TEXTO PARA DISCUSSÃO

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Fixprice analysis of labor-managed economies

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1. Introduction

Since the publication of Benjamin Ward's [1958] seminal work on the labor-managed firm the literature on the subject expanded rapidly. The hypothesis of the perversity of the firm's supply curve was extensively discussed (Meade [1972], Vanek [1970], Steinherr and Thisse [1979]), while another branch of the literature focused on the efficiency of general equilibrium (Drèze [1976], Ichiishi [1979, 1981]). Oddly enough, almost no attention was given to the macroeconomics of labor-managed economies. Some remarks on the problems of employment and inflation are scattered in the literature (Vanek [1970], Meade [1972], Tyson [1980]), but there was no attempt to elaborate a specific macro theory adapted to these economies. The aim of the present paper is to begin to fill this gap. In the spirit of Barro-Grossman [1971], Malinvaud [1977], Benassy [1982], and others, we construct a disequilibrium macro model with one (labor-managed) representative firm, and establish the possibility of different regimes (Classical, Keynesian, and Inflationary). The nominal balances-price levei parameter space is partitioned into four regions: three of these correspond to the different types of fixprice equilibria and in the fourth one no equilibrium exists. We also derive short-run comparative statics results for each equilibrium type.

It was conjectured by Vanek [1970] and Meade [1972] that the perversity of the labor-managed firm's supply curve would have adverse effects on macroeconomic stability. Meade, for example, claimed that "to rely on Keynesian policies to expand effective demand in times of unemployment would be at best ineffective, and at worst might lead to a reduction in output and employment". We analyze the question in the context of our model, and find out that while the conjecture is false as long as short-run analysis is concerned, it is correct when the long-run dynamic behavior of the economy is considered. More precisely, we find the same signs for short-run multipliers as those that would obtain in a capitalist economy. Nevertheless, when Classical unemployment prevails, the adoption of "expansionista" policies like increases – in govennement expenditure or in the money supply would cause an acceleration of a process of decline in output coupled with price inflation. It is true that our analysis establishes that such a process would take place anyway, as long as the Classical region would be entered. The adverse effects of the policies mentioned above consist then in the *acceleration* of the output reduction-price inflation process.

Given these results, and the implicit lack of stability of the (unique) Walrasian equilibrium, one is tempted to conclude that labor-managed economies are inferior to their capitalist counterparts from the macroeconomist's point of view. A more careful analysis of the menu of available macro policies reveals that this is not necessarily the case. When unemployment is of the Classical type tax increases, besides reducing the deficit, have desirable effects on output, employment and inflation, and at least in this case the governement may be able to solve three problems at once.

2. The Model

We consider an economy with three coramodities, a consumption good, labor, and money. The price of the consumption good is exogenously fixed in each period, while the wage rate is endogenously determined. No future contracts are allowed. Money is the only store of value. The market for the consumption good is cleared by quantity rationing on the long side. There is no labor market in the usual sense. Firms are labor-managed, which means they try to maximize revenue net of nonlabor costs per worker. This average net revenue is henceforth called the "wage" rate, since it plays a role similar to that played by the wage rate in a capitalist economy. Both the demand for labor and the desired (by firms) wage rate are thus functions of the price of the consumption good. This means labor demand is *not* a function of the wage rate. Labor supply is, as in a capitalist economy, a function of the prevailing rate. We can still speak of excess demand or supply of labor, but one such situation of disequilibrium does not affect the wage rate according to the mechanism usually called the Law of Supply and Demand.

Let m be total nominal balances, and p be the price of the consumption good. The labor-managed firm has a production function $f: R_+ \to R_+$, twice continuosly differentiable, monotonic, and strictly concave, and a fixed cost (in nominal terms) of K. The wage rate is then given by

$$w(L, p) = \frac{pf(L) - K}{L} \tag{1}$$

The notional labor demand $L^{P*}(p)$ is then given by arg $\max_L w(L, p)$, and the "notional" wage rate is $w^{P*}(p) = \max_L w(L, P)$. We also denote the notional supply of the consumption good by $y^{P*}(p)$. Of course, $y^{P*}(p) = f[L^{P*}(p)]$. Since L^{P*} is implicity given by

$$\frac{pf[L^{P*}(p)] - K}{L^{P*}(p)} = pf'[L^{P*}(p)] \tag{2}$$

We also have

$$w^{P*}(p) = \frac{pf[L^{P*}(p)] - K}{L^{P*}(p)} = pf'[L^{P*}(p)] \tag{3}$$

It is well-known that (cf. Ward [1958])

$$\frac{\partial L^{P*}(p)}{\partial p} < 0$$

$$\frac{\partial y^{P*}(p)}{\partial p} < 0$$

$$\frac{\partial w^{P*}(p)}{\partial p} < 0$$

Thus, the labor-managed firm has a perverse supply curve. It is easy to show that $L^{P*}(p)$ is uniquely defined. Indeed,

$$\frac{\partial w(L,p)}{\partial L} = \frac{pf'(L) - w(L,p)}{L} \tag{4}$$

$$\frac{\partial^2 w(L,p)}{\partial L^2} = \frac{pf''(L) - \frac{2\partial w(L,p)}{\partial L}}{L} \tag{5}$$

and hence $\frac{\partial w(L,P)}{\partial L} = 0$ implies that w is concave at the point L, and $w(.,p)$ can have no local minima. We also conclude that $w(.,p)$ is increasing and strictly concave in the interval $[0, L^{P*}(p)]$.

If we define $\underline{L}(p)$ by the equation $pf\left[\underline{L}(p)\right] = K$, we then clearly have

$$w(L,p) \gtreqless 0 \leftrightarrow L \gtreqless \underline{L}(p)$$

We can now define the effective labor demand $L^P(p,\overline{y})$ and the effective labor supply $y^P(p,\overline{L})$ as follows:

$$L^P(p,\overline{y}) = \begin{cases} L^{P*}(p) & if\ \overline{y} \geq y^{P*}(p) \\ f^{-1}(\overline{y}) & if\ f\left[\underline{L}(p)\right] \leq \overline{y} < y^*(p) \\ 0 & if\ \overline{y} < f[\underline{L}(p)] \end{cases} \tag{6}$$

$$y^P(p,\overline{L}) = \begin{cases} y^{P*}(p) & if\ \overline{L} \geq L^{P*}(p) \\ f(\overline{L}) & if\ \underline{L} \leq [\overline{L} < L^{P*}(p)] \\ 0 & if\ \overline{L} < \underline{L} \end{cases} \tag{7}$$

We also define two "effective" wage rates in the following way:

$$w^{PL}(p,\overline{L}) = w\{f^{-1}[y^P(p,L)]\} \tag{8}$$

$$w^{Py}(p,\overline{y}) = w[L^P(p,\overline{y}),p] \tag{9}$$

Now, let x be the consumption of the private sector, in real terms. We denote by $L^{C*}(p,w,m)$ and by $x^{C*}(p,w,m)$ the notional labor supply and good demand functions. These functions are assumed to satisfy the following conditions:

$$(CW) \qquad \frac{\partial L^{C*}}{\partial p} < 0 \qquad \frac{\partial L^{C*}}{\partial m} < 0 \qquad \frac{\partial L^{C*}}{\partial w} > 0$$
$$\frac{\partial x^{C*}}{\partial p} < 0 \qquad \frac{\partial x^{C*}}{\partial m} > 0 \qquad \frac{\partial x^{C*}}{\partial w} > 0$$

These properties could be derivd, in the context of an economy with many identical consumers, from properties of the utility function of these consumers, as done by Van den Heuvel [1983]. We prefer, however, to assume them directly.

