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Exchange rate overshooting: evidence from Brazil

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Abstract

The objective of this study is to review the existence of exchange rate overshooting in the Brazilian economy. The study will review the original model proposed by Dornbusch and a modern version using a DSGE model. Following that, a VAR model will be estimated to observe how the exchange rate responds to monetary policy. Robustness estimations will be made to justify the validity of the results found. Results show the existence of exchange rate overshooting as a consequence of interest rate shocks by the Brazilian Central Bank.

1 Introduction

A great interest in the study of macroeconomics is trying to understand the behaviour of macroeconomic variables to shocks. Though general effects are mostly known, comprehending how variables interact along different time frames is important to analyse the effects of different policies. This work intends to examine the effects, in the short and the long term, of interest rate decisions on exchange rates.

As present by Dornbusch in 1976, the Overshooting Model intends to understand this behaviour. This model explains that, as the money market adjusts faster than the goods market, the effect of monetary policy leads to a greater effect on the money market in the short-term than in the long-term. As the goods market is more rigid, the money market is the only one to adjust instantly to shocks. As the goods market begins adjusting, the impact on the money market is lessened.

Therefore, if the central bank makes a monetary expansion, the exchange rate depreciates to the point it clears all markets. As the effects in inflation and output begin to be felt, the pressure on the exchange rate decreases, which leads to a small appreciation. This characterizes the Overshooting, exchange rates respond in a greater magnitude in the short-term than in the long-term, thus, it overshoots.

In order to verify if this behaviour is present in Brazil, this study will utilize a Vector Autorregressive (VAR) model to estimate, with Brazilian data, how the exchange rate reacts, over time, to the central bank's action. To give more validity to the results, robustness estimations will also be made. For all the estimated, impulse response function will be plotted and the conclusions will be addressed.

2 Literature review

2.1 Original model - Dornbusch 1976

"Rudiger Dornbusch's masterpiece, "Expectations and Exchange Rate Dynamics" was published twenty-five years ago in the Journal of Political Economy, in 1976. The "overshooting" paper-as everyone calls it-marks the birth of modern international macroeconomics. There is little question that Dornbusch's rational expectations reformulation of the Mundell-Fleming model extended the latter's life for another twenty-five years, keeping it in the forefront of practical policy analysis." (Rogoff, 2001)

The model developed by R. Dornbusch in 1976 is based on a small economy in the global market with capital mobility. It is one of the most cited papers in the field of economics.

2.1.1 Capital mobility and expectations

According to the model, assets denominated in the foreign and the domestic currencies are considered perfect substitutes. The capital mobility, therefore, guarantees that the local interest rate r has to be equal to the global interest rate r^* plus the expected rate of depreciation of the local currency x. This happens because if there is an expectation that the local currency will depreciate, the local assets have to have interest rates higher than the foreign ones to keep the hypothesis that they are perfect substitutes.

$$r = r^* + x \tag{1}$$

It is important to note that the expected rate of depreciation of the local currency (x) is formed by the difference between the current exchange rate e and the long run exchange rate $\bar{e}$, which, according to the hypothesis of perfect foresight, is known, times a constant, called by Dornbusch as a coefficient of adjustment.

$$x = \theta(\bar{e} - e) \tag{2}$$

2.1.2 The money market

The domestic interest rate is set in the money market. The demand for real money is related to the output of the economy and to the interest rate. Therefore, we can assume that:

$\frac{M}{P} = f(r, Y)$

It is worth reminding that this is also the equation of real money demand from the IS-LM model.

Putting the equation above in a logarithmic format gives us:

$m - p = f(r, Y)$

As the increase in the interest rate reduces the demand for money, and the increase in the output increases the demand for money, we have that:

$$m - p = \phi y - \lambda r \tag{3}$$

With ϕ and λ being constants.

If we combine the equations (1), (2) and (3), we have that:

$$p - m = -\phi y + \lambda[r^* + \theta(\bar{e} - e)] \tag{4}$$

It is worth reminding that in the long-run:

$\lim_{t\to\infty} r_t = r^*$ and $\lim_{t\to\infty} \bar{e} - e_t = 0$

Thus, in the long-run:

$$\bar{p} = m + \lambda r^* - \phi\bar{y} \tag{5}$$

In which $\bar{p}$ is the long-run equilibrium price level.

By joining (4) and (5), it is possible to observe that the differential between the current and the long-run price levels is proportional to the differential between the current and the long-run exchange rates.

$p - \bar{p} = \lambda\theta(\bar{e} - e)$

Isolating e:

$$e = \bar{e} - \frac{p - \bar{p}}{\lambda\theta} \tag{6}$$

This is extremely important, as it allows one to find the current spot exchange rate if it is known the values long-run interest rate and price level converge to and the current price level.

Assuming the price levels and the local and global interest rates are known, thus also knowing the interest differential, there is only one value for the current exchange rate such that the rate of currency depreciation and the interest rate differential are equal.

An increase in the price level leads to a growth in the local interest rate results in a capital flow that appreciates the spot exchange rate until the new interest rate differential and the difference between long-run and spot exchange rates reach equilibrium.

2.1.3 Goods market

The domestic demand function is related to the output level and the domestic interest rate, as formulated in the IS-LM model, but also to the relative price of domestic goods $e-p$, as in the IS-LM-BP model. The increase in output increases demand. The increase in the interest rate decreases demand. The increase in the relative price of domestic goods decreases demand.

Therefore, in logarithmic, it is the following:

$$\ln D = u + \delta(e-p) + \gamma y - \sigma r \tag{7}$$

With u, δ, γ and σ being constants. u is a linear coefficient while all the others are angular coefficients.

The rate of increase of price levels is related to the excess demand D/Y or $d-y$:

$$\dot{p} = \pi \ln \frac{D}{Y} = \pi[u + \delta(e-p) + (\gamma - 1)y - \sigma r] \tag{8}$$

In the long-run:

$e \to \bar{e}; \quad p \to \bar{p}; \quad r \to r^*; \quad \dot{p} \to 0$

So, in the long-term:

$$\bar{e} = \bar{p} + \frac{\sigma r^* + (1-\gamma)y - u}{\delta} \tag{9}$$

Using (2), (8) and (9) the relation between $\dot{p}$, $\bar{p}$ and p can be found:

$$\dot{p} = \nu(\bar{p} - p) \tag{10}$$

While:

$\nu = \pi[\frac{\delta - \sigma\theta}{\theta\lambda} + \delta]$ thus, it is only a constant

2.1.4 Equilibrium exchange rates

As the money market can adjust faster to changes in price levels, interest rates, exchange rates and output than the goods market, we can use the equation of equilibrium in the money market as the short-time relation between price levels and the spot exchange rate in any given time. Thus, we can use the following equation to relate e and p:

$p = -\lambda\theta e + \bar{p} + \lambda\theta\bar{e}$

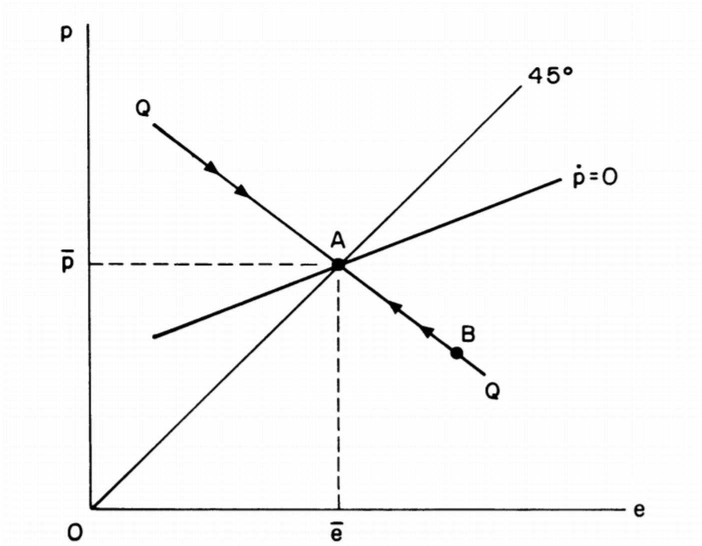
However, for both the money and the goods markets be in equilibrium, there is the condition that the current and the expected price levels have to be equal $p = \bar{p}$. Therefore, we have that $\dot{p} = 0$.

