agricultural and financially backward economies.

The next step is to use the estimates for 0 to compute the several concepts of the "optimal" inflation and compare them to the actual values. Table 2 reports five alternative definitions of the optimal inflation together with the average monthly rates and standard deviations actually observed during the period indicated. Several assumptions have been made as regards the computation of each formula as well as relating to the period under consideration. The first column is obtained simply by using the median estimate for 0 reported in Table 1. It is important to observe that the time dimension of the "optimal" inflation should be the same as the one of the data used for the estimation of 0, which in our case is monthly[31] .

---

[29] Durbin-Watson statistics for Cagan's model produced by M. S. Khan (1975), p. 358 and for Cagan's model corrected for problems related with structural specification see R. L. Jacobs (1977b), p. 343.

[30] As suggested in D. A. Nichols (1974).

[31] This is often overlooked leading to serious inconsistencies. Money demand equations generally use monthly, quarterly and annual data, so that the implied $\Pi^* = 1/\beta$ should have this same time dimension. Friedman himself seemed confused with the issue: while computing "optimal" inflation rates on an *annual* basis considered Cagan average estimates of β

We ruled out economic growth during the hyperinflation period, by virtue of which we ignored the formulations proposed by Friedman, Mundell and Aghevli, and modified Cathcart's formula accordingly. The parameter α, which represents the level of real money balances under zero inflation, introduces some difficulties related to units of measurement. For any reasonable choice of units, a should be a large number so that there would be no problem as regards Cathcart's formula as we will consider that its second term becomes negligibly small. It would appear that Barro's formulation, which also includes α, would be affected by the choice of units, but in fact a "proper" choice of the parameters of the government's loss function could avoid the problem. It turns out, however, that the choice of the parameters of the government's loss function is totally arbitrary; it actually represents a description of the government's policy motivations without which we are unable to obtain an empirically testable expression of his model[32].

Table 2

Alternative Definitions of the "Optimal" Inflation and Actual Inflations

Country	(1) $1/\beta$	(2) Cathcart	(3) Tot. Rev.	(4) Jacobs	(5) Barro	(6) Actual	(7) St. Dev.	(8) Period
Austria	20.2	9.61	16.0	48.6	84.0	32.6	35.7	Feb. 1921/Sep. 1922
Hungary	46.8	23.1	37.5	58.0	94.0	29.6	26.2	Jul. 1922/Feb. l924
Germany	24.9	12.4	19.5	45.6	154	41.8	61.0	Aug. 1920/Jul. 1923
Poland	39.5	28.8	32.5	28.8	139	52.8	61.7	Apr. 1922/Jan. l924

Sources and observations: Columns (1) βs from Table 4.1. Columns 2 and 3 are equations (5) and (11) in text, respectively. Column 4 is from R. L. Jacobs (1977a), p. 303. Column 5 is from R. G. Barro (1972), p. 984. Columns (6) and (8) from data in J. P. Young (1925), vol. I, p. 530 and vol. II, pp. 322, 349 and in J. Yan Walré de Bordes (1925), p. 83.

Column (3) of Table 2 report the concept of Π^* defined in equation (9), which correspond to maximization of total revenues. In order to obtain these values we solved numerically equation (9) considering specific values for α and also a specific functional form for the relation between taxes and inflation. As far as α was concerned we consider the stock of real balances held in a zero inflation

"converted" for an annual basis to be 0.5, from which the optimal annual inflation rate would be 200% , cf. M. Friedman (1971), p. 851. Cagan's average estimate for β was 4.68, as seen in Table 4.2, from which the "optimal" inflation would be of 20.4% *monthly*. Friedman's mistake was observed by Barro, cf. R. G. Barro (1972), pp.988-989, and has been reproduced here and there in the inflationary finance literature. See, for example, M. S. Khan/M. D. Knight (1982), p. 348, ff. 6.

[32] That takes away nothing from the conceptual value of the model, the problem being the policy parameters from which to obtain a "prediction" of what inflation would be. One possible alternative would be to equate his formula to the observed rates of inflation and then solve for the parameters of the loss function; since there are at least two such parameters in the formula their values would be overdetermined so that "reasonable" values could always be found. This procedure, however, would be like hypothesizing the conclusion, thus not by any means a "test" of Barro's model.

year of the post-hyperinflation period[33]. For the relation between taxes and inflation we considered the following functional form:

$$T(\Pi) = \tau_0 e^{-\tau_1 \Pi} \tag{10}$$

from which we could rewrite the first order conditions for the maximization of total revenues as:

$$\tau_1 \tau_0 e^{-\tau_1 \Pi} = (1 - \beta\Pi)\alpha e^{-\beta\Pi} \tag{11}$$

In order to obtain values for the parameters τ_0 and τ_1 and to be able to solve equation (11) numerically we estimated equation (10) empirically obtaining the results reported in Table 3.

Table 3 reports regressions of the real value of tax revenues on inflation for Germany, Poland and Austria. The Polish equation includes a trend term which has to do with the fact that Poland as a country was created in 1918; one observes during the first years of the new republic at least until the mid-1920s a continuous growth of the public sector as natural part of the process of nation building. No such a thing takes place in the other countries, for which government revenues and expenditures preserved the same order of magnitude before and after the hyperinflations.

A simple way to control for that is to introduce the trend term which appears in Table 3 as positive and significant.

Table 3

Regression of the Real Value of Tax Revenues on Inflation[1] (t statistics in parentheses)

Country	$\log \tau_0$	τ_1	Trend	R2	DW
Germany[2]	5.97 (76.5)	-1.08 (-8.63)	-	0.69	1.40
Austria[3]	1.33 (34.6)	-1.10 (-2.01)	-	0.60	0.98
Poland[4]	3.05 (36.3)	-0.49 (-1.91)	0.04 (5.97)	0.64	1.21

Sources and observations: (1) For sources of price data see Table 2. (2) Revenues in millions of gold marks from Republic of Germany – Krieglastenkommission (1924), p. 34. The period considered was January 1921 to July 1923. (3) Revenues including only direct taxes, as estimated by J. V. Yan Sickle (1931), p. 203. The period considered was January 1921 to December 1922. (4) Revenues in millions zloty from Republic of Poland (1926), pp. 173-177, not including the extraordinary property taxes of 1922 and 1924. The period considered vas January 1921 to April 1924.

[33] We considered values for the stock of real balances held about one gear after the stabilization, thus allowing the money supply to adjust to the increase in money demand generated by price stability. The values considered were 110 million dollars for Austria as of mid-1924, 550 million zloty for Poland (as of December of 1924) and 416 million pengö for Hungary and 4,000 million gold marks for Germany (both as of December of 1925), cf. J. P. Young (1925), vol. I, pp. 298 and 353 and B. R. Mitchell (1978), p. 358.

Table 3 shows a very significant negative association between the real value of tax revenues and 1nflation for the three countries indicated. The slope coefficient for Poland is lower than the ones for Germany and Austria, which is compatible with the more extensive analysis conducted in Franco (1986)[34]. The values for τ_0 represent proxies for the zero-inflation level of tax revenues, a very important magnitude as far as stabilization policy was concerned; the regressions indicate a zero-inflation level of revenues of about 391 million gold marks for Germany, about 600 thousand dollars for Austria (for direct taxes only) and around 40 million zloty for Poland. It is interesting to compare these values with the ones actually event, our purpose here is not to discuss the nature or the importance of fiscal reforms for the stabilization, a theme we developed in full detail in Franco (1986). The important point to keep in mind is that the zero inflation budget deficits were, on the assumption of expenditures not sensitive to inflation, much lower than indicated by a simple inspection of the budgetary conditions at the height of the hyperinflation. This is a very important theme we will deal more fully in the next section.