We can also define an effective labor supply function $L^C(p,w,m,\overline{x})$ and an effective good demand function $x^C(p,w,m,\overline{L})$ in the same way as these functions are defined for a capitalist economy. These functions are assumed to satisfy the following conditions:

$$(CE) \qquad \frac{\partial L^{C}}{\partial p} < 0 \qquad \frac{\partial L^{C}}{\partial m} < 0 \qquad \frac{\partial L^{C}}{\partial \overline{x}} > 0$$
$$\frac{\partial L^{C}}{\partial w} \geq 0 \qquad\qquad > if\ w > 0$$

$$L^C(p,0,m,\overline{x}) = 0$$

for all values of p, m and $\overline{x}$.

$$\frac{\partial x^{C}}{\partial p} < 0 \qquad \frac{\partial x^{C}}{\partial m} > 0 \qquad \frac{\partial x^{C}}{\partial \overline{L}} > 0$$
$$\frac{\partial x^{C}}{\partial w} \geq 0 \qquad\qquad > if w > 0$$

The functions $L^C(.)$ and $x^C(.)$ are themselves useless in the analysis of a labor-managed economy, where the wage rate is a function of the level of output and employment. We thus define the LM-effective labor supply function $\hat{L}^C(p,m,\overline{x})$ and the LM-effective good demand function $\hat{x}^C(p,m,\overline{L})$ as follows:

$$\hat{L}^C(p,m,\overline{x}) = L^C[p, w^{Py}(p,\overline{x}+g), m, \overline{x}] \qquad (10)$$

$$\hat{x}^C(p, m, \overline{L}) = x^C[p, w^{PL}(p, \overline{L}), m, \overline{L}] \tag{11}$$

where g is real government expenditure.

Some of the properties of the functions $\hat{L}^C$ and $\hat{x}^C$ are given by the following result:

Proposition 1: For all $(p, m, \overline{x})$ such that $\hat{L}^C(p, m, \overline{x}) > 0$,

$$\eta_w^L \geq -\eta_p^L \rightarrow \frac{\partial \hat{L}^C}{\partial p}(p, m, \hat{x}) > 0 \tag{12}$$

where

$$\eta_w^L = \frac{\partial L^C}{\partial w}\frac{w^{Py}}{L^c} \qquad \eta_p^L = \frac{\partial L^C}{\partial p}\frac{p}{L^c}$$

$$\frac{\partial \hat{L}^C}{\partial m}(p, m, \overline{x}) < 0 \tag{13}$$

$$\frac{\partial \hat{L}^C}{\partial \overline{x}}(p, m, \overline{x}) \geq \frac{\partial L^C}{\partial \overline{x}}[p, w^{Py}(p, \overline{x} + g), m, \overline{x}] > 0 \tag{14}$$

For all $(p, m, \overline{L})$ such that $\hat{x}^C(p, m, \overline{L}) > \hat{x}^C(p, m, o)$,

$$\eta_w^x \geq -\eta\frac{x}{p} \rightarrow \frac{\partial \hat{x}^C}{\partial p}(p, m, \overline{L}) > 0 \tag{15}$$

where

$$\eta_w^x = \frac{\partial x^C}{\partial w}\frac{w^{PL}}{x^C} \;\; and \;\; \eta_p^x = \frac{\partial x^C}{\partial p}\frac{p}{x^C}$$

$$\frac{\partial \hat{x}^C}{\partial m}(p, m, \overline{L}) > 0 \tag{16}$$

$$\frac{\partial \hat{x}^C}{\partial L}(p, m, \overline{L}) \geq \frac{\partial x^C}{\partial \overline{L}}[p, w^{PL}(p, \overline{L}), m, \overline{L}] > 0 \tag{17}$$

Proof: From (CE) and the definitions of $\hat{L}^C$ and $\hat{x}^C$, the inequalities (13) and (16) follow immediately. Indeed, we have

$$\frac{\partial \hat{L}^C}{\partial m} = \frac{\partial L^C}{\partial m} \quad and \quad \frac{\partial \hat{x}^C}{\partial m} = \frac{\partial x^C}{\partial m}$$

Using the chain rule, we find

$$\frac{\partial \hat{L}^C}{\partial \overline{x}} = \frac{\partial L^C}{\partial \overline{x}} + \frac{\partial L^C}{\partial w}\frac{\partial w^{Py}}{\partial \overline{y}}$$

and, from (CE), $\frac{\partial L^C}{\partial w} > 0$. Since $\frac{\partial w^{Py}}{\partial \overline{y}} \geq 0$, we conclude that (14) holds. A similar reasoning demonstrates (17).

Now, if $\eta_w^L + \eta_p^L \geq 0$, and since

$$\frac{\partial \hat{L}^C}{\partial p} = \frac{\partial L^C}{\partial p} + \frac{\partial L^C}{\partial w}\frac{\partial w^{Py}}{\partial p}$$

$$\frac{\partial w^{Py}}{\partial p}(p, \overline{x} + g) = \frac{\overline{x} + g}{f^{-1}(\overline{x} + g)} > \frac{w^{Py}(p, \overline{x} + g)}{p}$$

we have

$$\frac{\partial \hat{L}^C}{\partial p} > \frac{\partial L^C}{\partial p} + \frac{\partial L^C}{\partial w}\frac{w^{Py}}{p} = \frac{L^C}{p}\left(\eta_p^L + \eta_w^L\right) \geq 0$$

and the implication (12) is proved. The proof of (15) is similar. QED.