From (8) we have that if $\dot{p} = 0$:

$p = e + \frac{u + (\gamma - 1)y - \sigma r}{\delta}$

Finally, we have that the $\dot{p} = 0$ line has a positive slope. Points above the line mean that there is an excess in the supply of goods, which results in the decrease of price levels in the long run, that is, $\dot{p} < 0$ and $\bar{p} < p$. On the other hand, points below the line indicate there is an excess in demand, resulting in higher price levels in the future, $\dot{p} > 0$ and $\bar{p} > p$.

If the exchange rate increases, the domestic goods are going to become relatively cheaper, increasing demand for them. As a consequence, assuming the economy does not have enough time to adjust (increase) the supply, with higher demand and same supply, to achieve an equilibrium, the domestic price level will have to increase.



This graph is taken from Dornbusch's paper. He highlights that the $\dot{p} = 0$ line has a slope of less than 45 degrees because using an appropriate choice of units, the prices of $\bar{p}$ and $\bar{e}$ are equal.

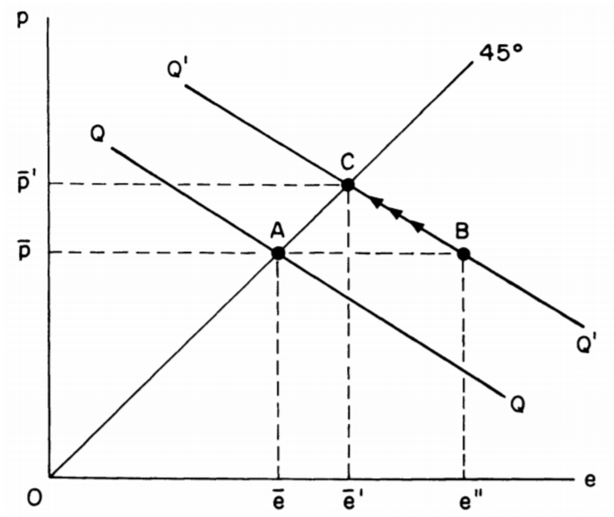
2.1.5 Effects of a monetary expansion

In this part, it will be analyzed how this model reacts to a monetary expansion given the initial condition that the economy was in equilibrium. A monetary expansion believed to be permanent will make the QQ equation dislocate to the right. It is worth remembering that the QQ equation is the condition of equilibrium in the money market. The line moves to the right because $\bar{p}$ adjusts instantly, $\bar{p} = m + \lambda r^* - \phi y$, that is, an increase in m leads to an increase in $\bar{p}$. And, given $p = -\lambda\theta e + \bar{p} + \lambda\theta\bar{e}$, an increase in $\bar{p}$ leads to a shift of the equation to the right. As the money market adjusts very rapidly, the economy will be at the money market equilibrium condition, so, it will be in the dislocated QQ line.

The sticky prices condition leads to the fact that, in the short-term, the price level p of the economy will remain the same. Because of that, the economy will have $p = \bar{p}$ in the short run and will be in the new QQ line. Therefore, it moves, in the short-term, from the previous money and goods market equilibrium point, A to point B, that respects the conditions explained in this paragraph.

It easy to observe that in point B, the goods market is not in equilibrium. It will only reach the equilibrium in the long run, with a growth in the domestic price levels. Given that, the economy will reach the money and goods market equilibrium, which can be clearly seen to be point C.

As the money and the goods market have vastly different times to adjust, after the monetary expansion, the economy will quickly go from point A to point B and then, slowly move along the new QQ equation to point C. The following graph is also taken from Dornbusch's paper.



Because of the increase in the money supply, there is a decrease in the interest rates and an anticipation of a currency depreciation, $\bar{e}_{t+1} > \bar{e}_t$. Consequently, the domestic assets become less attractive because of both factors and a flow of capitals to the rest of the world begins, and, further, a depreciation of the domestic currency. As the currency depreciation happens rapidly, it has to be enough to generate a expectation of appreciation before it ends.

That is, a depreciation enough to offset the reduction in the domestic interest rate. In summary, the monetary expansion leads to a spot currency depreciation with a higher magnitude than the expected long run devaluation. This happens because only in these circumstances the economy will anticipate a long-term appreciation, be compensated for the reduction in the national interest rate and, thus, neutralize the capital outflow to the rest of the world.

Considering that in $t = 1$:

$p = \bar{p}_{t=0}$ we have that $\frac{\partial p}{\partial m} = 0$

Rearranging (4):

$$e = \frac{m-p-\phi y}{\lambda\theta} + \frac{r^*}{\theta} + \bar{e}$$

Given the foreign exchange rates are unaffected by a change in the money supply or in the domestic interest rate following the hypothesis of a small open economy:

$$\frac{\partial r^*}{\partial m} = 0$$

If we look at (5) and (9), considering that y and r^* are unaffected by a change on m in the short-term:

$$\frac{\partial \bar{p}}{\partial m} = 1 \qquad \frac{\partial \bar{e}}{\partial \bar{p}} = 1 \qquad \Rightarrow \qquad \frac{\partial \bar{e}}{\partial m} = \frac{\partial \bar{e}}{\partial \bar{p}}\frac{\partial \bar{p}}{\partial m} = 1$$

This shows that:

$$\frac{\partial e}{\partial m} = 1 + \frac{1}{\lambda\theta}$$

Which confirms that in the short run there is an overshooting.

If there is a high interest response of money demand, that is, λ is very high, as it can be seen in (3), a variation in money supply will only generate a limited variation in the interest rates. On the other hand, if θ is very high, this means that a change in the domestic interest rates will only slightly change the difference between the spot, and the expected exchange rates $\bar{e} - e$. Consequently, it is possible to notice that the overshooting is larger as both λ and θ are smaller, which is clearly seen in the preceding equation.

This relies heavily on the assumption that the assets market adjusts faster than the goods market. Accordingly, a change in the nominal money supply in the short-term generates a change of the same magnitude in the real money supply. Under these conditions, the spot exchange rate is altered to put the money market back in equilibrium conditions. This is done by creating an expectancy of currency appreciation, after the overshooting, which outweighs the reduction in interest rates of the domestic assets.

It is important to realize that a monetary expansion leads to an excess demand for goods, which is generated by both a reduction on interest rates and by the depreciation in the spot exchange rate, making domestic good relatively cheaper than foreign ones. These constitute two independent ways in which the increase in money supply can affect the domestic goods demand.