For purposes of the computations of Table 2 we used the values of τ_1 estimated in Table 3; for Hungary we simply used which seems compatible with the sensitiveness of the Hungarian tax system to inflation relative to the Austrian one discussed in great detail in Franco (1986). The values for the parameter τ_o considered in Table 2 took into consideration the estimates of Table 3 and also the analysis of these countries tax systems offered in Franco (1986). For Germany it makes no difference, as far as the final estimate reported in Table 2 is concerned, to consider the estimate of τ_0 from Table 3 or the observed average for the first quarter of 1924. For Austria it should be noted that the estimates of Table 3 were based on figures for direct taxes only; for this reason, we opted to use the observed value for total tax revenues[35]. For Poland and Hungary we also used observed values immediately after the stabilization but excluding any new taxes introduced during the stabilization[36].

In view of all these assumptions, equation (11) was solved numerically and the values obtained are reported in column (3) of Table 2. These values are lower than $1/\beta$ and higher than the values obtaíned using Cathcart's formulation. Columns (4) and (5) report two concepts of seigniorage maximizing inflation rates that we did not discuss in the last section. On column (4) we report "optimal" inflations as computed by Rodney Jacobs in a model whose distinguishing features are that the money supply is endogenized and where the money demand is specified as $(m/d)^d = \alpha e^{-\beta\sqrt{\pi}}$[37].

[34] Esp. Chapter 8.

[35] We considered the average for the last three months of the first reconstruction period reported in Table 8-2 (approximately 10 million dollars). Considering, for instance, that all other revenues would be as sensitive to inflation as table 4-2 indicated that direct taxes were, and on the assumption of direct taxes accounting for 14% of total revenues (as assumed by the League for 1922), by using the estimate of τ_0 of Table 4-2 we would obtain a value of 8.5 million dollars.

[36] The values considered were 100 million zloty and 10 million dollars, respectively.

[37] His revenue maximizing inflation rate would be given by $\Pi^* = (2/\beta)^2$, cf. R. L. Jacobs (1977a), p. 300.

On column (5) we report the seigniorage maximizing level of inflation obtained by Barro using his "payments' period" model for the demand for money. Barro's model is the only one to account for the means of payments motive in holding money, by virtue of which he derived a very "inelastic" demand schedule for real balances, and consequently very high revenue maximizing inflation rates[38].

Next, a few observations should be made about the choice of the period to be considered for purposes of computation of the actual rate of inflation. Cagan, for example, compared his estimates of $1/\beta$ with the average compounded inflation rates but only for the hyperinflation period, according to his definition; this period is different from the one utilized for the estimation of β[39]. This inconsistency might become serious since the estimation of β using only the data for the few months defined by Cagan as correspondlng to the hyperinflation period could produce substantially different values for β. If one's interest in performing the comparisons we proposed is to test for the existence of a concern about seigniorage collection then there is no reason to consider only the hyperinflation period, much on the contrary one could go as far back as 1914. But, in fact, we would like to isolate the war and its very immediate aftermath and see whether there is any evidence for inflationary finance during the later period when economic life has been roughly normalized. Poland and Hungary, for example, faced armed convulsions until much later than 1918, so that the periods considered for purposes of estimation of money demand employed by Cagan, as indicated in Table 2, correspond more or less to a period in which economic life has been freed from the direct effects of wars and revolutions. For Austria the same holds, even though hostilities ended in 1918, mostly because the hyperinflation had a very early start. For Germany we considered the same period Cagan did and we preserved his procedure of not considering the last four observations (August-November of 1923). The inclusion of these would raise the average monthly inflation in 1310% rendering all concepts of "optimal" inflation meaningless[40].

There are two basic facts to be reconciled with the theories of "optimal" inflation discussed in the last section: one that the observed inflations were by and large higher than the "optimal" levels and the other that inflations display an enormous variability. On account of the former, the several definitions of "optimal" inflation, with the exception of Jacobs' and Barro's, produced values generally lower than the observed ones, except for Hungary. Thomas Sargent conjectured that this

---

[38] R. J. Barro (1972).

[39] He defined the hyperinflation period as starting the month in which prices rose by more than 50% and as ending at the month it falls below that number and remains so for at least a year. Cagan used longer period for his estimates of β generally including more observations for the months preceding the hyperinflation period. This meant to include 10 more observations for Austria, 8 more for Hungary and Poland and 23 more for Germany, cf. P. Cagan (1956) pp. 26, 43 and 59.

[40] Cagan omitted these last four observations from the period considered for his money demand estimation only because they created an overwhelming problem of serial correlation in his equations. For our purposes, it seems that in these four months something qualitatively different was taking place as the inflation rate "exploded".

"Paradox" could have been due to bad estimates of β[41], but an inspection of Table 1 suggests that there seems to be nothing overwhelmingly problematic about these estimates. As observed above there is consistency among the various estimates, which is quite remarkable in view of the variety of methodologies utilized.

Milton Friedman detected the same "Paradox" more generally in his "casual observation" of the experience of developing countries. His justification for that was the "shortness of time perspective" of governments that, in view of the existence of lags in the adjustment of real balances and expectations, would always be tempted to increase their "inflation tax" collection temporarily despite the fact that the total collection would be smaller in a longer run horizon[42]. This leads us directly to the issue of the variability of inflation which, as observed in Table 2, was incredibly high. There seems to be no way to reconcile this fact with any notion of "optimal" inflation for all concepts predict a constant rate of inflation independently of the functional form chosen for the money demand schedule. The only exception is the seigniorage maximizing inflation rate suggested in Barro's more recent work, which corresponds to equation (7) in last section. This formulation includes a stochastic term that derives from the assumption that the government's willingness to pursue inflationary finance is described by a random walk; there seems to be no reasonable justification for these governments' inflationist tendencies to exhibit such variability[43].

The high inflation variability is consistent with models predicting unstable inflations, such as Sjaastad's for example, and with situations in which the government needs to "surprise" the public more or less consistently in order to extract "inflation tax" beyond the maximum steady state level. Both theories have been strongly questioned on theoretical grounds as incompatible with equilibrium under rational expectations, yet no alternative – if one wants to preserve the claim that the hyperinflations we "purely monetary" phenomena – has been made available. The next step is, therefore, to verify whether the models associated with the need for "surprise inflation" – and the most notable is Cagan's model – provide useful descriptions of the dynamics of inflation in the hyperinflation episodes.

4. The Monetary Dynamics Revisited

In this section we will discuss two versions of Cagan's benchmark model of hyperinflation in conjunction with an alternative description of the dynamics of government finances under high inflation emphasizing the endogeneity of fiscal and monetary policies. Again our focus is not on

[41] T. Sargent (1977), p. 429.
[42] M. Friedman (1971), p. 853.
[43] Barro's "optimal" inflation formula would actually include another stochastic term derived from the assumption that α, in our notation, would also follow a random walk . This appears even less justifiable than the other stochastic term.

theoretical detail but on displaying the empirically testable implications of each theory.