The government expenditure g is assumed to be fixed. Let r be the government revenue, and τ the (constant) average income tax rate. We assume that the fixed costs K of labor-managed firms consist of payments to the government. Then we have

$$r = K + \tau w L \tag{18}$$

The savings of consumers are given by

$$ps = wL(1 - \tau) - px \tag{19}$$

and the government's deficit is financed by money creation.

$$\dot{m} = pg - r \tag{20}$$

It is then easy to show that

$$\dot{m} = ps \quad (21)$$

To complete the model, we have to define an adjustment function for the price of the consumption good. We simply assume that this price moves in the direction of the excess demand for the good.

3. The Problem of the Existence of Equilibrium

The existence of a fixprice equilibrium has been established under quite general conditions for a capitalist economy (cf. Bohm [1978]). For labor-managed economies, clearly there will be situaticns in which no equilibrium exists. The reason for this phenomenon is the assumption of the existence of the fixed costs K. Take, for example, the extreme case in which K is so large that even if employment attained its upper bound the firm would be making losses: $f(L_{max}) < K$ (where L_{max} is an upper bound to the labor supply). Clearly no equilibrium may exist. Of course, less extreme examples of nonexistence can be constructed. The underlying story is always that at no levei of aggregate good supply the supply of labor is sufficient to produce that output. Before we proceed, we need a formal definition of equilibrium.

Definition 1: We say that $(\hat{L}, \hat{x})$ is an equilibrium for the economy defined by (p, m, g, K) if there are $\overline{L}, \overline{x}$ such that

$$\hat{L} = \min\{L^P(p, \overline{x} + g), \hat{L}^C(p, m, \hat{x})\} \quad (22)$$

$$\hat{x} = \min\{y^P(p, \overline{L}) - g, \hat{x}^C(p, m, \hat{L})\} \quad (23)$$

and

$$\begin{aligned} \overline{L} &= \hat{L} && \text{if } \hat{L} = \hat{L}^C(p, m, \hat{x}) \\ \overline{L} &= \infty && \text{otherwise} \\ \overline{x} &= \hat{x} && \text{if } \hat{x} = \hat{x}^C(p, m, \hat{L}) \\ \overline{x} &= \infty && \text{otherwise} \end{aligned}$$

Definition 2: Let $(\hat{L}, \hat{x})$ be an equilibrium for the economy defined by (p, m, g, K). We say that $(\hat{L}, \hat{x})$ is:

i) A classical equilibrium if $\hat{L} = \hat{L}^C(p, m, \hat{x})$ and $\hat{x} < \hat{x}^C(p, m, \hat{L})$

ii) A Keynesian equilibrium if $\hat{L} < \hat{L}^C(p, m, \hat{x})$ and $\hat{x} = \hat{x}^C(p, m, \hat{L})$

iii) An Inflationary equilibrium if $\hat{L} = \hat{L}^C(p, m, \hat{x})$ and $\hat{x} < \hat{x}^C(p, m, \hat{L})$

iv) A Walrasian equilibrium if $\hat{L} = \hat{L}^C(p, m, \hat{x}) = L^{P*}(p)$ and $\hat{x} = \hat{x}^C(p, m, \hat{L}) = y^{P*}(p) - g$

(v) A Keynesian-Inflationary equilibrium if $\hat{L} = \hat{L}^C(p, m, \hat{x})$ and $\hat{x} < \hat{x}^C(p, m, \hat{L})$[2]

Notice that the above definition of equilibrium satisfies the conditions of voluntariness and efficiency. That is, no agent buys or sells more than he wants and only the short side of each market faces a quantity constraint.

The next result States that if at some level of output y the corresponding labor supply is sufficient to produce y, then an equilibrium exists.

Theorem 1: If (p, m, g, K) is such that for some $\overline{x} \geq 0$, $\hat{L}^C(p, m, \hat{x}) \geq f^{-1}(\overline{x} + g)$, then an equilibrium for the economy defined by (p, m, g, K) exists.

Rather than providing a formal proof of theorem 1, which would involve an argument similar to the one developed in Bohm [1978], we provide a diagram. In Figure 1, there is a point A of the curve $\hat{L}^C$ that lies to the right of the curve $f(L) - g$.

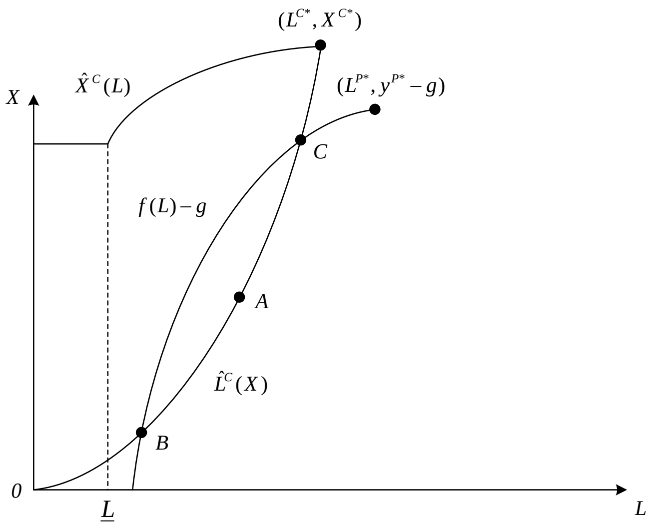

Figure 1

In this case there are *two* Inflationary equilibria, given by points B and C. In Figure 2 we see a situation of coexistence of one Inflationary equilibrium (point D) and one Classical equilibrium (point E).

[2] According to these definitions every Walrasian equilibrium is a $K - I$ equilibrium. Clearly our classificarion exhausts the possibilities. It turns out that finer definitions are not needed for our purposes.

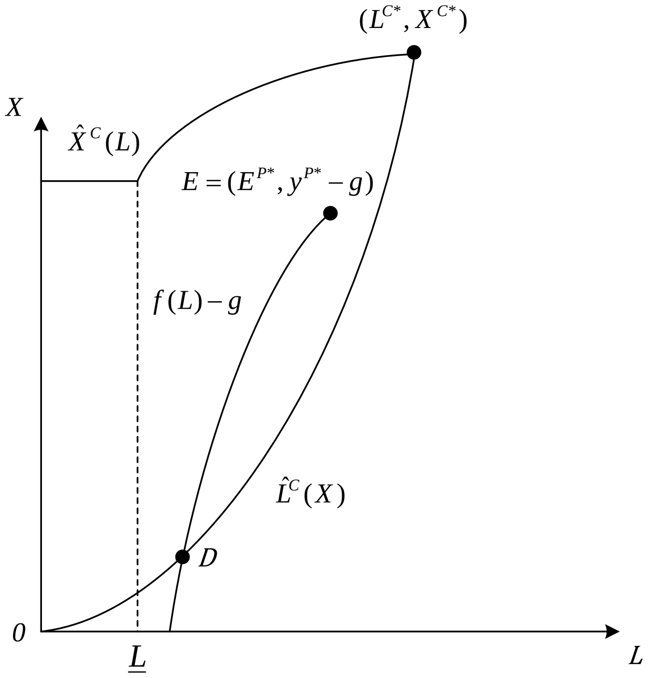

Figure 2

Finally, in Figure 3 we see a situation in which there exists a unique equilibrium, of the Keynesian type (point F).