Following the affect described in the preceding paragraph, the goods market has to start adjusting in the direction of achieving equilibrium. The spot exchange rate depreciation generates an inflationary pressure instead of a rise in the supply of goods. After having a reduced interest rate and relatively cheaper goods, the rise on price level starts to rise the domestic interest rates, even though in a much lesser magnitude than the reduction observed in the short run.

Also, as thoroughly described already, after the overshooting, the exchange rate starts to appreciate mildly. The increase in the price level also helps in the currency appreciation as it reduces the real money supply and increases interest rates. The rise in interest rates results in a capital flow from to rest of the world which is compensated by a currency appreciation.

In the long run, we can see that $r_{t=0} = r_{t=\infty}$ because $\bar{e}_{t=0} = e_{t=0}$ and $\bar{e}_{t=\infty} = e_{t=\infty}$. So the effect in the interest rates in this model is only real as long as the price level does not converge to the new equilibrium price level. From the graph and from (8), considering $r_{t=0} = r_{t=\infty}$, we observe that there is a proportional increase on both price levels and exchange rate. $\Delta e = \Delta p$, or in log, $e_{t=\infty} - e_{t=0} = p_{t=\infty} - p_{t=0}$.

2.1.6 Short run adjustments in output

In this part, we are going to relax the hypothesis of a fixed level of output at full employment used in the previous part of the literature review. Therefore, there will be variations in y in the short-term, but in the long run, the output always converges to full employment.

The condition that arises from this relaxation is that the output in the short run is equal to the demand:

$$y = \ln D = u + \delta(e - p) + \gamma y - \sigma r$$

And $\bar{y}$ is the output at full employment. Which results in a different form of (9):

$$\dot{p} = \pi \ln \frac{D}{Y} = \pi(y - \bar{y})$$

This shows a relationship between the difference between the output and the output of full employment and the price level. It reaffirms the earlier statement that an increase in demand creates an inflationary pressure. In fact, most of the relations observed earlier will stand despite the variations in output.

The increase in the nominal money supply, because of the sticky prices hypothesis, will increase the real money supply. Consequently, there will be a growth in domestic output which will lead to the start of rise in price levels. In the long run, however, the real money supply will return to its initial level, as well as the domestic output.

On the other hand, the spot exchange rate will behave very differently compared to what was observed previously. There is a possibility of an exchange rate depreciation that has a lesser magnitude instead of a higher one. This succeeds from the possibility that the growth on output resultant of the monetary expansion can make the domestic interest rates actually increase. Given this condition, there would be an expectation of currency depreciation to outweigh the national and foreign interest rate differential. In this case, the spot exchange rate would be less than the expected exchange rate.

However, the cases in which a monetary expansion makes the interest rate increase are extremely limited and require high income elasticity of money demand, high price elasticity and high income elasticity of demand for domestic goods.

It is interesting to note that, in case the interest rate rises as a consequence of a monetary expansion, the spot exchange rate depreciation is the only channel for transmission of the monetary policy. This means, in these conditions, the growth in aggregate demand and, thus, on output will be caused exclusively by the exchange rate depreciation, with the interest rate rise dampening the monetary policy transmission by reducing the increase in demand and output.

The effects of a monetary expansion are only temporary because the rise in price levels make real prices converge back to their initial value, the output starts reducing and returns to full employment output and the interest rate goes back to its initial rate given that $\Delta\bar{e} = \Delta\bar{p}$ and the effects on $\bar{e}$ and $\bar{p}$ cancel each other.

Given the model with and without the possibility of variation of output in the short-term, Dornbusch tries to answer the question about which model is more useful. He states that there is no doubt the model with fixed output is more useful in a very-short-term. As the exchange variations in response to monetary policy occur within days, and various studies show central banks' policies only have meaningful effects on output months after its implementation, a fixed output approach is much more realistic to analyze currency overshooting after monetary policies.

However, in the intermediary run, it is more relevant to use the model with an output that changes in response to the monetary policy. In this case, it is expected that both output and prices will adjust to the increase in aggregate demand. Therefore, each model remains relevant depending in the time horizon of the analysis.

2.2 Overshooting with inflation targeting

In the subsequent decades, many criticisms of Dornbusch's proposition were made. Most significantly, a wide range of models from the 1970s were challenged by the Lucas critique. It stated that it was naive to utilize models based exclusively in historical data to predict the effects on the economy. Furthermore, after the proposition of the Phelps island model, by Lucas (1973), the hypothesis of sticky prices was questioned. It provided the theoretical basis for arguing that the effects of monetary policy could be understood without Keynesian nominal rigidities, according to Rogoff (2001).

Therefore, new formulations were required to maintain the relevance of not only the model, but also, some of the fundamental hypotheses from New Keynesian economics. In response, the use of microeconomic fundamentals became standard in the following publications and was crucial in confirming these premises. This kept the idea of the overshooting at the forefront of studies on exchange rate depreciations while making slight changes to formulations to incorporate changes in monetary policy implementations since the first version was published.

The following part of the literature review intends to explain the rationale from a recent version of the overshooting incorporating microfoundations and tries to explain the model from de Gregório (2006). It reviews Exchange Rates in a Dynamic New-Keynesian (DNK) General Equilibrium Model.

2.2.1 Households

Preferences

Consider a continuum of of measure 1, j $\in$ [0,1] of households in which the jth household maximizes the following lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[C_t(j) + h\left(\frac{M_t(j)}{P_t}\right) - v(Y_t(j)) \right] \tag{11}$$

In which β represents the discount factor, E_0 is the expected value when $t = 0$, $C_t(j)$ is the consumption of the jth household at time t, $\frac{M_t(j)}{P_t}$ is the real money balance, and $Y_t(j)$ the production of the jth good. Finally, v is the disutility from production given the product differentiation of each good.

The variable $C_t(j)$ is defined as the CES composite consumption index:

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{12}$$

In which η is the elasticity of intratemporal substitution between a bundle of Home bundle of Foreign goods, γ is the share of the household's consumption expenditure allocated to foreign goods. It is assumed that $\eta > 0$ and $0 < \gamma < 1$. $C_{H,t}$ and $C_{F,t}$ are defined by by the CES composite consumption indexes as follows:

$$C_{H,t} = \left[\int_0^1 [C_{H,t}(j)]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[\int_0^1 [C_{F,t}(j)]^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \tag{13}$$

In which $\theta > 0$ is the price elasticity of demand faced by each monopolist, $C_{H,t}$ and $C_{F,t}$ are the quantities all households combined consumed of domestic and foreign goods, respectively.

The individual household constraint is the following:

$$\int_0^1 \left[P_{H,t}(j)C_{H,t}(j) + P_{F,t}(j)C_{F,t}(j)\right] dj + M_t(j) + E_t\left[F_{t,t+1}B_{t+1}(j)\right] = \quad (14)$$

$$= (1-\tau)P_{H,t}(j)Y_{H,t}(j) + M_{t-1}(j) + B_t(j) + TR_t$$

Consumers can store domestic non-interest bearing money or state-contingent claims. Therefore, the international financial markets are complete, with no need for having foreign assets, meaning that transitory shocks do not have permanent consequences according to de Gregório (2006).

TR_t are lump sum transfers received, B_t are payoffs received from a portfolio held in $t-1$, τ is the income tax rate and $F_{t,t+1}$, the stochastic discount factor.