In its simplest form Cagan's model assumes an exogenously given budget deficit, which is entirely financed by money creation, and a perfectly dichotomized economy where actual inflation is determined by solving the equilibrium conditions in the money market. Cagan's model does predict a hyperinflation, or an "explosive" inflation, when certain conditions are met, so that the relevance of the model as far as the hyperinflations are concerned is obviously associated with the empirical verification of these conditions. In this connection we will consider the nature of the latter in some detail. Consider an adaptative expectations process given by:

$$\hat{\pi}^e = \lambda(\Pi_t - \Pi e_t) \tag{12}$$

where λ corresponds to the speed to which expectations of inflation adjust in response to surprises. Now by differentiating the demand for real balances given in equation (1) with respect to time we obtain the following relation:

$$\theta - \Pi_t = \beta\hat{\pi}^e \tag{13}$$

where θ stands for the rate of growth of the money supply. Using (12) and (13) we obtain:

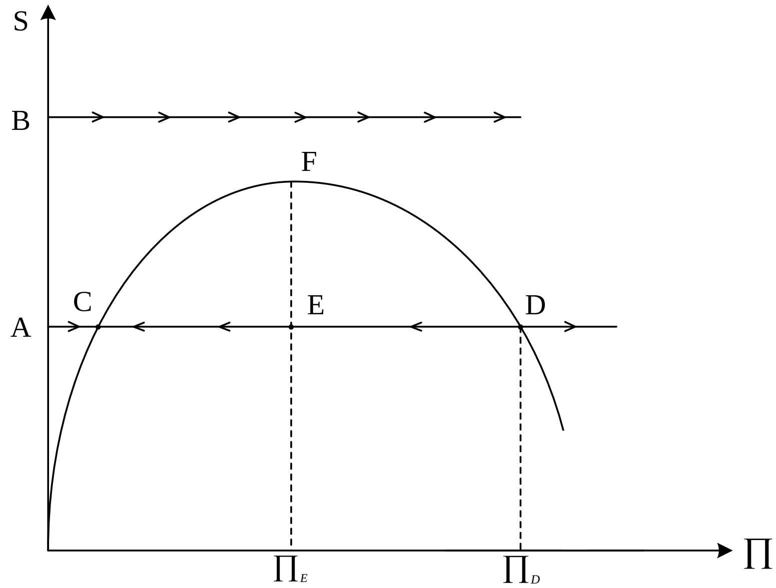
$$\Pi_t = \left[\frac{1}{1-\beta\lambda}\right][\theta - \beta\lambda\Pi e_t] \tag{14}$$

which expresses actual inflation as function of expected inflation.

Now consider the situation depicted in Graph 2 where we plot the revenues from the collection of seigniorage and alternative levels for the exogenously given budget deficit. As indicated by the arrows in the graph for any point inside the seigniorage curve the economy moves to the left – eventually finding an equilibrium on the left side, or on the rising portion of the curve. In a point such as point F, for example, the amount of seigniorage collected is given by $\Pi_E \alpha e^{-\beta\Pi_E}$; at point E, or for a budget deficit equal to A, the amount of seigniorage effectively collected is equal to $\Pi_E \alpha e^{-\beta\Pi^e}$. It follows that for points inside the curve we have $\Pi_E \alpha e^{-\beta\Pi_E} > \Pi_E \alpha e^{-\beta\Pi^e}$, or that $\Pi^e > \Pi_E$. This means that agents overestimated inflation, so that, from equation (12), inflationary expectations would be adjusted downwards; the economy would move towards point C, where $\Pi^e = \Pi_E$ and an equilibrium obtains.

Graph 2

Dynamics of Inflation under the Basic Cagan Model



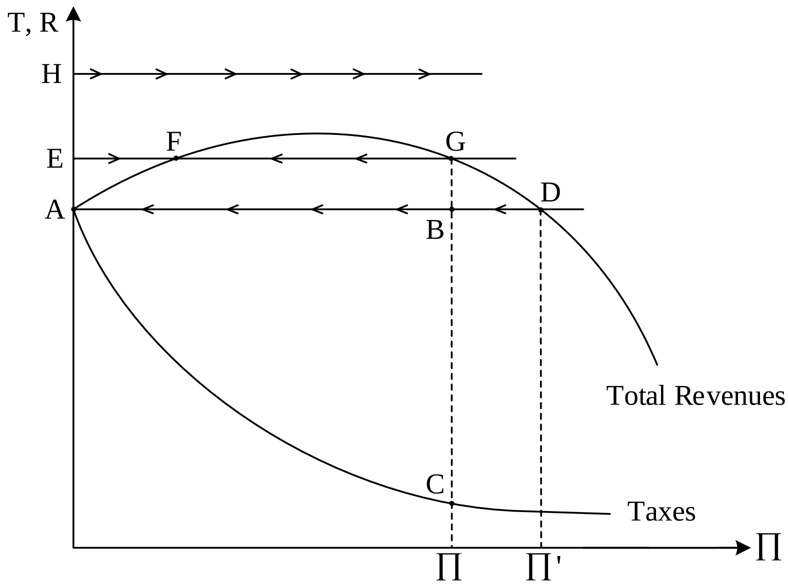
There are basically two ways by which a hyperinflation could be generated in this model: one is to consider a budget deficit that is larger than the maximum steady state seigniorage collection given by point F, and the other is when the economy is outside the curve at its right side or for points, for instance, to the right of D. In either case the government is collecting seigniorage above the steady state level: at D, for example, this means $\Pi_D \alpha e^{-\beta \Pi_D} < \Pi_D \alpha e^{-\beta \Pi^e}$, or $\Pi_D > \Pi^e$, meaning that inflation was larger than expected, which would induce an upward revision of expectations. The "inflation tax" base is reduced so that a larger inflation rate is now required to finance the same budget deficit given by A; an accelerating rate of inflation would obtain.

The first type of hyperinflation, represented by point B in Graph 2, corresponds to a strictly monetary hyperinflation in the sense of no outside influence intervening in the process: once the budget deficit is fixed at B, inflation would accelerate continuously. In contrast, a hyperinflation of the second type needs some outside shock to drive the economy out of the seigniorage curve to the instability region. In fact, it needs more than that for in this model an equilibrium on the right side of the seigniorage curve can only be obtained by assumption. The dynamics of the model is such that for points inside the curve an equilibrium is found on the left side of the curve and for points outside inflation is accelerating; the right side of the curve is like hidden in the shade. In this regard, for an hyperinflation of the second type, the model not only does not inform on the nature of the shock that places the economy on the instability region, but it also provides no indication on how the initial levels of inflation, as for example point D in Graph 2, were reached. This is actually very serious for at points to the left of the maximum of the seigniorage curve the levels of inflation are extremely high, as it can be seen from Table 2. By taking the latter by assumption one is really doing away with the most important and perhaps the only relevant part of the problem, namely how inflations of 30 or 40% a month were reached. If Cagan's model is not capable of explaining that, then its relevance as regards the hyperinflations should be questioned even if does explain what happens after that, which we are not sure about. In this connection, it is generally assumed that the hyperinflations of the 1920s corresponded to a process of the first type, but no empirical support has been given to this claim.

One serious shortcoming of this model is the assumption of an exogenously given fiscal deficit; as seen in the last section and especially in Franco (1986) tax revenues were very strongly affected by inflation so that the budget deficit was not invariant to changes in the rate of inflation. The consideration of this fact represents a considerable addition to Cagan's model in terms of relevance and it does not change any of the conclusions of the model as regards the dynamics of inflation. It actually provides a more comprehensive version of the model that can be more realistically confronted with experience. The dynamics of this extended model with endogenous taxes can be seen with the help of Graph 3.

Graph 3

Dynamics of Inflation under the Extended Cagan's Model



Graph 3 is similar to Graph 1 as we measure total revenues (seigniorage plus tax revenues), tax revenues and also government expenditures – all in real terms or as percentages of GNP – against inflation. The graph also shows three possible levels of government expenditure, indicated by the points A, E, and H, that are financed by different combinations of taxation and money creation. If expenditure is fixed at A, for example, the budget is balanced at zero inflation, but as inflation accelerates a growing budget deficit is observed: if inflation is equal to Π, for example , there would be a fiscal deficit equal to BC. It is clear then that for different levels of inflation the same level of government expenditure could be financed by a sharply different mix of seigniorage and tax revenues: the expenditure corresponding to A, for example, could be financed entirely by taxes at zero inílation or mostly by seigniorage at Π. Similarly, a level of expenditure equal to E could be obtained at F or at G.

The first important point to observe is that independently of the "fundamental" fiscal position, or the value of the budget deficit at zero inflation, for all three configurations of government expenditure we would observe very large fiscal deficits at high levels of inflation, so that the mere observation of large budget deficits when inflation is high does *not* provide in itself any *a priori* indication on whether the level of expenditure correspond to "sound" positions like at A or "unsound" ones like at E and H.