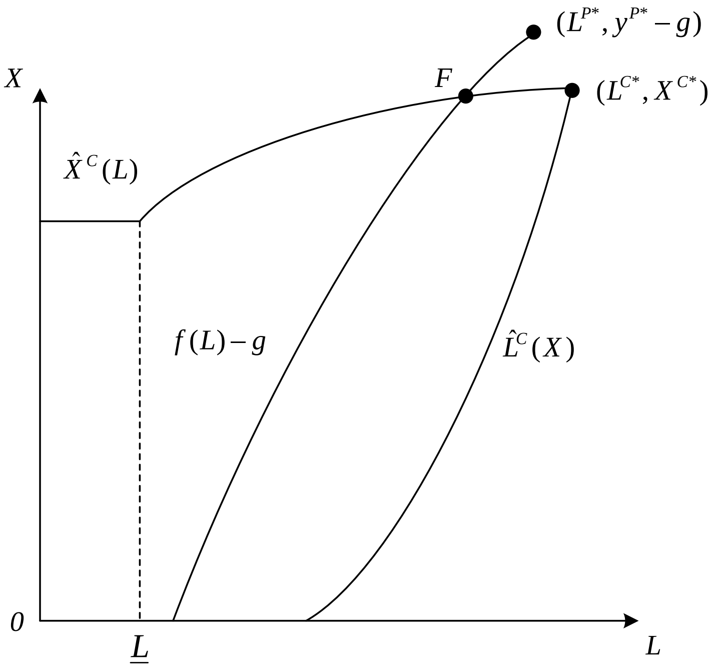

Figure 3

Figure 4 show a case of nonexistence of equilibrium. Notice that the condition of Theorem 1 is not satisfied.

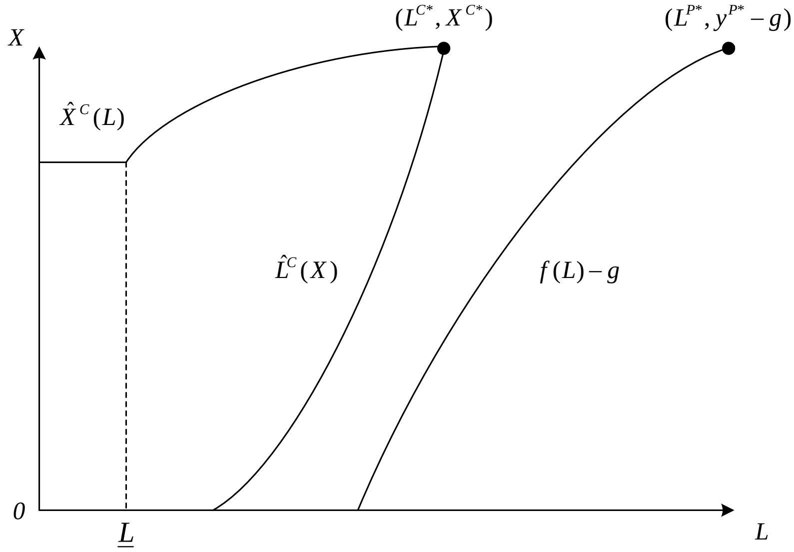

Figure 4

Clearly, the situation of Figure 4 occurs when g or K (or both) are too large. An equilibrium does not exist if the burden imposed by the existence of the government on the private sector is too heavy. The same phenomenon should clearly occur in a capitalist economy: there is no essential difference between the two types of economic organization, in this respect. Our next result shows that if the parameters p, m, g, and K, are such that there is some output level y at which a capitalist firm can pay workers their marginal revenue product and still make a profit, while the labor supply corresponding to that wage rate and output level y suffices to produce y, then there exists an equilibrium for the labor-managed economy defined by the same parameters.

We first define the functions $w^W(.)$ and $\pi(.)$ by:

$$w^W(L,p) = pf'(L) \tag{24}$$

$$\pi(L,p,K) = pf(L) - K - Lw^W(L,p) \tag{25}$$

Definition 3: We say that the economy defined by (p,m,g,K) is *feasible* if there are $L > 0$ and $x > 0$ such that

$$f(L) = x + g$$

$$\pi(L,p,K) \geq 0$$

$$L^C[p, w^W(L,p), m, x] \geq L$$

Intuitively, an economy is feasible if workers supply an amount of labor in excess of L when they receive their marginal revenue product $pf'(L)$ as a wage, and face a quantity constraint at a level

$f(L) - g$ in the good market, while the firm can employ a labor force of size L, pay workers their marginal revenue product, and still make a profit.

Theorem 2: If the economy defined by (p, m, g, K) is feasible then a fixprice equilibrium for that economy exists.

Proof: Let $\overline{L}$ and $\overline{x}$ satisfy the conditions of Definition 3. We have

$$0 \leq \frac{\P(\overline{L})}{\overline{L}} = \frac{pf(\overline{L}) - K}{\overline{L}} - w^w(\overline{L}, p) = w(\overline{L}, p) - w^w(\overline{L}, p) \leq w^{PL}(p, \overline{L}) - w^w(\overline{L}, p)$$

and hence $w^{PL}(p, \overline{L}) \geq w^w(\overline{L}, p)$. Now, it is easy to show that $w^{Py}(p, \overline{x} + g) = w^{PL}(p, \overline{L})$, so we have

$$w^{Py}(p, \overline{x} + g) \geq w^w(\overline{L}, p) \quad (26)$$

We conclude that

$$L^{-C}(p, m, \overline{x}) = L^C[p, w^{Py}(p, \overline{x} + g), m, \overline{x}] > L^C[p, w^w(\overline{L}, p), m, \overline{x}] \geq \overline{L} = f^{-1}(\overline{x} + g) \quad (27)$$

where the first inequality follows from (CE) and *(26)*, and the second one from the hypothesis of the theorem. From (27), $\overline{x}$ satisfies the condition of theorem 1, and therefore an equilibrium exists. QED.