The demand functions after maximization are:

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\theta} C_{H,t}, \quad C_{F,t}(j) = \left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{-\theta} C_{F,t} \quad (15)$$

In which the price indexes for domestic and foreign goods are:

$$P_{H,t} = \left[\int_0^1 \left[P_{H,t}(j)\right]^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \quad P_{F,t} = \left[\int_0^1 \left[P_{F,t}(j)\right]^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \quad (16)$$

Consequently, the demand allocations are the following:

$$C_{H,t} = (1-\gamma)\left[\frac{P_{H,t}}{P_t}\right]^{-\eta} C_t, \quad C_{F,t} = \gamma\left[\frac{P_{F,t}}{P_t}\right]^{-\eta} C_t \quad (17)$$

And the home and foreign consumer price indexes are, respectively:

$$P_t = \left[(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{18}$$

$$P_t^* = \left[(1-\gamma^*){P_{H,t}^*}^{1-\eta} + \gamma^* {P_{F,t}^*}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{19}$$

Therefore, the budget constraint becomes:

$$P_t C_t + M_t(j) + E_t\left[F_{t,t+1}B_{t+1}(j)\right] = (1-\tau)P_{H,t}(j)Y_{H,t}(j) + M_{t-1}(j) + B_t(j) + TR_t \tag{20}$$

Finding the first order conditions from de Gregório (2006), the household maximizes the utility by setting the bond and money allocations according to the individual household constraint. These are:

$$\beta E_t\left[\frac{u_c(C_{t+1})}{u_c(C_t)}\frac{P_t}{P_{t+1}}\right] = E_t\left[F_{t,t+1}\right] \tag{21}$$

$$u_c(C_t) = h_m\left(\frac{M_t}{P_t}\right)\frac{1}{P_t} + \beta E_t\left[u_c(C_{t+1})\frac{P_t}{P_{t+1}}\right] \tag{22}$$

For the rest of the world, the problem is analogous, resulting in the following Euler equation:

$$\beta E_t\left[\frac{u_c^*(C_{t+1}^*)}{u_c^*(C_t^*)}\frac{P_t^*}{P_{t+1}^*}\frac{E_t}{E_{t+1}}\right] = E_t\left[F_{t,t+1}\right] \tag{23}$$

Finally, we have the relationship between domestic and foreign consumption, in which κ is a constant representing initial wealth differences and Q_t is the real exchange rate:

$$u_c(C_t) = \kappa u_c^*(C_t)Q_t \tag{24}$$

The real exchange rate is a function of the spot exchange rate S_t:

$$Q_t = S_t \frac{P_t^*}{P_t} \tag{25}$$

2.2.2 Government

In the model, the government balances its budget giving the following budget constraint:

$$\tau P_{H,t} Y_{H,t} - TR_t + M_t - M_{t-1} = 0 \tag{26}$$

2.2.3 Prices

Given log prices and defining π as the inflation in log, we have:

$$p_t = (1-\gamma)p_{H,t} + \gamma p_{F,t}, \quad \pi_t = (1-\gamma)\pi_{H,t} + \gamma \pi_{F,t} \tag{27}$$

$$\pi_t = p_t - p_{t-1} \tag{28}$$

Conversely, the log real exchange rate is:

$$q_t = s_t + p_t^* - p_t \tag{29}$$

The model considers that foreign consumption of goods from the home country are negligible, giving $p_t^* = p_{F,t}^*$.

2.2.4 Log-linearized model

Solving the model, we get the following equation for the Aggregate Demand of the economy:

$$x_t = E_t[x_{t+1}] - \frac{1}{\sigma}i_t + \phi_\pi E_t[\pi_{H,t+1}] - \phi_s(E_t[s_{t+1}] - s_t) - (1-\rho_z)z_t \tag{30}$$

It relates output and interest rates as in the traditional IS curve, but also relates the output to current and expected spot exchange rates. z_t is the natural output level and x_t the output gap.

The Aggregate Supply solving the model and considering foreign prices constant is:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda_x x_t + \lambda_q q_t \tag{31}$$

$$\pi_t = (1-\gamma)\pi_{H,t} + \gamma(s_t - s_{t-1}) \tag{32}$$

The equation that relates the difference between foreign and domestic interest rates and the exchange rate, the Uncovered Interest Parity is the following:

$$i_t = i_t^* + E_t[s_{t+1}] - s_t \tag{33}$$

On the other hand, the monetary policy rule is given by the next equation. It considers that the central bank tries not to make sharp interest rate changes, so it considers not only the current rate but also a previous to set the rate for the period ahead ($i_{t+1} = (1 - \rho_i)\bar{i_t} + \rho_i i_{t-1}$). ρ_i is a smoothing parameter and $\bar{i_t}$ is the interest rate of full employment which the central bank targets.

$$i_t = \rho_i i_{t-1} + v_\pi E_t[\pi_{t+1} - \bar{\pi}] + v_x x_t + v_s s_t \tag{34}$$

Finally, the inflation target, foreign interest rate, potential output and cost push shocks follow:

$$\bar{\pi}_t = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \varepsilon_t^{\bar{\pi}} \tag{35}$$

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_t^{i^*} \tag{36}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \tag{37}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{38}$$

With this model, it is possible to utilize a Vector Autoregressive (VAR) model to capture the behaviour of the variables given an economic shock. In the empirical evidence section, more attention will be given to the estimation of the response from a shock.

3 Empirical Strategy

This section will be divided in four parts. The first one intends to explain the models used to evaluate the results in exchange rates of monetary policies, foreign and domestic. The second will elaborate on the data utilized and how it will be used. The third is going to present the empirical results obtained and the fourth will contain a summary of what was done and the conclusions taken from the results.

3.1 Model

The model chosen to observe the effects of monetary policy on the exchange rate is a VAR model. This follows the theoretical framework analysed in the literature review, more specifically, the Dynamic New-Keynesian model from de Gregório (2006). The variables utilized to find empirical results will be the domestic interest rate, set by the Brazilian Central Bank, the US interest rate, set by the Federal Reserve as the foreign interest rate, Brazilian gross domestic product, the consumer price index of Brazil and the real exchange rate between the US Dollar (USD) and the Brazilian Real (BRL). More details about the variables are given in the following pages.

The model stipulates that the behaviour of these variables can be modelled by a finite-order Vector Autoregressive model:

$$Y_t = \sum_{n=1}^{p} A_n Y_{t-n} + u_t$$

Y_t is a vector consisting of the variables in the model in time t. Y_{t-n} is a vector consisting of these variables in the same order n periods before the time t. The square-matrix A_n captures the interaction between the variables in time $t-n$ and t. u_t is the error parameter and measures the forecast errors in time t. It is important to note that u_t has a linear mapping with the structural shock in the model ε_t. Thus:

$$u_t = S\varepsilon_t$$

The choice is to use a structural VAR, which results in the problem of how to identify the model. The decision was to specify it in the following way, which is also the most common specification utilized in the literature.

$$\begin{bmatrix} i^* \\ y \\ \pi \\ i \\ q \end{bmatrix} = B(L) \begin{bmatrix} S_{11} & 0 & 0 & 0 & 0 \\ S_{21} & S_{22} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 \\ S_{41} & S_{42} & S_{43} & S_{44} & 0 \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix} \begin{bmatrix} \varepsilon^{i^*} \\ \varepsilon^{y} \\ \varepsilon^{\pi} \\ \varepsilon^{i} \\ \varepsilon^{q} \end{bmatrix} \tag{39}$$

Several reasons lead to the indication that it is, indeed, the best specification. First of all, variables such as Brazilian GDP, inflation, interest rate and USD/BRL exchange rate have no effect on the setting of US interest rates by the Federal Reserve. That is, the Fed independently sets up its monetary policy. Therefore, there is no contemporaneous effect from other variables in the SVAR identification.