Therefore, when taxes are adversely affected by inflation, the need for fiscal restraint or tax "reform" as part of a stabilization effort should not be established by the size of the observed budget deficit, but by some measure of the zero-inflation-budget or the "inflation-corrected" deficit. As it is indeed observed and described in detail in Franco (1986), tax revenues reacted strongly to the sudden end of the hyperinflations, revealing "true" budget deficits that were actually very small; in Germany and Hungary even small surpluses are observed. In these conditions fiscal reforms would be hardly an important factor for the stabilization and most likely not necessary; in fact it is shown in Franco (1986) that no significant, let alone drastic, fiscal reforms effectively took place, except possibly at the rhetorical level[44]. This is certainly not a general feature of all high inflation episodes, though the adverse influence of inflation over taxes is very commonly observed. The size of the "inflation-corrected" deficit, and consequently the need for tax reform, is an empirical matter that depends on the specific tax system being considered.

The monetary dynamics underlying this extended version of Cagan's model is basically the same of the basic model. Consider a point like B in Graph 3; the total revenues collected at this level of inflation, which are given by G, are greater than the level of expenditure given by point A, or that $T(\Pi) + \Pi\alpha e^{-\beta\Pi} > A$. The fiscal deficit $BC = A - T(\Pi)$ is smaller than the amount of seigniorage effectively collected, which is given by $\Pi\alpha e^{-\beta\Pi^e} = BG$. But since the deficit should be equal to $\Delta M/P$, which is equal to $\theta\alpha e^{-\beta\Pi^e}$, it follows that at B we have $\Pi > \theta$ which, by equation (13) implies that $\hat{\pi}^e < 0$ or that $\Pi^e > \Pi$. The economy would then move from B towards A with a falling inflation as indicated by the arrows in the graph; this would be the case for all points on the A line to

[44] The issue will be subject to very detailed consideration in Chapter 8, yet it is useful to have in mind the main characteristics of the fiscal "reforms" simultaneous with the stabilizations: nearly insignificant new sources of taxation were created before or shortly after the stabilization, except in Poland, in which a new property tax was enacted. In Poland, however, the budget was never balanced. The levels of government expenditure were not reduced in any significant amount in any of the cases, quite on the contrary small increases are observed. Significant shifts in the direction of expenditure are indeed observed.

the left of D. Similarly, for expenditure equal to E we would have an equilibrium at F, to be reached from any point to the left of G and to the left of F.

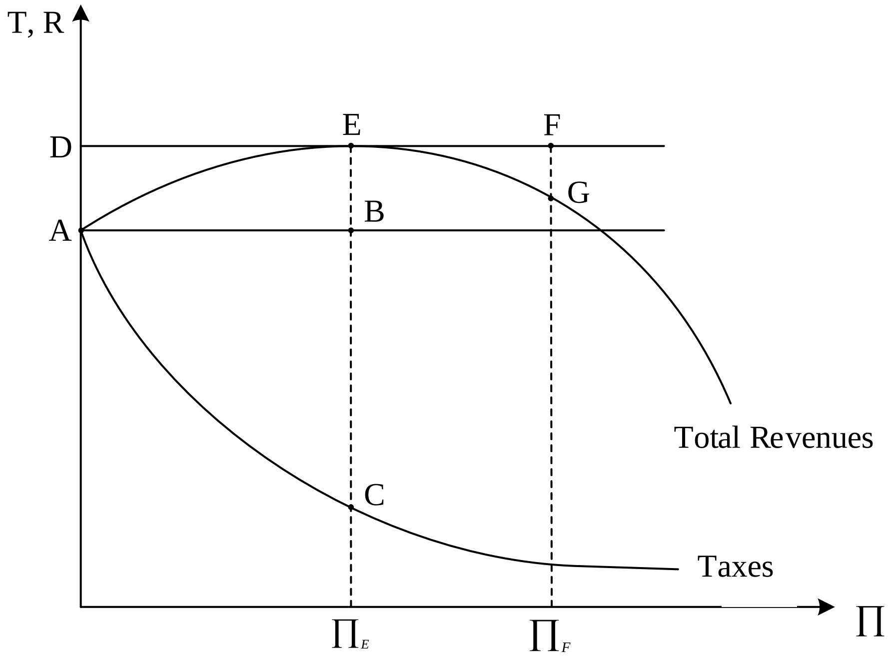
As in the case of the basic Cagan's model an hyperinflation can be generated if the economy is outside the total revenues curve: the purely monetary hyperinflation would obtain if the level of government expenditure is above the maximum total revenues at a steady state or at points like H in Graph 3. The other alternative considers levels of expenditure such as given by A or E and that the threshold levels of inflation Π and Π' respectively are, for some reason outside the model, surpassed. These two possibilities, and also the corresponding ones relative to the basic Cagan's model, will be subject to empirical verification in the next section. But, before we turn to that, however, it would be very interesting to discuss a "null hypothesis", or an alternative dynamic process relating inflation, money and the budget, against which the predictions of Cagan's model should be tested.

Consider that significant inflationary shocks are generated outside the monetary sector so that jumps in prices (in the exchange rates, for example) catch *both* the public and the central bank by surprise. Note that, in these conditions, prices have gone up *before* the increases in the money supply necessary to accommodate the shock have been implemented so that the "surprise" does not imply in any extra seigniorage collection. As the monetary authority provides accommodation for the shock the growth of money supply is basically determined by the speed to which agents adjust their money balances to desired levels. The basic point is that for these types of shocks we do not observe the collection of seigniorage above the steady state levels; if inflation is predominantly governed by non-monetary shocks we should observe the economy mostly inside the seigniorage (or the total revenues) curve. The implied dynamics can be considered with the help of Graph 4.

Consider that the government maintains a level of expenditure given by point A defining a balanced budget under zero inflation. A succession of external shocks then drive inflation to Π_E at which a budget deficit equal to BC is observed. Consider that the central banks monetizes this deficit so that an amount of money equal to BC is printed and put into circulation. It turns out that the private sector's demand for money at Π_E (and at the going real interest rate) is such that if the central bank does not print additional amounts of money to place the economy at point E a monetary stringency would ensue. From point B the central bank would issue notes against the discounting of "legitimate" trade bills until an amount of seigniorage corresponding to BE is collected. This means that for these non-monetary shocks we would observe that the amount of seigniorage generated would be greater than the government budgetary needs. "Surplus" funds in cash would be observed at the central bank which would create an incentive for the government to increase expenditure in order to capture all or part of the "unused" seigniorage revenue BE. In this case government expenditure could increase even up to the level corresponding to point D, in case of which all the additional seigniorage produced in accommodating Π_E is appropriated by the government. Most likely, however, a combination of the two would ensue, namely government expenditure and the printing of money against private bills would both adjust to changes in the inflation rate. In this case, both the level of government expenditure and the amount of seigniorage plus taxes actually collected would be inside the total revenues curve.

Graph 4

Monetary Dynamics for Non-Monetary Shocks



For inflation rates above Π_E this picture is somewhat changed. If an inflationary shock drives the economy from point E off the curve to a point like F, the amount of seigniorage generated in the accommodation process is no longer enough to finance the current budget deficit. Government expenditure might simply be revised downwards as the "surplus" funds are no longer available. Previous increases from A to D had been basically transitory so that, at least in theory, there seems to be no reason to presume that the same flexibility would not be displayed on the downwards direction. If this is not the case, or if the government insists in maintaining expenditure above what is determined by the curve, the only way to finance it would be to issue money above the "needs of trade" or engaging into a pattern of Cagan-like surprise increases in the money supply; these are the conditions under which a Cagan hyperinflation of the second type would most likely result.

