Pontifícia Universidade Católica
do Rio de Janeiro

Angelo Francisco Sirtoli Delamare

The Transmission of Foreign Demand Shocks in the Brazilian Economy: Insights from a HANK Model

Dissertação de Mestrado

Masters dissertation presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Yvan Becard

Rio de Janeiro
April 2025

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Prof. Yvan Becard
Advisor
Departamento de Economia – PUC-Rio

Prof. Carlos Viana de Carvalho
Departamento de Economia – PUC-Rio

Prof. Miguel Bandeira
Departamento de Economia – Insper

Rio de Janeiro, April 11th, 2025

Angelo Francisco Sirtoli Delamare

B.A. in Economics, Universidade Federal do Rio Grande do Sul (UFRGS), 2021.

Bibliographic data

Delamare, Angelo Francisco Sirtoli

The Transmission of Foreign Demand Shocks in the Brazilian Economy: Insights from a HANK Model / Angelo Francisco Sirtoli Delamare; advisor: Yvan Becard. – 2025.

73 f: il. color. ; 30 cm

Dissertação (mestrado) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia, 2025.

Inclui bibliografia

1. Economia – Teses. 2. Demanda Externa. 3. Agentes Heterogêneos. 4. Canais de Transmissão. 5. Desigualdade de Renda e Riqueza. I. Becard, Yvan. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. III. Título.

CDD: 004

Acknowledgments

I thank my parents, Cristina and Francisco, for their love and support. Without their belief in me and constant encouragement, none of this would have been possible. Their sacrifices and dedication have always inspired me to keep pushing forward, and I am profoundly thankful for everything they have done.

I am grateful to PUC-Rio for providing such a stimulating and enriching environment, which has greatly contributed to my academic and personal development. I am especially thankful to my advisor Yvan Becard for his exceptional guidance, patience, and encouragement throughout this project. His mentorship has been invaluable. I also want to thank Professor Márcio Garcia for his precious insights, stimulating discussions, and continuous support. My appreciation further extends to the committee members, Professors Carlos Viana and Miguel Bandeira, for their careful review and valuable comments.

I am thankful for the wonderful friends I have had the privilege of meeting in my cohort. The camaraderie, laughter, and shared experiences have made the Master's journey far more enjoyable. I could not have asked for a better group to share this experience with.

Lastly, I would like to thank CNPq, CAPES, and PUC-Rio for their financial support, without which this work would not have been possible.

This study was partially financed by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001 - and partially financed by the Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq.

Abstract

Delamare, Angelo Francisco Sirtoli; Becard, Yvan (Advisor). **The Transmission of Foreign Demand Shocks in the Brazilian Economy: Insights from a HANK Model**. Rio de Janeiro, 2025. 73p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

We develop a heterogeneous-agent New Keynesian model to analyze the transmission of foreign demand shocks in the Brazilian economy. This is achieved by incorporating incomplete markets into a seminal open-economy model. Under the assumptions of sticky wages, flexible prices, and a constant real interest rate, we analytically show that our model features two indirect transmission channels, whereas the original model includes only the direct channel. We extend our model by introducing investment, price rigidity, a Taylor rule, and a segmented international financial market. In this extended version, foreign demand shocks are transmitted through the direct channel and five indirect channels. By calibrating the model with Brazilian macroeconomic and distributional data, we show that the direct channel plays a minimal role in the transmission process. When examining how foreign demand shocks affect different wealth percentiles, we find that the poor are more influenced by labor dynamics, while the rich are more influenced by financial dynamics.

Keywords

Foreign Demand; Heterogeneous Agents; Transmission Channels; Income and Wealth Inequality.

Resumo

Delamare, Angelo Francisco Sirtoli; Becard, Yvan. **Transmissão de Choques de Demanda Externa na Economia Brasileira: Insights de um Modelo HANK**. Rio de Janeiro, 2025. 73p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Desenvolvemos um modelo Novo Keynesiano de agentes heterogêneos para analisar a transmissão de choques de demanda externa na economia brasileira. Isso é realizado pela incorporação de mercados incompletos em um modelo seminal de economia aberta. Sob as premissas de salários rígidos, preços flexíveis e uma taxa de juros real constante, mostramos analiticamente que o nosso modelo apresenta dois canais de transmissão indiretos, enquanto o modelo original contém apenas o canal direto. Estendemos o nosso modelo introduzindo investimento, rigidez de preços, uma regra de Taylor e um mercado financeiro internacional segmentado. Nesta versão estendida, os choques de demanda externa são transmitidos pelo canal direto e cinco canais indiretos. Ao calibrar o modelo com dados macroeconômicos e distributivos brasileiros, mostramos que o canal direto desempenha um papel mínimo no processo de transmissão. Ao examinar como os choques de demanda externa afetam diferentes percentis de riqueza, descobrimos que os pobres são mais influenciados pela dinâmica do trabalho, enquanto os ricos são mais influenciados pela dinâmica financeira.

Palavras-chave

Demanda Externa; Agentes Heterogêneos; Canais de Transmissão; Desigualdade de Renda e Riqueza.