Secondly, as thoroughly stated in closed economy macroeconomic models in the literature, output and inflation do not react contemporaneously to monetary policy. The effect from the interest rate only reaches these variables with a lag, and in the medium-term. Also, with the hypothesis of some price-stickiness in the short-term, a shock in output immediately generates a change in prices. Thus, the specification allows for a contemporaneous effect of output on inflation but not the other way around.

Thirdly, the central bank uses monetary policy and sets interest rates in order to control inflation and promote economic growth. This is in line with the Taylor rule, for example. Hence, shocks in output and inflation lead to a short-term reevaluation of the monetary policy. If inflation rises or GDP contracts, central banks tend to take immediate action to put the economy in a stable growth path. This is in line with the specification used, allowing the interest rates to react contemporaneously to output and inflation.

Finally, the Brazilian Central Bank's policy of floating exchange rates mean that it cannot set interest rates with an objective of targeting a level of exchange rate. As explained by Dornbusch, the money market adjusts almost instantaneously to shocks in the economy. So, the identification lets the exchange rate react contemporaneously to any shock in the other variables.

3.2 Data

Variable	Source
i = Average Annual Interest Rate (Effective SELIC)	Brazilian Central Bank
i^* = Average Annual Federal Funds Rate	Federal Reserve
π = Natural Log of 12 Month Accumulated Inflation Rate (IPCA)	IBGE
q = Natural Log of Average of Current Month Real Exchange Rate	Brazilian Central Bank
y = Natural Log of Current Month Real Gross Domestic Product	Brazilian Central Bank

3.2.1 Descriptive analysis

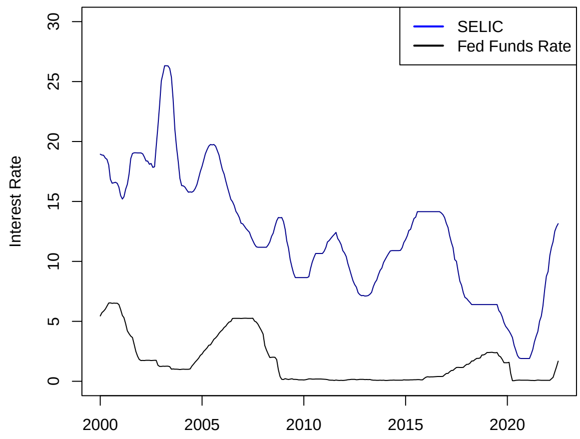
This section will be used to comment on some of the data and show the behavior of the relevant variables in the model since the year 2000.

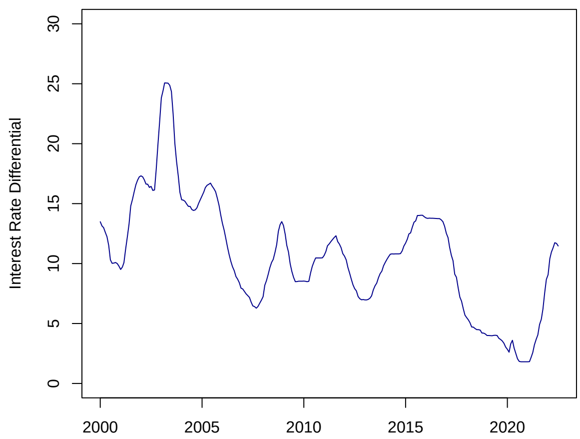
As thoroughly explained, interest rates, both domestic and foreign are relevant in exchange rate variations of the pair USD/BRL. The Brazilian Central Bank kept the rates considerably high during the century. In the early 2000s, it was used to combat inflation and consolidate the currency, which had been created in 1994 after a context of hyperinflation, and it incorporated some risk premiums from the departure from the dollar peg, following the Russian and the Asian crises and from Lula's election, which generated fears of populist handling of the economy.

Beginning in 2005, it was possible for the central bank to start lowering the rates as inflation was controlled and its expectation started anchoring. When the 2008 crisis hit, unlike in the USA, in Brazil, the rates were raised as fears of capital flight mounted. In the following decade, while the Federal Reserve kept its rates at zero, the BCB did not go below 7%. In 2014, Brazil had its worst economic crisis in history and the BCB was marked by its lack of independence and was used by president Rousseff for populist policies. The policy had to be reverted after a sharp rise in inflation and only in the next government it resumed to have credibility.

After repeated reductions of the interest rates, from 2016 to 2018, inflation converged to target. In early 2020, the COVID-19 crisis broke out and the BCB pushed rate to their lowest levels since the Real plan in 1994. As inflation picked up in 2021, Brazil quickly started a series of rate hikes, much earlier in comparison with the Fed.

The following graphs show the interest rates for both Brazil and the USA, the interest rate differential and the increase in rates from one month to the other, the former will be used in the VAR estimation:


SELIC
Fed Funds Rate
Interest Rate
0
5
10
15
20
25
30
2000
2005
2010
2015
2020


Interest Rate Differential
0
5
10
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25
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2000
2005
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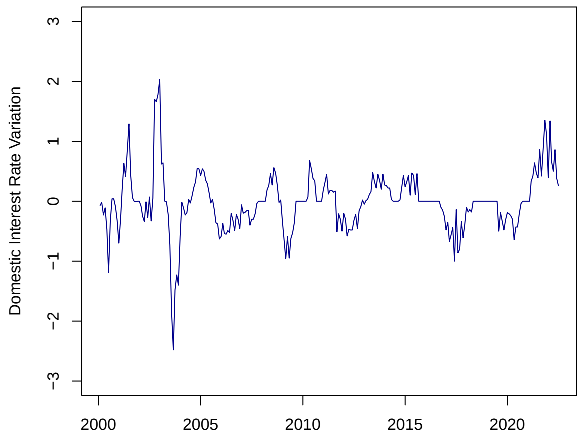
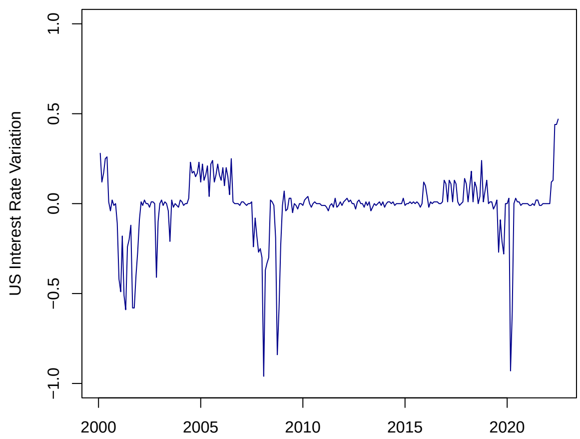
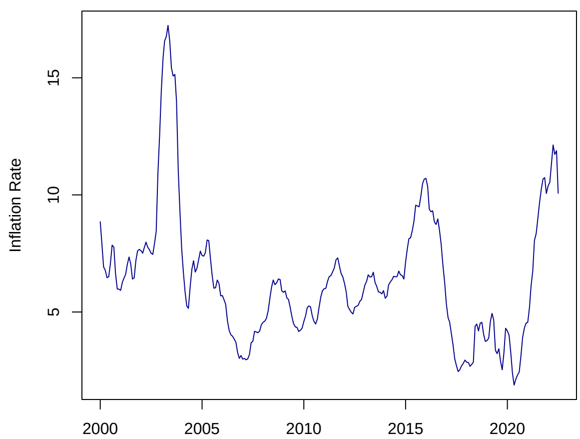
2020