In sum, if inflationary shocks are generated outside the monetary sector – meaning that inflation was not in any sense created by monetary expansion – the levels of expenditure and of total revenues actually collected would lie inside the total revenues curve for the most part of the inflationary process and would most likely be on the curve as inflation reaches exceptionally high levels. These are exactly the opposite predictions of Cagan's model, for which inflation is created because the economy is outside the seigniorage (total revenues) curve and the dynamics of the model indicates that the economy will tend to drift further off the curve. The predictions of these alternative models are very easily distinguishable empirically, in fact by determining the position of the economy relative to the total revenues curve we are able to identify the "type" of shock that is behind the inflationary process. This is the agenda of the next section.

5. The Monetary Dynamics Examined

This section offers empirical verifications of the alternative monetary dynamics considered in the last section. We will consider first the basic Cagan model with an exogenous budget deficit and verify whether seigniorage effectively collected was in excess of the maximum steady state levels, as predicted in the case of a purely monetary hyperinflation, and also whether we had hyperinflations of the second type, or in which seigniorage collection was greater than steady state levels for high inflation rates. Similarly, we will consider Cagan's extended model with an explicit government budget and verify whether total revenues actually collected (including taxation and seigniorage) and government expenditure were greater than the maximum steady state revenues and if they fall outside the total revenues curve. Then, we consider last section's "null hypothesis" more closely; in this respect we check whether there was monetary expansion in excess of government budgetary needs and whether government expenditure conformed with Cagan's exogeneity (with respect to inflation) assumption or with the adjustment process consistent with the "null hypothesis".

Consider first the ratios reported in Table 4 corresponding to the amount of seigniorage effectively collected as a proportion of the maximum steady state seigniorage collection. The former considered increases in the money supply deflated with exchange rates on a monthly basis and averaged for the respective quarter; the latter was estimated considered the assumptions made in connection with the computations of Table 2[45].

The resulting ratios are generally low for all countries. For Hungary they are everywhere low and for Poland values somewhat higher, though never greater than one, are observed at later quarters. The Austrian ratios are on the high side but, like the German ones, a value significantly greater than one is only observed in the very last quarter of the hyperinflation. These ratios indicate quite clearly that these countries did not collect seigniorage above the maximum steady state level. This means that in the framework of the basic Cagan model without a government budget, there is no empirical

[45] We assumed a Cagan type money demand equation with the values for the parameter β as given by the medians in Table 4-1; for the zero inflation demand for money, or the parameter α, we considered the observed values in a "normal" post-stabilization year, generally one year after.

base to support the claim that these countries experienced a Cagan type hyperinflation of the first sort, namely a purely monetary hyperinflation.

Table 4

Seigniorage Actually Collected as a Proportion of the Maximum Steady State Levels[1]

(quarterly averages-the number in parentheses indicate monthly average inflation rates in the respective quarter)

Date	Germany[2]	Hungary[3]	Poland[4]	Austria[5]
1921 - I	-0.10 (-3.6)	0.09 (-52)	0.75 (15.0)	0.79 (6.6)
II	0.29 (0.7)	-0.12 (-82)	0.64 (22)	0.65 (2.2)
III	0.31 (15.3)	0.25 (15.3)	0.36 (19.5)	0.40 (8.6)
IV	0.70 (20.0)	0.17 (7.4)	0.40 (-1.4)	0.88 (64.0)
1922 - I	0.19 (16.4)	0.13 (9.6)	0.12 (8.9)	0.72 (14.8)
II	0.35 (9.1)	0.15 (11.4)	0.25 (6.1)	0.80 (35.5)
III	0.58 (61.0)	0.38 (27.4)	0.58 (20.5)	1.39 (85.5)
IV	0.76 (76.0)	0.17 (8.3)	0.55 (31.5)	-
1923 - I	0.95 (59.0)	0.15 (272)	0.75 (43.3)	-
II	0.67 (66.8)	0.37 (302)	0.58 (26.7)	-
III	1.76 (1942)	0.76 (59.8)	0.89 (57.8)	-
IV	-	0.32 (12.9)	0.91 (177.0)	-
1924 I	-	0.35 (42.9)	-	-
II	-	0.56 (2.1)	-	-

Sources and observations: (1) Seigniorage collected is change in money supply deflated with exchange rates. Maximum steady state seigniorage collection is computed from "optimal" rates of inflation from Table 4-2 and with zero inflation levels of money demand as indicated below. (2) Money supply figures from J. P. Young (1925), vol. I pp. 527-529, converted in gold marks with exchange rates from F. Graham (1930), pp. 156-158. Zero inflation demand for money assumed equal to 4200 million gold marks and maximum seigniorage considered was 690 million gold marks. (3) Money supply figures from J. P. Young (1925), vol. II pp. 321-322 and exchange rates from L. L. Ecker-Rácz (1933a), p. 61. Zero inflation demand for money assumed equal to 394 million gold crowns and maximum seigniorage considered was 68.5 million gold crowns. (4) Money supply figures from J. P. Young (1925), vol. II pp. 347-348 and exchange rates from Republic of Poland (1926), pp. 126 and 173. Zero inflation demand for money assumed equal to 550 million zloty and maximum seigniorage considered vas 80.4 million zloty. (5) Money supply and exchange rates figures from J. van Walré de Bordes (1924), pp. 46-50 and 116-139. Zero inflation demand for money assumed equal to 542.5 million gold crowns and maximum seigniorage considered was 40.6 million gold crowns.

It might be that these episodes characterized hyperinflations of the second sort, namely that exploding inflations were observed after these economies were driven outside the seigniorage curve at some point in the process. In Table 5 we verify this possibility by measuring the ratios between the seigniorage effectively collected and the steady state seigniorage collection at the observed rate of inflation.

The first thing to note is that a number of high ratios are observed early in the hyperinflation period; except for the first quarter in Austria and Poland all these quarters correspond to periods of price stability or deflation so that these are situations in which the economy was outside the seigniorage curve but at the left side. Other than that we observe consistently low ratios for Hungary as in Table 4, and ratios on the high side in Poland and Austria. In Germany after 1922-1V and in 1921-IV Austria we observe ratios greater than one that coincide with jumps in inflation, yet an explosive inflation is not observed after that; Austria would return to the inside of curve the next quarter and the German ratios would show declining values until 1923-III. This seems consistent with our "null hypothesis", namely that these economies would return to the curve if driven off. Polish and Hungarian ratios shows a high value in the very last quarter though for the latter this corresponded to the post stabilization remonetisation of the economy. Only in for the very last quarter in Germany, Poland and Austria there is indication of these economies being well out of the curve; but only in Germany an unambiguous inflationary "explosion" is observed.

Table 5

Seigniorage Actually Collected as a Proportion of Steady State Levels

(quarterly averages - the number in parentheses indicate monthly average inflation rates in the respective quarter)

Date	Germany	Hungary	Poland	Austria
1921 - I	0.23	-0.27	1.07 (15)	1.24 (6.6)
II	3.93 (0.7)	0.21	4.50 (2.2)	2.70 (2.0)
III	0.35	0.39	0.44	0.53
IV	0.72	0.47	-4.07 (-1.4)	1.33 (64)
1922 - I	0.21	0.29	0.26	0.77
II	0.51	0.29	0.70	0.69
III	0.69	0.43	0.70	3.67 (85)
IV	1.83 (76)	0.42	0.57	-
1923 - I	1.60 (59)	0.16	0.76	-
II	1.37 (67)	0.40	0.62	-
III	$8.1\text{x}10^{31}$ (1942)	0.79	0.97	-
IV	-	0.55	6.87 (177)	-
1924 - I	-	0.35	-	-
II	-	4.78 (2.1)	-	-

Sources and observations: (1) Seigniorage collected is change in money supply deflated with exchange rates from sources described in Table 4. Steady state seigniorage collection is computed assuming a Cagan type money demand function with semi-elasticities β given by medians from Table 1 and for α, the zero inflation demand for money we considered the values used in Table 4.