4. The Problem of the Unigueness of Equilibrium

In Figures 1 and 2 situations in which there is a multiplicity of equilibria are depicted. It is easy to see that this phenomenon may prevail even when the parameters that define the economy are such that there exists a unique fixprice equilibrium for the associated capitalist economy at each fixed level of the wage rate w. Given that there may exist several equilibria, it is interesting to distinguish among them those that are stable from the point of view of a tâtonnement process in quantities. It should be obvious that the equilibria corresponding to the points B and D in Figures 1 and 2 should be unstable equilibria of such a process, while the equilibria C and E in the same Figures are stable. One possible definition of a tâtonnement process in quantities would be given by

$$\begin{aligned} x(t) = f[L(t)] \quad & -g \\ \dot{L}(t) > 0 \; if \quad & \hat{x}^C[L(t)] > x(t) \\ & \hat{L}^C[x(t)] > L(t) \\ & L(t) < L^{P*} \end{aligned}$$

$$\dot{L}(t) = 0 \; if \quad \hat{x}^C[L(t)] \geq x(t)$$

$$\hat{L}^C[x(t)] \geq L(t)$$

$$L(t) \leq L^{P*}$$

at least one of the inequalities
holding as an equality

$$\dot{L}(t) < 0 \; if \quad \text{otherwise}$$

on the interval $[f^{-1}(g), \infty]$

A fixprice equilibrium $(\hat{L}, \hat{x})$ is said to be *irregular* if $(\hat{L}, \hat{x})$ lies on the graph of one of the functions $L^C(.)$ or $\hat{x}^C(.)$, and $f'(\hat{L})$ equals the slope of that curve at that point, otherwise $(\hat{L}, \hat{x})$ is said to be *regular*. Generically, all equilibria are regular, and there is a finite number of equilibria. It is easy to see that, for the tâtonnment process in quantities defined above:

i) All Classical equilibria are regular and stable;

ii) A regular Keynesian equilibrium $(\hat{L}, \hat{x})$ is stable if and only if $f'(\hat{L}) > \frac{\partial \hat{x}^C}{\partial \overline{L}}(\hat{L})$;

iii) A regular Inflationary equilibrium $(\hat{L}, \hat{x})$ is stable if and only if $\frac{1}{f'(\hat{L})} > \frac{\partial \hat{L}^C(\hat{x})}{\partial \overline{x}}$

iv) A Keynesian-Inflationary $(K - I)$ equilibrium is stable if and only if it satisfies the conditions in ii) and iii).

The following assumption guarantees the existence of a unique regular stable equilibrium, but does not rule out the possibility that another equilibrium, regular but unstable, may exist.

(U) For all w and $\overline{L}$,

$$\frac{\partial x^C}{\partial \overline{L}}(p, w, m, \overline{L}) < \frac{w}{p} \tag{28}$$

$$\frac{\partial x^C}{\partial w}(p, w, m, \overline{L}) < \frac{\overline{L}}{p} \tag{29}$$

and the function $h: \mathbb{R}^2_+ \to \mathbb{R}$ defined by

$$h(w, \overline{x}) = L^C(p, w, m, \overline{x}) - f^{-1}(\overline{x} + g) \tag{30}$$

is concave.

Remark: A similar, but weaker form of the first part of Assumption (U) was used by Bohm [1978] to insure uniqueness of equilibrium. Schulz [1983] proved the global uniqueness of fixprice equilibria *à la Benassy* for exchange economies using this type of assumption. The intuition behind (28) is that the increase in income resulting from an increase in employment is not wholly spent in the consumption good. The condition (28) is thus similar to the assumption that the marginal propensity to consume is less than one»Similar remarks apply to (29). As for the last part of the assumption, first notice that f is concave function, and thus so is $-f^{-1}$. Then a sufficient condition for the concavity of h is the concavity of $L^C(.)$ in w and $\overline{x}$. It is reasonable to assume that L^C is a concave function of each one of the variables w and $\overline{x}$, taken individually. Given this, L^C is a concave function of both of these variables if the cross-effects $\frac{\partial^2 L^C}{\partial w \partial \overline{x}}$ are not too strong.

Theorem 3: Suppose that the economy defined by (p, m, g, K) is feasible, and that assumption (U) is satisfied. Then there exists a unique regular stable fixprice equilibrium. There may also exist a unique regular unstable Inflationary equilibrium.

Proof: First, we have

$$\frac{\partial \hat{x}^C}{\partial \overline{L}} = \frac{\partial x^C}{\partial \overline{L}} + \frac{\partial x^C}{\partial w}\frac{\partial w^{PL}}{\partial \overline{L}} < \frac{w^{PL}}{p} + \frac{p}{\overline{L}}\left[f'(\overline{L}) - \frac{w^{PL}}{p}\right]\frac{\partial x^C}{\partial w} = \frac{x^{PL}}{p}\left[1 - \frac{p}{L}\frac{\partial x^C}{\partial w}\right] + \frac{p}{L}f'(\overline{L})\frac{\partial x^C}{\partial w}$$
$$\leq f'(\overline{L})\left[1 - \frac{p}{L}\frac{\partial x^C}{\partial w}\right] + f'(\overline{L})\frac{p}{L}\frac{\partial x^c}{\partial w} = f'(\overline{L})$$

where the first inequality follows from (28), and the second from (29) and $pf'(\overline{L}) \geq w^{PL}$, which holds on $[0, L^{P*}(p)]$. Hence, every Keynesian equilibrium is regular and stable and there is at most one Keynesian equilibrium. It is easy to see that the same is true of $K - I$[3] and Classical equilibria. Also, it is impossible that the following pairs of equilibria may exist for a same economy: Keynesian-$(K - I)$, Keynesian-Classic, or $(K - I)$-Classic. This follows immediately from the fact that the curve $f(.) - g$ can cross the curve $\hat{x}^C(.)$ at most once.

Now, define $\hat{h}: \mathbb{R}_+ \to \mathbb{R}$ and $g: [0, L^{P*}(p)] \to \mathbb{R}_+^2$ as follows:

$$g(L) = [w^{PL}(p, L), f(L) - g]$$
$$\hat{h} = h_o g$$

Then we have

$$\hat{h}(L) = L^C[p, w^{PL}(p, L), m, f(L) - g] - L,$$
$$= \hat{L}^C[p, m, f(L) - g] - L,$$

[3] Except in the special case where the $K - I$ equilibrium lies in the $(K - I)$-No-equilibrium boundary.

and at any equilibrium $(\hat{L}, \hat{x})$ we must have $\hat{h}(\hat{L}) \geq 0$. Also, an equilibrium $(\hat{L}, \hat{x})$ with $\hat{h}(\hat{L}) = 0$ is regular if and only if $\hat{h}'(\hat{L}) \neq 0$, and stable if and only if $\hat{h}'^{(\hat{L})} < 0$. Since h and the component functions of g are concave, and h is monotonically nondecreasing, $\hat{h}$ is concave.