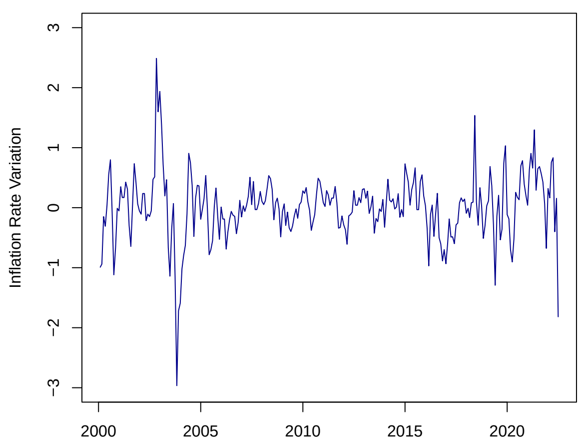




In regards to inflation, it remained consistently higher than in the developed world during this century. It is worth mentioning that since 2005, inflation remained close to the central banks target with the exception of two moments. The first began in 2015 and was consequence of populist policies of energy price controls in 2014, that led to an unsustainable situation in the energy market, forcing a price readjustment in 2015. Other policies of fiscal stimulus contributed as well. The second was the inflation shock experienced by the world after the COVID-19 crisis. The BCB increased the interest rates significantly and with government policies (tax cuts) on fuel prices, it seems that inflation is being controlled.

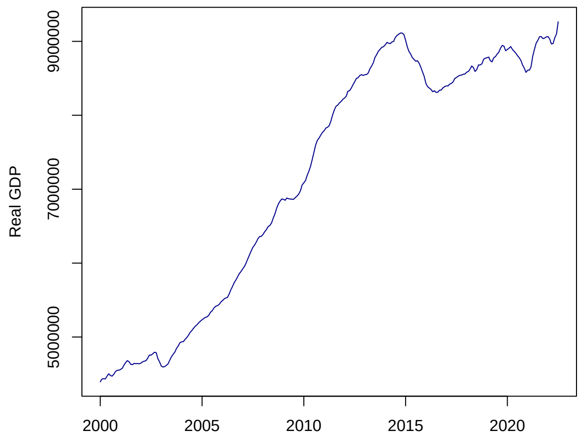
The following graphs show the 12-month inflation rate in Brazil and the its first-difference, which will be used in the VAR estimation:



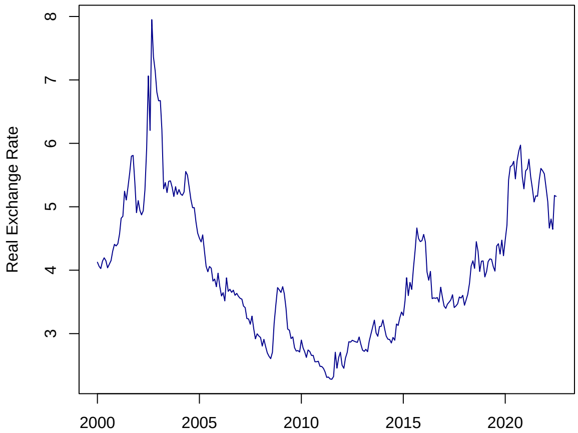


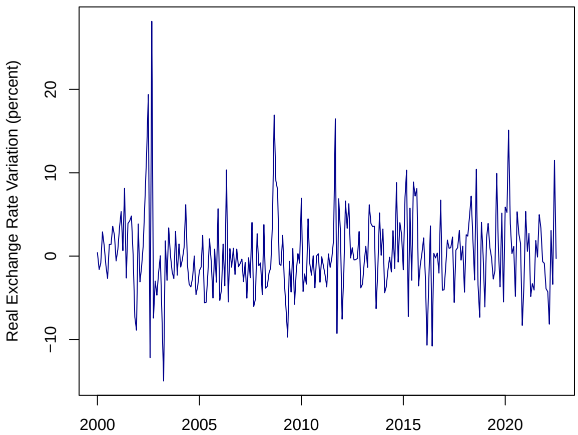
The Brazilian economy grew rapidly from 2000 to 2014, suffering only a limited effect from the global financial crisis of 2008. In 2015, the country experienced its largest economic crisis in history and there was a significant drop in GDP. Beginning in 2017, the economy started to recover, but growth rates were limited, not returning to pre-crisis levels. In 2020, the global COVID-19 downturn affected Brazil and there was another recession. However, this time, the economy recovered in 2021 and GDP reached pre-pandemic levels in this year.

The following graphs show the real GDP in Brazil and the 12-month growth rate by month, which will be used in the VAR estimation:



Finally, regarding the real exchange rate, in 2002, with the election of Lula and great uncertainty about his economic plan, the Real suffered its greatest devaluation. During Lula's government, after doubts about his economic policy were erased and coupled with current account surpluses, accumulation of foreign exchange reserves and increase in commodity prices, the Brazilian currency significantly appreciated. There was a major depreciation during the Great Recession, with capital flight, that was rapidly eased, with the currency returning to its previous levels. In the end of 2014 and beginning of 2015, with the perception of the incoming recession, high inflation and lack of credibility of the president of the Central Bank and of the government, the Real began a cycle of devaluations. This was only eased with the change in government. With the arrival of the pandemic, and rate cuts in Brazil, the currency suffered another significant devaluation. However, in 2022, after strong rate hikes to combat inflation and a favourable economic scenario, in real terms, the Real got close to its pre-pandemic levels.





3.3 Results

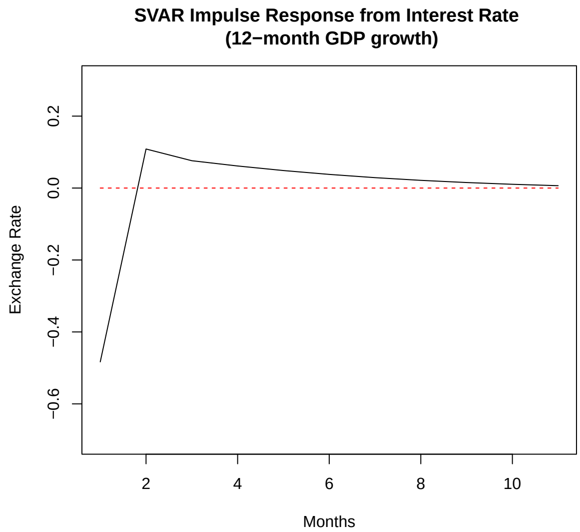
In the model estimation, the period analyzed is the post-Great Recession period, the data utilized starts in January 2009 and goes up to the last month available when this research began to be written, in August 2022.