In sum, the ratios reveal that these countries did not experience Cagan hyperinflations of the second sort except at the very end; that means basically that the model does not explain how the extremely large level of inflation of the pre-explosion quarters (61% for 1922-III Germany, 42.9% for 1924-I Hungary, 57.8% for 1923-III Poland and 35.5% for 1922-II Austria) were reached. Cagan's model would predict decelerating inflations if these economies remained inside the seigniorage curve so that the evidence seems unambiguous that the inflationary process was mostly determined from the outside.

Table 6

Total Revenues (Seigniorage plus Taxes) Actually Collected

as a Proportion of the Maximum Steady State Levels

(quarterly averages - the number in parentheses indicate monthly average inflation rates in the respective quarter)

Date	Germany[1]	Hungary[2]	Poland[3]	Áustria[4]
1921 - I	0.60	0.22	0.50	0.66
II	0.71	0.08	0.45	0.59
III	0.54	0.35	0.31	0.47
IV	0.61	0.30	0.33	0.70
1922 - I	0.39	0.28	0.30	0.56
II	0.52	0.29	0.41	0.59
III	0.52	0.40	0.45	0.88
IV	0.49	0.25	0.42	0.49
1923 – I	0.64	0.24	0.53	-
II	0.53	0.39	0.60	-
III	1.09 (1942)	0.68	0.64	-
IV	-	0.38	0.60	-
1924-1	-	0.40	-	-
II	-	0.54	-	-

Sources and observations: For sources and observations on money supply and exchange rate figures see Table 4. The parameters necessary to compute maximum steady state revenues are discussed in section 3. (1) Tax revenues from Republic of Germany-Krieglastenkommission (1924), p. 32, but with figures for 1923-III from Franco (1986). Optimal level of revenues considered was 690 million gold marks. (2) For actual tax revenues we considered the annual values proportionally distributed throughout the respective gear. The value of optimal revenues considered was 100.7 million gold marks. (3) Tax revenues from Republic of Poland (1926), p. 173. Tax revenues for 1921 were estimated on the basis of the annual figures reported in J. P. Young (1925), vol. II, p. 183 redeflated by the correct average exchange rate for the period as reported in Republic of Poland (1926b), p. 126 and uniformly distributed for the year. The optimal value for revenues considered was 163.9 million zloty. (4) Actual tax revenues obtained from the annual values in Table 8-1 equally distributed through the corresponding year. The optimal value for revenues vas of 84.4 million gold marks.

Now let us consider if these results carry over to the extended Cagan model with an explicit government budget. Table 6 shows ratios between the total revenues actually collected and our estimates of the maximum steady total revenues. Actual total revenues correspond to seigniorage collection as of Tables 4 and 5 and actual tax collection. Maximum steady state revenues considered the formulas of section 3, namely a Cagan type money demand and a tax function as given by equation (10) with the parameter values used in the computations associated with Table 2.

Table 6 replicates in full the results of Table 4 for the basic Cagan model; the ratios are everywhere low which does confirm that we do not have hyperinflations of the first sort in any of these countries except for Germany in the very last quarter, which is precisely the moment in which inflation did show an "explosive" behaviour. That means that, in the framework of the extended

Cagan model with endogenous taxes, we found no support for the conjecture that these hyperinflations were purely monetary phenomena.

Next we check whether in the extended model there is indication of hyperinflations of the second sort, or determined by total revenues actually collected being outside the total revenues curve. Table 7 reports ratios between revenues actually collected and the revenues that could be collected at that rate of inflation in a steady state, i.e., the relevant point of the total revenues curve.

The ratios of Table 7 behave almost exactly like the ones of Table 5. There are some high values at early periods when prices were stable, but only for Germany we observe the economy off the total revenues curve. The ratios are generally low for Poland and Hungary except for the very last quarter. In Germany and Austria, the economy is driven out of the curve in 1922-IV and 1921-IV, respectively, when inflations registered a large jump. Yet, as we also observed in Table 5, the tendency was for these economies to retum to the curve; in 1923-II Germany appeared to be nearly on the curve and Austria well inside in 1922-II. Again an unambiguously "explosive" behaviour is only observed in 1923-III Germany.

Table 7

Total Revenues (Seigniorage plus Taxes) Actually

Collected as a Proportion of the Steady State Levels

(quarterly averages - the number in parentheses indicate monthly average inflation rates in the respective quarter)

Date	Germany	Hungary	Poland	Austria
1921 - I	1.77 (-3.6)	0.77	0.55	0.74
II	1.17 (0.7)	0.53	0.66	0.83
III	0.55	0.42	0.32	0.51
IV	0.61	0.43	0.58	1.12 (64)
1922 - I	0.40	0.37	0.37	0.56
II	0.58	0.37	0.53	0.59
III	0.64	0.41	0.47	2.01 (85)
IV	1.06 (76)	0.35	0.42	-
1923 - I	1.01 (59)	0.25	0.54	-
II	0.97	0.40	0.61	-
III	$2.6x10^9$ (1942)	0.73	0.71	-
IV	-	0.46	1.87 (177)	-
1924 - I	-	0.40	-	-
II	-	0.96	-	-

Sources and observations: see Table 4-6 for sources and methodology.

Since total revenues actually collected, if correctly calculated, should be exactly equal to the portion of government expenditure not financed by debt, the results of Tables 6 and 7 can be double-checked by recalculating the relevant ratios using figures for government expenditure, instead of those for actual total revenues collected. This is done in Table 8 and 9, in which we considered the monthly data on government expenditure that is available for Germany and for Poland after 1921, and considered annual values uniformly distributed in the four quarters for the other countries and for 1921 Poland.

The fact that we do not have monthly figures for government expenditure for Austria and Hungary does not allow us a clear verification of the results of the past tables, yet ratios significantly lower than one are observed in both cases as we did have in Table 6. For Poland and Germany, the results of Tables 6 and 8 are very similar – except for the first two quarters for Germany – which is a powerful indication that our estimates for total revenues actually collected seem to be correct. That means basically that our conclusions as regards the presence of hyperinflations of the first type are not altered in any meaningful way.

Table 8

Government Expenditure as a Proportion of the Maximum

Steady State Levels of Total Revenues (quarterly averages)

Date	Germany[1]	Hungary[2]	Poland[3]	Austria[4]
1921 - I	0.96	0.47	0.34	0.67
II	1.05	0.47	0.34	0.67
III	0.87	0.32	0.34	0.67
IV	0.59	0.32	0.34	0.67
1922 - I	0.48	0.32	0.33	0.66
II	0.56	0.32	0.35	0.66
III	0.39	0.20	0.32	0.66
IV	0.43	0.20	0.34	-
1923 - I	0.47	0.20	0.47	-
II	0.58	0.20	0.53	-
III	0.97	0.34	0.51	-
IV	-	0.34	0.43	-
1924 - I	-	0.34	-	-
II	-	0.34	-	-

Sources and observations: (1) Expenditure figures from Republic of Germany-Krieglastenkommission (1924), p. 32. We deducted from the total the values of the increases in government debt outside the Reichsbank as of *ibid.*, p. 63. (2) Expenditure figures are annual values from Franco (1986) divided equally between quarters. (3) Expenditure figures from Republic of Poland (1926b), p. 173. The values for 1921 consider the annual figures reported in J. P. Young, vol. II, p. 123 redeflated as described in Table 4. (4) Expenditure figures are annual values as of Table 8-1 divided equally between quarters.

Table 9 reports the ratio of government expenditure over steady state values for total revenues. The picture is very much the some we had in Table 7: Poland shows consistently low ratios, except for the very last quarter; for Hungary we also observe low ratios but, like in Germany, we observe high values for the first two quarters when inflation was negative[46]. The ratios for Austria and Germany approach one at some points but like it was observed in Table 7, the tendency seems to be for these economies to return to the inside of the curve; the exception is again 1923-III Germany.