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List of abbreviations

AR – Autoregressive

DSGE – Dynamic Stochastic General Equilibrium

FOC – First-Order Condition

HANK – Heterogeneous-Agent New Keynesian

IRF – Impulse Response Function

RANK – Complete-Market Representative-Agent New Keynesian

SAMBA – Stochastic Analytical Model with a Bayesian Approach

TFP – Total Factor Productivity

UIP – Uncovered Interest Parity Condition

1
Introduction

With the trade liberalization of the early 1990s, Brazil began a significant transformation that allowed it to integrate more deeply into the global economy. By exploiting its comparative advantages, Brazil's export basket became dominated by soybeans, oil, iron ore, raw sugar, coffee, meat, and corn. Given the nature of these industries, the country became more sensitive to foreign demand shocks, which can significantly affect both economic stability and overall well-being. A positive example of this is the commodities boom of the 2000s, which contributed to a period of increased foreign direct investment, poverty reduction, and robust economic growth.

Understanding the different channels through which external demand shocks propagate within an economy is a crucial step in comprehending the potential impacts of such disruptions. In this context, we ask the question: how do foreign demand shocks transmit to the Brazilian economy? We address this by employing a heterogeneous-agent New Keynesian (HANK) model. The advantage of this framework is that it uncovers new transmission mechanisms stemming from household inequality.

This thesis identifies six potential transmission channels, organized into three categories. First, two trade channels: (i) the direct effect, which represents the increase in production resulting from a positive change in external demand; (ii) the expenditure switching channel, which reflects the expansion of foreign demand for domestic goods due to a decrease in their relative prices. Next, two labor channels: (iii) the multiplier effect, which accounts for the increase in employment and capital accumulation; (iv) the real income effect, which captures the rise in real wages. Finally, two wealth channels: (v) the interest rate channel, which expresses the changes in monetary policy; and (vi) the revaluation channel, which accounts for unexpected capital gains or losses that occur during the shock period. Of these channels, the multiplier, real income, and revaluation effects do not manifest in a representative-agent model, as they influence macroeconomic aggregates due to wealth inequality.

Our argument proceeds in two steps. In the first step, we incorporate incomplete markets and heterogeneous agents into the seminal open economy model of Galí and Monacelli (2005). Inspired by the work of Auclert et al. (2024) on the transmission of foreign interest rate shocks, we assume sticky wages, flexible prices, and a constant real interest rate in order to derive analytical results. Given that the interest rate remains unchanged, consumption stays constant in the seminal model, while output and net exports are influenced solely by the direct channel. In contrast, our model fits two additional indirect channels: the multiplier effect and the revaluation effect.

The analytical results reveal that the influence of these three channels is shaped by the degree of economic openness. Specifically, for output, a higher level of trade openness results in a greater relative importance of the direct channel. For consumption and net exports, this parameter affects all channels equally. The quantitative results indicate that the indirect channels generate a positive response in consumption. This, in turn, leads to a larger expansion in production and a smaller increase in net exports. The primary driver of these new dynamics is the multiplier effect.

In the second stage of our analysis, we extend our basic model in four directions in order to incorporate three new transmission channels. First, investment is included to better understand how the transmission of the shock differs between output and its individual components. Second, imperfect exchange rate pass-through is introduced, which, by breaking the law of one price, gives rise to the expenditure switching and real income channels for the foreign demand shock. Third, a Taylor rule is incorporated to generate the real interest rate channel. Finally, a segmented international financial market is introduced to break the Uncovered Interest Parity (UIP) condition. As a result, the real exchange rate appreciates further, which helps to dampen the interest rate and its impact on investment.

We calibrate the model to match a set of Brazilian aggregate and distributional moments, then simulate a foreign demand shock. Our results show that only 22% of the output variation is explained by the direct channel. This contrasts with the representative-agent model, where the direct channel dominates. Breaking down the transmission, we have that trade channels contribute 55% to the output response, labor channels account for 24%, and wealth channels represent 21%. For consumption, labor and wealth channels are equally important. In the case of investment, the interest rate channel is the primary driver. Regarding net exports, the direct channel acts as the positive force, while the expenditure switching channel serves as the negative force.

Finally, we investigate the heterogeneous behavior of households by analyzing the responses of consumption, savings, and disposable income across different wealth percentiles. The results indicate that the poor are the most negatively affected in terms of consumption and disposable income, while the rich emerge as the main beneficiaries. Decomposing the effects for consumption, we find that the poor are more influenced by labor dynamics, while the rich are more influenced by financial dynamics.

Related Literature: This thesis contributes to the emerging literature on HANK models for small open economies (Aggarwal et al., 2023; Guo, Ottonello and Perez, 2023; Zhou, 2022; De Ferra, Mitman and Romei, 2020; Oskolkov, 2023; Hong, 2023). The aforementioned Auclert et al. (2024) also incorporates incomplete markets and heterogeneous agents into the seminal model of Galí and Monacelli (2005), but analyzes the transmission of foreign interest rate shocks. Unlike our work, they only decompose the effects in the basic model, while their extended version of the model, which does not include investment and segmented financial markets, focuses solely on the aggregate responses of economic variables.