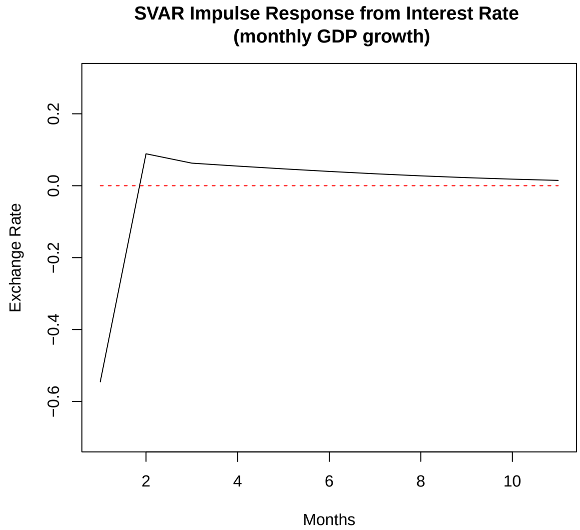
This time frame was selected because the early 2000s was a period of extremely high interest rates, caused by high risk-premiums and served as a moment of stabilization for the Real, created in 1994. Also, during the 2008 financial crisis, sharp movements in interest rates were observed. Then, exchange rates were mostly linked to the economic situation of countries, not necessarily monetary policy.

The stabilization in the early 2000s had two main focuses, inflation and exchange rate. Given a history of hyperinflation, Brazil had a policy focused on taming inflation despite great costs. Inflation accelerated in 1999 and increased until 2003, a fear that this could lead to the inviability of the Real Plan led to extremely high interest rates. On the other hand, after several exchange rate crises such as Mexico (1994), Asia (1997), Russia (1998), in 1999, the BCB was forced into allowing the exchange rate to freely float, the first occasion since the establishment of the new currency. Fears of trustworthiness crises, as observed after Lula's election, were another fear. His election lead to a massive devaluation and sharp increase in interest rates as a means of containing the exchange rate movement.

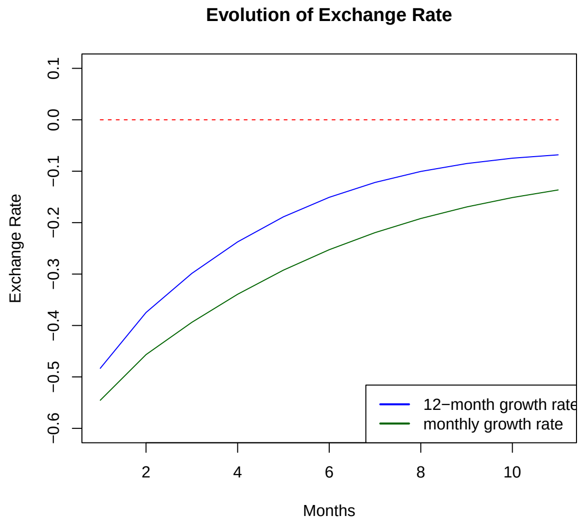
As for the Great Financial Crisis, as already stated, interest rates movements diverged significantly in Brazil and the rest of the world. While the Federal Reserve decreased rates to close to zero, the BCB increased rates. This happened in attempt to minimize the capital flights from Brazil widely seen in emerging markets. Flight-to-quality offset the interest rate differential effect in exchange rates.

The model is estimated for Brazil, using monthly data and the variables already described. To make all variables in the model stationary, the first-difference of inflation rate, and domestic and foreign interest rates are used. For exchange rate and output, the first-difference of the log of these variables is computed. However, as monthly GDP growth rates are not widely available and do not necessarily indicate the economic condition of the country, another model was also estimated. The output data used is the 12-month accumulated output growth rate instead of the monthly growth rate, while the other variables remain the same. Furthermore, the lag-order selection was made using Akaike information criterion (AIC), Hannan–Quinn (HQ), Schwarz criterion (SC) and Final Prediction Error (FPE) tests, all of which indicated one lag as the best selection for both estimations. The impulse response functions follow below:





The impulse response functions show that in the month following a positive shock on interest rates, there is an appreciation in the exchange rate. In the following months, there are depreciations of much smaller magnitudes until equilibrium is reached and exchange rate variation returns to zero. The graph above shows the percentage variation on exchange rates by month following monetary policy. Using these results, it is possible to evaluate the reaction in the exchange rate itself and not on the percentage variation by month. The impact on exchange rate itself is exposed below:



From the estimation of model previously described, it is possible to observe the response on the exchange rates from the monetary policy. An increase in the interest rates, causes a sharp appreciation in the exchange rate (USD/BRL) in the following month and, subsequently, depreciations in the following months of much less magnitudes. That is, the exchange rate appreciates significantly in the short-term and converges to a level more appreciated than before the increase in rates but more depreciated than the level reached in the first month.

This is consistent with the original behaviour explained by Dornbusch. In the short-term, monetary policy has only a minimal impact in both the inflation and the output. Therefore, in an open economy, the money market equilibrium has to be reached via the exchange rate almost exclusively. The higher interest rates make domestic assets more attractive and generate a net capital flow to the country, appreciating the domestic currency.

In the medium-term, however, effects of the interest rate hike that in the short-term were only felt in exchange rates begin to impact other variables such as inflation and output. Inflation and output decrease. Consequently, the magnitude of the effect on exchange rate is lessened, leading to a currency more appreciated than before the rate change, but more depreciated than in the moment it absorbs exclusively the rate shock. This leads to a small depreciation of the currency afterwards. Likewise, given the exchange rate is appreciated, net exports decrease. For net exports to reach an equilibrium, it is necessary, hence, that the currency begins weakening.

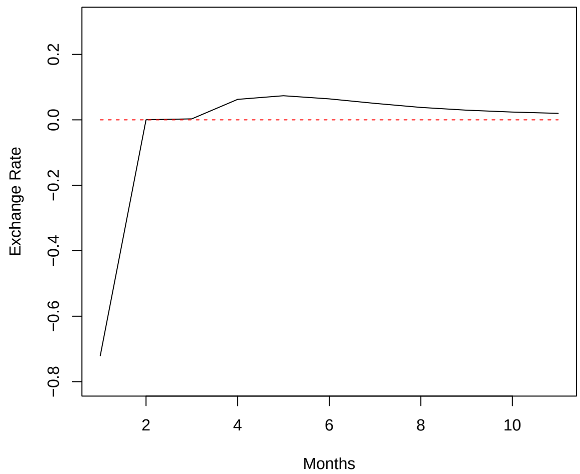
Following this line of thought, the uncovered interest parity condition is respected. As the domestic to foreign interest rate differential widens, the expectation of a currency depreciation follows. This depreciation can only happen if the exchange rate instantly appreciates in the short-term and overshoots, to later depreciate and reach its long-term equilibrium.

3.4 Robustness

To check the robustness of the results, other model estimations will be made. As stated in Bjornland (2009), there are three dimensions that deserve being analysed: specification of the VAR, choice of variables in the VAR and choice of restrictions used to identify the VAR. These will be analysed in the following paragraphs.