Since $\hat{h}$ is concave, it may have at most two zeros. The assumption that the economy is feasible implies that $\hat{h}(L) > 0$ for some $L \geq 0$. Then all equilibria with $\hat{h}(\hat{L}) = 0$ must be regular. At any Inflationary equilibrium $(\hat{L}, \hat{x})$, we must have $\hat{h}(\hat{L}) = 0$. Thus, there may be at most two Inflationary equilibria, all such equilibria must be regular, and if there are two of such equilibria, one of them is stable ($\hat{h}' < 0$) and the other one unstable ($\hat{h}' > 0$).

If there is an unstable Inflationary equilibrium (L, x), then $\hat{h}'(L) > 0$. Then any $\tilde{L}$ such that $\hat{h}(\tilde{L}) > 0$ must satisfy $\tilde{L} > L$, and clearly another equilibrium exiust exist. So if there is a unique equilibrium anl this equilibrium is Inflationary then it must be stable.

If a Classical, $K - I$, or Keynesian equilibrium coexists with an Inflationary equilibrium, then the latter is necessarily unstable. To see this, first notice that the Inflationary equilibrium (L, x) must satisfy $L < \hat{L}$. Given that $L < \hat{L}$, $\hat{h}(L) = 0$, and $h(\hat{L}) \geq 0$, it follows from the concavity of $\hat{h}$ that $\hat{h}'(L) > 0$, and the Inflationary equilibrium at (L, x) is unstable.

Finally, we conclude that there are at most two equilibria. For suppose this is not the case. Then two of these equilibria must be Inflationary. Otherwise there would be a pair of equilibria whose possibility was excluded above (eg. a Classical-Keynesian pair). One of the Inflationary equilibria must then be stable. Also, since there are at most two Inflationary equilibria, there is some equilibrium of another type. But this is contradiction of the previous paragraph. QED.

5. Representation of Equilibria in Parameter Space

Assuming that the conditions of Theorem 2 are satisfied, we can depict the regions in $m - p$ space where the different types of stable equilibria obtain.

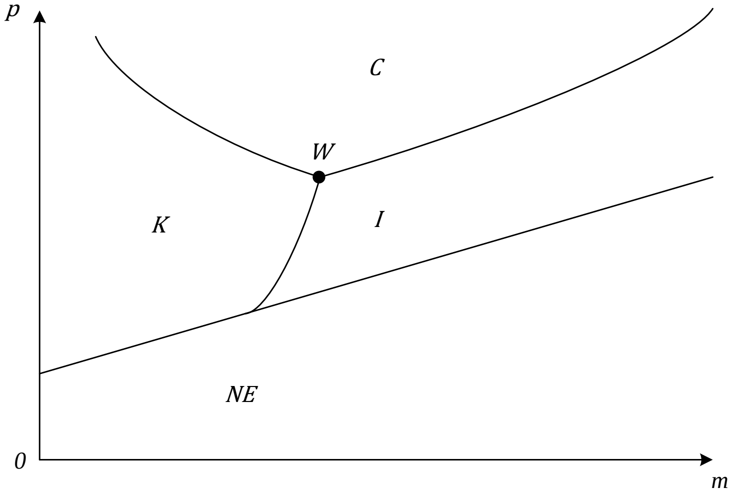

Figure 6

In the region NE, the values of the parameters m and p are such that no equilibrium exists. In the regions C, K, and I, Classical, Keynesian, and Inflationary stable equilibria respectively obtain. Point W corresponds to the Walrasian equilibrium. As mentioned above, it is possible that an unstable equilibrium of the Inflationary type also exists.

The following results justify Figure 6 as an adequate representation of the regions in $m-p$ space that correspond to the different regimes. We first show that the Walrasian equilibrium is unique if the worker's supply and demand curves are homogeneous of degree one.

Proposition 2: Suppose that

$$L^{C*}(p,w,m) = L^*(\frac{w}{p},\frac{m}{p}) \tag{31}$$

$$x^{C*}(p,w,m) = x^*(\frac{w}{p},\frac{m}{p}) \tag{32}$$

and that (CW) holds. Then there is at most one pair (m^*,p^*) that is associated to a Walrasian equilibrium.

Proof: we define implicity the functions $m^L(.)$ and $m^x(.)$ by

$$L^C[p,w^{P*}(p),m^L(p)] - L^{P*}(p) = 0 \tag{33}$$

$$x^{C*}[p,w^{P*}(p),m^x(p)] - y^{P*}(p) = -g \tag{34}$$

Using (31), (32), and the fact that, from the Envelope theorem

$$\frac{\partial w^{P*}}{\partial p} = \frac{f[L^{P*}(p)]}{L^{P*}(p)} = \frac{w^{P*}}{p} + \frac{K}{pL^{P*}(p)}$$

and differentiating (33) and (34) we obtain

$$\frac{\partial m^L}{\partial p} = \frac{m}{p} + \frac{p\frac{\partial LP^*}{\partial p} - \frac{K}{pL^{P*}}\frac{\partial L^*}{\partial(w/p)}}{\frac{\partial x^*}{\partial(m/p)}}$$

It follows immediately from our hypotheses that

$$\frac{\partial m^L}{\partial p} - \frac{\partial m^*}{\partial p} > 0 \qquad (35)$$

Now, clearly (m^*, p^*) is associated to a Walrasian equilibrium if and only if the graphs of $m^L(.)$ and $m^x(.)$ intersect at (m^*, p^*), that is, if

$$m^* = m^L(p^*) = m^x(p^*)$$

From (35), these graphs can intersect at most at one point. QED.

We now analyze the shapes of the boundaries between boundary between the regions in $m - p$ space. Let $\left.\frac{\partial p}{\partial m}\right|_{CI}$ be the slope of the boundary between the Classical and Inflationary regions and define similarity $\left.\frac{\partial p}{\partial m}\right|_{CK}$, $\left.\frac{\partial p}{\partial m}\right|_{KI}$ and $\left.\frac{\partial p}{\partial m}\right|_{NEI}$.

Proposition 3: The region NE intersects the vertical axis. The Inflationary region lies "to the right" of the classical and Keynesian regions, in the sense that along the CI and KI boundaries, an increase in m determines a movement into the Inflationary region. Similarly, the No-Equilibrium and Classical regions lie respectively to the right of the Inflationary and Keynesian regions. If the function $L^C(.)$ can be written as

$$L^C(p, m, w, \overline{x}) = L^C(\frac{m}{p}, \frac{w}{p}, \overline{x}) \qquad (36)$$

then $\frac{\partial p}{\partial m} > 0$ along the boundaries NEI, NEK, KI and CI. If similarly x^C is homogeneous of degree zero in (p, w, m) and the real-balance effect $\frac{\partial x^C}{\partial(m/p)}$ is small then the KC boundary is negatively sloped.