Table 9

Government Expenditure as a Proportion of Steady State Levels of Total Revenues

(quarterly averages - the number in parenthesis indicate the monthly average inflation rate in the quarter)

Date	Germany	Hungary	Poland	Austria
1921 - I	2.87 (-3.6)	1.65 (-5.2)	0.37	0.75
II	1.73 (0.7)	3.19 (-8.2)	0.50	0.93
III	0.89	0.38	0.35	0.72
IV	0.59	0.46	0.59	1.06 (64)
1922 - I	0.48	0.42	0.40	0.67
II	0.62	0.41	0.45	0.66
III	0.48	0.21	0.34	1.55 (85)
IV	0.93	0.28	0.33	-
1923 - I	0.74	0.21	0.48	-
II	1.06 (67)	0.20	0.53	-
III	2.3×10^9 (1942)	0.37	0.56	-
IV	-	0.41	1.34 (177)	
1924 - I	-	0.34	-	-
II	-	0.61	-	-

Sources and observations: see Tables 6 and 8 for sources and methodology.

In sum, Tables 4 through 9 provide unambiguous indications that none of the hyperinflations was produced by the systematic necessity to collect seigniorage above steady state levels. The evidence is clear that in no case we found support for a Cagan type hyperinflation of the first sort, namely for which the levels of government expenditure (or budget deficit) were above the maximum steady state levels. The evidence as regards hyperinflations of the second sort is also unfavourable. There are indications that at some points these countries entered the "explosive" inflation region, but there seemed to be a tendency for these economies to return and to remain on the steady state revenues curve as proposed in our "null hypothesis". Only at the very last quarter these economies seem to be well in the instability region, yet an inflationary "explosion" is only observed in Germany. In any event, Cagan's model falls to explain how the inflationary process got started and reached the very high pre-explosion levels; it is also at odds with the fact that a truly "explosive" inflation is only

[46] It is interesting to observe as regards Germany that in these three quarters, in contrast with other periods, the increases in the floating debt held at the Reichsbank exceeded very significantly the increases in the money supply, as shown in Table 4-10 ahead. Considering government expenditure net of this non-monetized portion of the floating debt, we would not observe the increases these high ratios during the first three quarters of 1921.

observed in one case.

The fact that there are economies remained inside the total revenues curve much of the time, and that they seemed to stay on the curve for some period, is very much consistent with what our "null hypothesis" or with the idea that the inflationary shocks were generated from the outside. In this connection the pattern would be one of government expenditure and note issuing against credit creation being slowly increased as inflation accelerated – which would characterize movements from inside up; for higher levels of inflation, the adjustment would be from the outside down, so that it would be consistent with ratios fluctuating around one. If this is indeed the case, we should observe money creation systematically exceeding the budgetary needs of the government, as explained in Graph 3, at least in the earlier stages of the process. This can be very easily checked with the help of Table 10 which reports the relation between the increases in government indebtedness with the central bank as a proportion of the total increase in the money supply.

Table 10

Increases in the Floating Debt as a Proportion of the Increases in the Money Supply

(quarterly averages)

Date	Germany[1]	Hungary[2]	Poland[2]	Austria[3]
1921 - I	-	-	1.38	-
II	1.91	-	1.29	3.20
III	1.75	-	0.95	1.05
IV	0.84	0.21	0.56	0.74
1922 - I	0.80	0.51	0.52	0.75
II	0.94^	0.67	0.06	0.51
III	0.74	0.07	0.65	0.60
IV	0.60	0.46	1.01	0.54
1923 - I	0.56	0.80	1.03	-
II	0.71	0.43	0.72	-
III	1.01	0.38	0.95	-
IV	-	0.43	0.88	-
1924 - I	-	0.50	-	-
II	-	0.53	-	-

Sources and observations: (l) Considering only floating debt held at Reichsbank as proportion of the increases in note circulation plus demand deposits, from J. P. Young (1925). vol. I, pp. 527-529. (2) Figures from J. P. Young (1925) vol. II, pp. 347-348 and 321-322. (3) Figures from J. van Walré de Bordes (1924), p. 54.

For Hungary the ratios are low indicating that "private inflation" or note issuing against credit creation through discounting of "legitimate" trade bills was the more important source of money creation during the whole period[47]. This is also true for Austria; the high ratios observed in 1921-II and III are due to the fact that up to this point the discounting of Treasury bills was not distinguished from private bills and when this started to be done we observe a large "artificial" increase in the floating debt at the central bank[48].

For Poland we observe high numbers for the first three quarters of 1921 and after 1922-IV. For the earlier period this could still be associated with aftermath of the war with the Soviet Russia, and for the later period the higher ratios are consistent with the significant increase in government investment expenditure and budget deficits after the collapse of the Michalski's stabilization experiment and the annexation of Upper Silesia in the summer of 1922[49]. Government deficits were the main determinant of money creation from then on, but for the last three quarters of 1923 "private inflation" was still significant. For Germany we observe especially high ratios in 1921-II and III, a period in which money creation was held off but reparations payments under the London schedule precluded the corresponding fiscal restraint. The ratios are generally high for Germany indicating that government spending was indeed the major determinant of money creation; but as for Poland "private inflation" remained quite significant.

Table 10 reveals that monetary expansion in excess of the government budgetary needs is very often observed; yet, especially in Germany and Poland government expenditure appears to play the major role as the determinant of money creation. This fact by itself is not inconsistent with inflationary shocks being generated from the outside for what might be taking place, as explained in section 4, is that government expenditure would be adjusting to the availability of funds created by the seigniorage generated in accommodating the current levels of inflation. This can also be subject to verification and it is fortunate that Poland and Germany are precisely the two countries for which we have figures for government expenditure on a monthly basis.

From the empirical point of view there are two basic indications that the adjustment process suggested above is taking place, one connected with government expenditure and the other with "private inflation", or money creation against "legitimate" private bills. If it is the former that does the adjustment, we should expect government expenditure to be always on the total revenues curve; the relation between government expenditure and inflation would be described by a parabola. "Private inflation" would play a residual role and a negative correlation should be observed between government spending and money creation against "real bills". If, on the other hand, government expenditure adjusts only partially, then both government expenditure and "private inflation" could move in the same direction so as to approach the economy to the total revenues curve. In this case,

[47] This finding is in full accordance with the detailed account of E. Boross (1985) of the activities of the Hungarian Note Institute during the inflation period, esp. pp. 208-209.
[48] As explained in J. van Walré de Bordes (1924), p. 54 ff.
[49] These episodes are examined in more detail in Chapter 10.

"private inflation" and government expenditure could be positively correlated. All these claims, together with the exogeneity of government expenditure implied in Cagan's model, could be given an empirical check by means of the estimation of an equation like:

$$G(\Pi) = a\Pi^2 + b\Pi + c + d\frac{\Delta B}{P} \tag{15}$$

commercial bills and advances extended to the private sector. All equations include dummy variables accounting for a few outliers observed in the series for G.

The regression results for Germany provide full support for the hypothesis of government expenditure adjusting to changes in inflation rates as predicted in our "null hypothesis"; the signs are exactly the ones predicted, namely $a < 0$ and $b > 0$, and all coefficients are significantly different from zero at 1%.