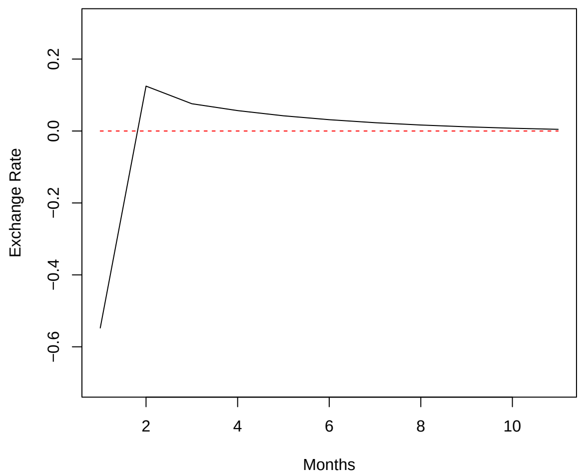
Firstly, regarding the specification of the VAR, to check the robustness of lag selection, the model will be estimated with two lags instead of one used previously. The results are presented below:

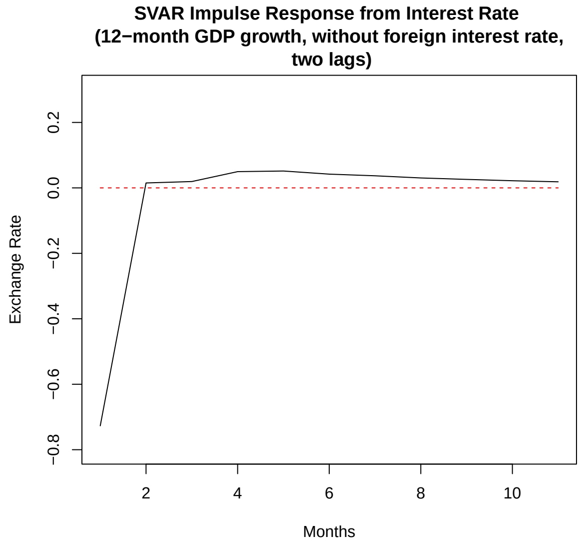
SVAR Impulse Response from Interest Rate
(12−month GDP growth, two lags)

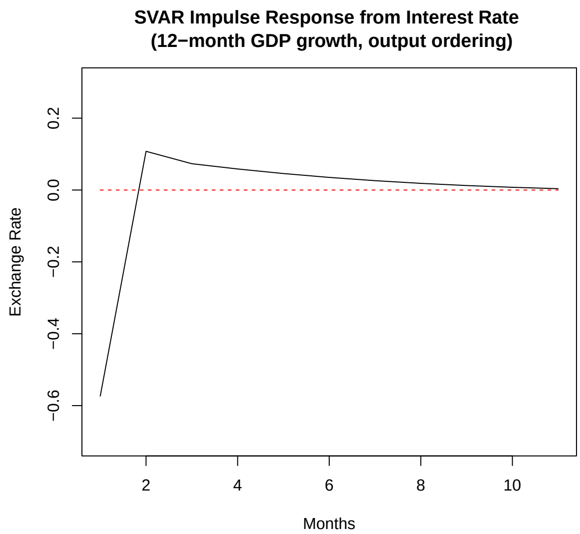


Secondly, regarding the model identification, two other models will be estimate. The first one excludes the foreign interest rate. This is because the Fed Funds Rate remained at near zero for the majority of the post-2009 period. The second follows an idea from Bjornland (2009), considering GDP as being contemporaneously affected by all other variables, with interest rates and exchange rates being affected only with a lag to GDP shocks. This makes sense since output is only published with a lag. The results are the following:

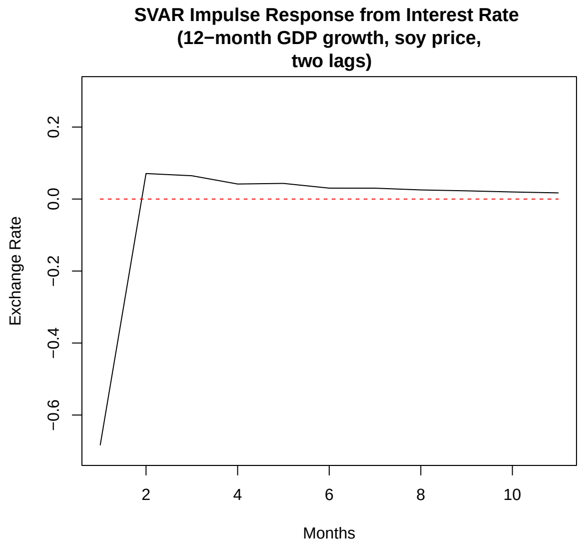
SVAR Impulse Response from Interest Rate
(12−month GDP growth, without foreign interest rate)

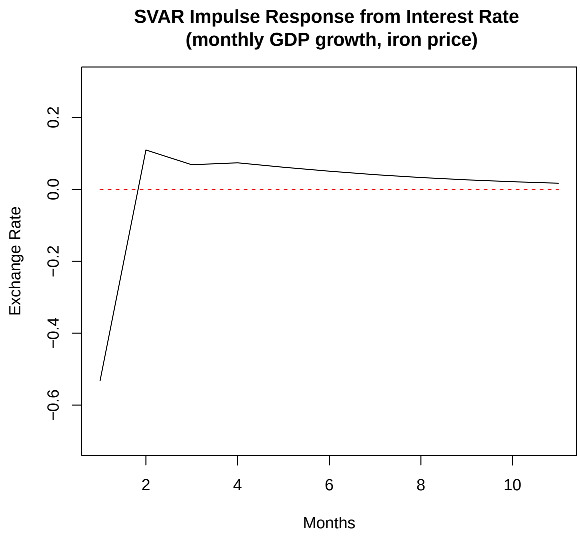






Thirdly, regarding the variables selected, several papers criticize the lack of robustness of models to the inclusion of other variables. Therefore, to check the robustness of the model to the inclusion of other variables, two different models will be estimated. The first will substitute foreign interest rates with iron ore price and the second with soybean price. The oil price is also significant to energy prices and inflation, so central banks might react to it. However, after attempting to estimate, the result was computationally singular, so it as not possible to calculate the SVAR. Iron ore and soybean prices are significant because these commodities are the two most significant exports of Brazil. They generate large surpluses to Brazil's current account balance and are significant sources of foreign exchange to the country. Thus, variations in these prices are likely to affect exchange rates. The results are presented below:





In all different specifications estimated to verify the robustness of the results previously described, it is possible to observe the same behaviour. That is, in the month following an increase in domestic interest rates, the exchange rate appreciates. In the following months, there are depreciations of much smaller magnitude. Therefore, compared to before the monetary policy action, the exchange rate is appreciated. But, the appreciation is maximum in the month following the action by the central bank.

4 Conclusion

In this study, it is attempted to identify the occurrence of exchange rate overshooting as proposed by Dornbusch in the 1970s. The study tries to expose the theoretical approach to why overshooting happens, both the original proposition and a modern one using a DSGE model and considering inflation targeting, currently used by most central banks.

In the empirical part of the study, the goal is to evaluate whether or not this behaviour is observed using Brazilian data. The empirical model, together with robustness estimations, indicate that, indeed, the exchange rate overshoots in the short term after a shock in interest rates.

Analysing the impulse response functions, it is observed that there is a sharp monthly appreciation on the exchange rate following an increase in domestic interest rates. In the subsequent months, there are depreciations of a much less significant magnitude. In the long-term, the exchange rate remains more appreciated than before policy action, but not as appreciated as in the month following it.

The empirical model chosen was robust to changes in the specification of the VAR, addition of different variables and changes to the data used. Therefore, it is possible to conclude that results obtained give confidence to conclude that the exchange rate overshoots in Brazil after BCB policy.

Finally, understanding the behaviour of economic variables and its responses following monetary policy can help to better suit central bank actions to the economic environment. As a country that underwent strong exchange rate movements in its history with mostly detrimental effects, studies using Brazilian data can help prevent and mitigate adverse effects of currency devaluations.

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