Proof: When p is very small the labor supply is not sufficient to produce for the government demand, so no equilibrium exists, no matter what the level of m may be. Hence, the region NE intersects the vertical axis.

An increase in m does not affect the Walrasian supply of goods or demand for labor, but determines a contraction in the labor supply and an expansion of the demand for the good. It is easy to see then, graphically, that the Inflationary region lies to the right of the CI boundary etc.

If a pair (m, p) lies in the NEI boundary then we have the following associated picture.

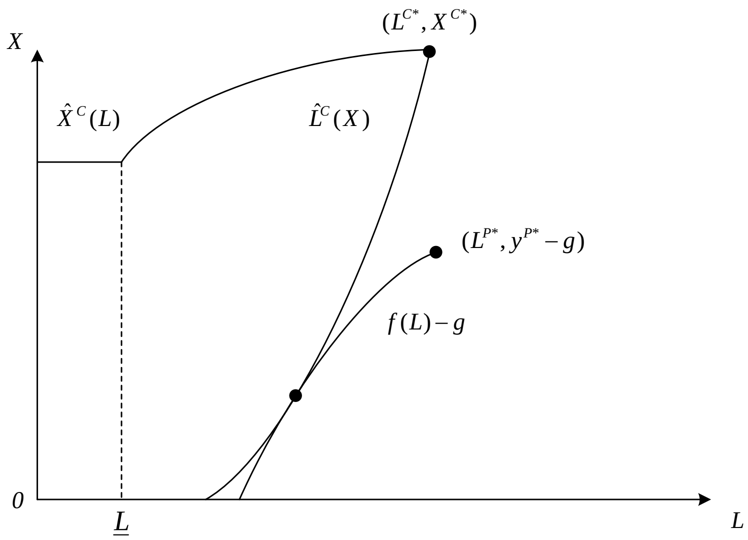

Figure 7

Point P is the unique Inflationary equilibrium. If m increases, then the curve $\hat{L}^C(.)$ moves to the left, and $(m + \Delta m, p)$ lies in the NE region, if $\Delta M > 0$. Suppose now that $\frac{\partial \hat{L}^C}{\partial p} > 0$. Then by increasing p, together with m, we can move to another point $(m + \Delta m, p + \Delta p)$ on the NEI boundary, and hence $\left.\frac{\partial m}{\partial p}\right|_{NEI} > 0$. A similar argument shows that $\left.\frac{\partial m}{\partial p}\right|_{NEK} > 0$ and $\left.\frac{\partial m}{\partial p}\right|_{KI} > 0$.

Still assuming that $\frac{\partial \hat{L}^C}{\partial p} > 0$, notice that the CI boundary is given by the following equation:

$$\hat{L}^C[p, m, c^{P*}(p)] = L^{P*}(p)$$

It follows that

$$\left.\frac{\partial m}{\partial p}\right|_{CI} = -\frac{\frac{\partial \hat{L}^C}{\partial p} + \frac{\partial L^{P*}}{\partial p}\left\{\frac{\partial \hat{L}^C}{\partial \bar{x}} f'[L^{P*}(p)] - 1\right\}}{\frac{\partial \hat{L}^C}{\partial m}}$$

From Proposition 1 and the fact that we are dealing with a regular stable equilibrium, we conclude that $\left.\frac{\partial m}{\partial p}\right|_{CI} > 0$. It remains only to show that $\frac{\partial \hat{L}^C}{\partial p} > 0$ or, considering (12), that $\eta_w^L \geq \eta_p^L$. From (36),

$$\frac{\partial L^C}{\partial w} = \frac{1}{p}\frac{\partial L^C}{\partial (w/p)}$$

$$\frac{\partial L^C}{\partial p} = -\frac{w}{p^2}\frac{\partial L^C}{\partial (w/p)} - \frac{m}{p^2}\frac{\partial L^C}{\partial (m/p)}$$

whence

$$\eta_w^L = -\eta_p^L - \frac{m}{pL^C}\frac{\partial L^C}{\partial (w/p)}$$

Now $\eta_w^L \geq -\eta_p^L$ follows from $\frac{\partial L^C}{\partial (m/p)} \leq 0$. QED.

6. Comparative Statics; Short-Run

In this Section we derive the comparative statics effects of changes in m, g and K on the equilibrium output level y. It turns out that Meade's [1972] conjecture that expansive fiscal policies never have a positive effect on output level is wrong, as far as the short-run is concerned. In the Keynesian region the effects of increasing government expenditure or the money supply are similar to those that would prevail in a capitalist economy. Also, although in the Inflationary region an increase in government expenditure has an adverse effect on output, the same would occur in a capitalist economy with elastic labor supply. As we shall see in the next section, expansive policies may indeed have adverse effects, but only in the long-run, when price changes are considered.

Classical equilibria are given by the equations

$$\hat{y} = f(\hat{L})$$

$$\frac{p\hat{y} - K}{\hat{K}} = pf'(\hat{L})$$

It is well-known (cf. Ward [1958]) that

$$\left.\frac{\partial \hat{y}}{\partial K}\right|_C > 0 \qquad (37)$$

and also, obviously,

$$\left.\frac{\partial \hat{y}}{\partial m}\right|_C = \left.\frac{\partial \hat{y}}{\partial g}\right| = 0 \qquad (38)$$

In the Keynesian region equilibria are defined by the equations

$$\hat{y} = f(\hat{L})$$
$$\hat{y} = x^C\left(p, \frac{p\hat{y} - K}{\hat{L}}, m, \hat{L}\right) + g$$

We have then, in a stable Keynesian equilibrium

$$\left.\frac{\partial \hat{y}}{\partial m}\right|_K = \frac{1}{\Delta}\frac{\partial \hat{x}^C}{\partial m} > 0 \qquad (39)$$
$$\left.\frac{\partial \hat{y}}{\partial g}\right|_K = \frac{1}{\Delta} > 0 \qquad (40)$$
$$\left.\frac{\partial \hat{y}}{\partial m}\right|_K = \frac{-1}{\Delta\hat{L}}\frac{\partial \hat{x}^C}{\partial w} < 0 \qquad (41)$$

where $\Delta = 1 - \left[\frac{\frac{\partial \hat{x}^C}{\partial \hat{L}}(p,m,\hat{L})}{f'(L)}\right]$ is positive since the equilibrium is stable.