Table 11

Regressions of Government Expenditure on Inflation and "Private Inflation"

(t statistics in parentheses)

Country	$a\Pi^2$	$b\Pi$	$const.$	$\frac{d\Delta B}{P}$	$dummy$	$trend$	R^2	DW
Germany[1]	-0.07 (-4.88)	22.3 (4.93)	459.7 (10.1)	-1834 (-2.63)	394.8 (4.96)	-	0.751	1.42
Poland[2]	5.15 (1.56)	-29.5 (-3.42)	45.6 (11.5)	25.0 (1.72)	41.9 (4.34)	2.13 (5.54)	0.875	1.71
Poland[2]	-	-18.5 (-5-21)	42.1 (10.9)	-	39.7 (3.93)	2.59 (10.3)	0.844	1.63
Poland[2]		-21.6 (-4.60)	19.1 (1.94)	-59.3 (-1.12)	44.2 (4.39)	4.16 (4.38)	0.875	1.69

Sources and observations: (l) Government expenditure data in millions of gold marks deflated with exchange rates. The period considered was January 1921 to June 1923. The figures are from Republic of Germany-Krieglastenkommission (1924), p. 32. The "private inflation" variable was the real value of the change in the stock of bills discounted and advances extended to the private sector as reported in J. P. Young (1925), vol. I, pp. 526-529 deflated with exchange rates. (2) Government expenditure data in millions of zloty for January 1922 to April 1924 from Republic of Poland (1926), pp.173-177. The "private inflation" variable considered was the real value of the change in the stock of bills discounted to the private sector from J. P. Young (1925), vol. II, pp. 348-349 deflated with wholesale prices. (4) For sources for price data, see Table 2.

lf the relation between G and Π is described by a parabola, or that it looks like the total revenues curve shown in Graph 1, then we should have $a < 0$ and $b, c > 0$. The coeficient for the "private inflation" variable – here measured as the real value of credit extended to the private sector ($\Delta B/P$) – would be negative, i.e., $d < 0$ if the economy is at or close to the total revenues curve and could be positive if the economy is inside the curve. Alternatively, if the hyperinflation is governed by a process such as the one suggested by Cagan, then G should show no correlation with inflation, so $a = b = 0$, and neither with the amount of credit given to the private sector. Cagan's model would not be incompatible with a positive correlation between G and Π, or $a = 0$ and $b > 0$, if we assume that the monetary dynamics he suggested could be superimposed to other factors influencing the rate of inflation and the level of expenditure. In this circumstance, shocks of whatever nature could be causing Π and G to vary but Cagan's dynamics would induce inflation to be always on the left side of the total revenues curve, or at points like A or F on Graph 2[50]. All these hypotheses are very simply confronted with the data by the estimation of equation (15) the results of which are reported in Table 11 below.

Table 11 reports regressions for Germany and Poland; the Polish equations includes a trend term whose justification is basically the same underlying its inclusion in the equation for Polish taxes. The "private inflation" variable was the real value of the change in the stock of government expenditure – which is characteristic of Cagan's model and also of Sargent and Wallace's 1973 paper – is very obviously rejected. The German equation also shows a significant negative correlation between credit creation to the private sector and government expenditure. This is consistent with an economy on the total revenues curve, in case of which credit creation would be playing a residual role in meeting the constraint.

The first Polish equation reveals that a and d are not significantly different from zero; the second equation omit the corresponding variables without significant loss in explanatory power. We observe a clear negative correlation between G and Π that is totally opposed to what would be predicted by Cagan's model. This result would be consistent with Sargent and Wallace more recent work[51]; but more realistically it seems to be generated by the fact that our sample does not include but a couple of observations for low inflations, so that it is like we are observing only the downward sloping segment of the total revenues curve. The weak positive correlation between G and the credit creation variable is consistent with an economy that is inside the total revenues curve and for which both government expenditure and credit creation can adjust to different levels of inflation without touching the total revenues curve. In Tables 5, 7 and 9 it seems clear that Poland remained very much inside the curve, or at least more to the inside than Germany. But Poland seems definitively closer to the total revenues curve later in the hyperinflation period. The third Polish regression in Table 11 considered only observations relative to the last quarter of 1922 and for 1923; it is interesting that the

[50] An alternative hypothesis has been suggested more recently in T. Sargent and N. Wallace (1984), according to which the economy would always tend to find an equilibrium on the "wrong" side of the total revenues curve. In principle no correlation should be observed between G and Π, for their model, like Cagan's presumes a constant G; but if G is, for some reason, changed the model would predict that G and Π would be negatively correlated or $a = 0$ and $b < 0$. Neither Sargent/Wallace or Cagan would there be any reason to presume any correlation between inflation and "private inflation".
[51] T. Sargent/N. Wallace (1984).

coefficient for the credit creation variable appears with a negative sign (although not significant), as expected from an economy closer to the curve. In sum, the relationship between government expenditure and inflation that emerges from these regressions seems to indicate that Poland and in Germany were indeed changing their levels of government expenditure in order to remain on the total revenues curve, or they adjusted spending according to the availability of funds. This is fully consistent with our hypothesis of exogenously determined inflationary shocks and quite at odds with Cagan's hypothesis of exogenous government expenditure[52].

6. Summary and Conclusions

In this paper we reviewed the monetarist theories of inflation that are usually applied for the hyperinflations. We surveyed briefly the theories of "optimal" inflation in section 2 and then we confronted the predictions of these theories with the hyperinflation figures in section 3. As it has been often observed these theories underpredict actual inflations and, perhaps more important than that, these theories are at odds with the incredible variability observed in the inflation data. Then we turned our attention to a second group of models associated with Cagan's benchmark model. The dynamic process implied by this model is discussed in detail in section 4 in conjunction with an extended version including an explicit government budget in which taxes are adversely affected by inflation. We also discussed in section 4 the type of monetary dynamics that is triggered by inflationary shocks determined from the outside of the monetary sector and to which the monetary authority provides accommodation. The predictions of these models are empirically compared in section 5. It is shown that seigniorage collection was not in excess of the maximum steady state levels, as predicted by the basic Cagan model in the case of a hyperinflation generated without any sort of non-monetary influence. The corresponding hypothesis was also rejected in the context of the extended Cagan model with endogenous taxes, namely total revenues actually collected and government expenditure were never greater than the maximum steady state total revenues. We also considered the weaker form of a Cagan hyperinflation, by means of which an "exploding" inflation is observed because these economies would have fallen outside to the right of their total revenues (seigniorage) curves by means of some non-monetary shock. It was shown that this was not the case except for the very last quarters of the process; in moments when these economies seem to enter the instability region the tendency was to return to the curve rather than to "explode" so that the evidence does not support a Cagan hyperinflation of this second sort. The only exception is Germany, for whom indeed an "explosive" inflation is observed in the last quarter when the economy was well in the instability

[52] See also T. Sargent/N. Wallace (1973), p. 422.

region. On the other hand, the model does not explain how similar "explosions" did not take place in other countries and neither does it explain how the "pre-explosion" rates of inflation were reached at all (German inflation averaged between 60% and 70% *a month* for the year before she fell into the instability region), for economies inside the total revenues (seigniorage) curve should experience falling rates of inflation. In sum, except for the very last quarter of the German inflation, Cagan's model is in absolute contradiction with these hyperinflation episodes.

Our basic conclusion is that inflation was predominantly, if not totally, determined from the "outside". This is consistent with the evidence mentioned above, namely that these economies remained inside the total revenues (seigniorage) curve most of the time. It is further verified that monetary expansion was very often far in excess of the government budgetary needs, and that government expenditure in Poland and Germany seem to conform with the type of adjustment proposed in our "null hypothesis". In sum, the evidence presented in this paper provides no support for the monetarist theories of the determination of inflation in the context of the hyperinflations of Germany, Poland, Hungary and Austria. The monetarist presumption that the collection of seigniorage was the one and only purpose of inflation appears to be quite inappropriate for these hyperinflations. These governments not only did not maintain inflation at their "optimal" levels in whatever criteria, as shown in section 3, but they also failed even to reach the total revenues (seigniorage) curve. If the whole purpose of the episode was to produce an increase in the spending power of the government then there seems to be no doubt that the attempt was an astounding failure. These conclusions certainly help to do away with preconceived ideas about the hyperinflations, but they provide little information on how these episodes were generated. This is a challenge yet to be properly faced.