Finally, the Inflationary region equilibria are given by

$$\hat{y} = f(\hat{L})$$
$$\hat{L} = \hat{L}^C(p, m, \hat{y} - g)$$

and hence

$$\left.\frac{\partial \hat{y}}{\partial m}\right|_I = \frac{-f'(\hat{L})}{\Omega}\frac{\partial \hat{L}^C}{\partial m} < 0 \qquad (42)$$
$$\left.\frac{\partial \hat{y}}{\partial g}\right|_I = \frac{f'(\hat{L})}{\Omega}\frac{\partial \hat{L}^C}{\partial \overline{x}} < 0 \qquad (43)$$

$$\left.\frac{\partial \hat{y}}{\partial K}\right|_I = \frac{-f'(\hat{L})}{\Omega \text{L}}\frac{\partial L^C}{\partial W} < 0 \tag{44}$$

where $\Omega = 1 - f'\big(\hat{L}\big)\frac{\partial \hat{L}^C}{\partial \hat{x}}(\hat{x}) > 0$ since the equilibrium is stable.

The following table sumarizes the results about signs of derivatives:

Variable	Region		
	C	K	I
m	0	+	-
g	0	+	-
K	+	-	-

7. Long-run Tendencies

The short-run comparative statics analysis developed in the previous section did not lend support to the hypothesis that expansionist fiscal policies might have perverse effects in a labor-managed economy. In this section we analyze the dynamic behaviour of the economy, focusing on the long-run effects of Keynesian policies and identify some conditions under which perverse effects do occur. When Ciassical unemployment prevails a spiral of inflation and increasing unemployment unfolds, and this process can, under certain conditions, be aggravated by expansionist policies. Demand pressure causes a rise in prices, which in turn determines a contraction of output and aggravates the initial situation of excess demand. Next, we formalize these statements.

We define a function $\hat{L}: \mathbb{R}_+^2 \to \mathbb{R}_+$, where $\hat{L}(p,m)$ is the equilibrium employment level that corresponds to the pair (p,m). Also, we let $\hat{x}(p,m) = f\big[\hat{L}(p,m)\big] - g$. As stated in Section 2, the evolution of (p,m) is then given by a pair of equations:

$$\dot{p} = \Phi\{\hat{x}^C[p,m,\hat{L}(p,m)]\} + g - f[\hat{L}(p,m)] \tag{45}$$

$$\dot{m} = (1-\tau)(pg - K) - \tau p\hat{x}(p,m) \tag{46}$$

where Φ is continuous, increasing, and satisfies $\Phi(0) = 0$. Suppose that initially the economy is at the Walrasian equilibrium, and τ and g are such that the budget is balanced. Then $\dot{p} = \dot{m} = 0$ at time zero. If an exogenous small shock, then throws the economy into the Classical region then throws the economy into the Classical region then an inflationary process in started. As prices go up supply contracts, and so do output and demand. This inflationary process can only finish after the economy crosses the KC boundary. For as long as the economy stays in the Classical or Inflationary regions p must be increasing. In order to find out if the KC boundary will ever be crossed we need to consider its slope and the signs of $\dot{p}$ and $\dot{m}$ in the Classical region. First, clearly $\dot{p} > 0$ in that region, and also

$$\hat{x}(p,m) = f[L^{P*}(p)] - g \tag{47}$$

Thus, $\hat{x}$ and hence $\dot{m}$ do not dependo on m in C. From (46),

$$\frac{\partial \dot{m}}{\partial p} = (1-\tau)g + \tau(\eta - 1)\hat{x}(p,m) \tag{48}$$

where $\eta \equiv -\frac{\partial \hat{x}}{\partial p}\frac{p}{\hat{x}} > 0$ from (47).

If $\eta = 0$, then

$$\frac{\partial \dot{m}}{\partial p} > 0 \Leftrightarrow \frac{g}{\hat{x}} > \frac{\tau}{1-\tau} \Leftrightarrow g > \tau y$$

and if $\dot{m} \geq 0$ then

$$g \geq \tau y + (1-\tau)\frac{K}{p} > \tau y$$

We conclude that $\frac{\partial \dot{m}}{\partial p}$ will be positive if $\eta = 0$ and $\dot{m} \geq 0$, and a fortiori this will also be true if $\dot{m} \geq 0$ and $\eta \geq 0$. Our next result follows easily:

Proposition 4: In the Classical region $\dot{p} > 0$ must hold. Therefore, if the KC boundary has a negative slope then a small shock that throws the economy into the Classical region triggers an endless process of inflation accompanied by increasing unemployment.

Proof: Clearly $\dot{p} > 0$ in C. We have shown above that $\dot{m} \geq 0$ implies $\frac{\partial \dot{m}}{\partial p} > 0$, and that $\frac{\partial \dot{m}}{\partial m} = 0$. It follows that if a small shock, which must necessarily involve a rise in p, throws the economy into Classical region, then initially $\dot{m} > 0$. This is because $\dot{m} = 0$ at the Walrasian equilibrium. The inflationary process thus triggered could only stop if the KC boundary were crossed. This would involve $\dot{m}$ assuming a negative value, since the slope of KC has been assumed to be negative. But as long as $\dot{m} \geq 0$,

$$\ddot{m} = \frac{\partial \dot{m}}{\partial p}\dot{p} > 0$$

and hence $\dot{m}$ can never become negative. QED.

From Proposition 3 we know that the KC boundary is indeed negatively sloped if the real-balance effect is sufficiently small. What are the consequences of the adoption of expansionist policies under these circunstances, when Classical unemployment prevails? A discrete increase in government expenditure may throw the economy into the Inflationary region, thereby aggravating the unemployment problem. The same can be said of a discrete increase in the money supply. However, such actions would have similar consequences in a capitalist economy, and the mechanisms in operation would be the same. In the case of an increase in g there remain less to be bought by workers and conseguently the labor supply declines. If m is increased workers become wealthier and again contract their labor supplies. On the other hand, discrete variations in m or g have no effect on output and employment as long as the economy remains in the Classical region (cf. Section 6). We must then look for the adverse effects of expansionist policies in labor-managed economies in the realm of dynamics. We do not undertake the task of analyzing in detail the long-run dynamics detemined by equations (45) and (46). In fact, it is highly unlikely that implicit or explicit parameters like τ, g, K or the expectations that underlie the functions x^C and L^C would remain invariant during such a process. Nevertheless, it is clear that increases in g or m would speed up the inflationary process and therefore accelerate the rate of decline of employment. This is true in the Classical region, and also in the Inflationary region when p is above the Walrasian equilibrium level. When unemployment is of the Keynesian type expansionist policies have the usual beneficial effects both in the short and long run.

8. Conclusions

According to the analysis developed above labor-managed economies should be plagued by high levels of unemployment coupled with inflation. Standard Keynesian policies have adverse effects, since the underlying problem is excess demand for goods. Measures that would expand supply (increases in K), or contract demand (increases in τ) would be more appropriate. From this point of view, it may well be that the problems pointed above turn out to be a blessing. For unemployment, as long as it is of the Classical type, may be fought with measures that produce a budget surplus for the government.

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