Druedahl et al. (2024) analyze external demand shocks but employ a model with tradable and non-tradable sectors, where the external economy is represented as a vector autoregressive system. In their framework, an increase in foreign demand is accompanied by a rise in the foreign interest rate. The main distinctions from our work are that they do not calibrate the model to a specific country and do not investigate the impacts of the shock on different wealth percentiles. Therefore, our contribution lies in demonstrating how a HANK model explains the transmission of an increase in external demand to an emerging economy characterized by extreme inequality, such as Brazil.

Our thesis also contributes to the literature analyzing the impact of foreign shocks on the Brazilian economy. Empirical studies indicate that commodity price shocks significantly affect the country (Shousha, 2016; Fernández, González and Rodríguez, 2018), with variations in the terms of trade accounting for a substantial portion of business cycle fluctuations (Schmitt-Grohé and Uribe, 2018; Ben Zeev, Pappa and Vicondoa, 2017; Fernández, Schmitt-Grohé and Uribe, 2017). Furthermore, positive shocks in external demand lead to increases in output, prices, and interest rates, accompanied by exchange rate appreciation (Canova, 2005; Horvath and Zhong, 2019; Feldkircher and Huber, 2016). In this context, our contribution consists of demonstrating the impact of external demand shocks through a micro-founded model, which allows for an analysis of the behavior of different agents and an assessment of how different social classes are affected.

Outline: The remainder of the thesis is organized as follows. Chapter 2 introduces the basic heterogeneous-agent model. Chapter 3 presents the analytical and quantitative results for the basic model. Chapter 4 discusses the extended version of the model. Chapter 5 details the data and calibration strategy, as well as the results concerning foreign demand shocks in the Brazilian economy. Finally, Chapter 6 concludes.

2
Basic Model

We incorporate incomplete markets and heterogeneous agents into the seminal open-economy model of Galí and Monacelli (2005). To obtain analytical results, we assume sticky wages, flexible prices, and a constant real interest rate. The economy consists of five types of agents: households, mutual fund, firms, labor unions, and government. Time is discrete and runs from $t = 0$ to infinity. Although there is no aggregate uncertainty, households face uninsurable idiosyncratic risk.

The world economy is modeled as a continuum of small open countries. Each country has zero measure, so its domestic policy decisions have no impact on the rest of the world. The key distinction is that the domestic economy is now subject to idiosyncratic productivity shocks and sticky wages, while the other countries in the world economy present a representative agent as well as flexible prices and wages.

The economy starts in the steady state. At time $t = 0$, a foreign demand shock occurs. We study perfect foresight transition paths back to the steady state. To do so, we employ the solution method from Auclert et al. (2021), which, through certainty equivalence, yields impulse responses identical to those of the model with aggregate risk. This entire structure mirrors that used in Auclert et al. (2024). The difference is that we analyze foreign demand shocks, while they examine foreign interest rate shocks.

2.1
Domestic Households

The domestic economy is populated by a continuum $i \in [0,1]$ of households which enjoy utility over consumption $c_{i,t}$ and labor $n_{i,t}$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \psi \frac{n_{i,t}^{1+\varphi}}{1+\varphi} \right\}, \tag{2-1}$$

where σ is the inverse elasticity of intertemporal substitution, and φ denotes the inverse Frisch elasticity of labor supply. Consumption is allocated between home goods $c_{i,H,t}$ and foreign goods $c_{i,F,t}$

$$c_{i,t} = \left[\alpha^{\frac{1}{\eta}} c_{i,F,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} c_{i,H,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2-2}$$

where α represents the degree of economic openness, and η is the elasticity of substitution between home and foreign goods.

Households are subject to idiosyncratic productivity shocks e_{it}, which follow a first-order Markov chain with mean $\mathbb{E}[e_{it}] = 1$. They earn labor income $e_{i,t}\frac{W_t}{P_t}n_{i,t}$ and invest their savings $a_{i,t}$ in a domestic mutual fund with real return $1 + r_t^p$. Their period-t budget constraint is

$$\frac{P_{F,t}}{P_t}c_{i,F,t} + \frac{P_{H,t}}{P_t}c_{i,H,t} + a_{i,t} = (1 + r_t^p)a_{i,t-1} + e_{i,t}\frac{W_t}{P_t}n_{i,t}, \tag{2-3}$$

where $P_{H,t}$ ($P_{F,t}$) is the price of home (foreign) goods in domestic currency units, W_t is the nominal wage, and $P_t = \left[\alpha P_{F,t}^{1-\eta} + (1-\alpha)P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the consumer price index.

Given the presence of incomplete markets, households are also subject to a borrowing constraint

$$a_{i,t} \geq 0. \tag{2-4}$$

The labor supply decision is delegated to a labor union due to frictions in the labor market, as specified below. Therefore, given hours worked, household i selects a path of consumption $c_{i,t}$ and a path of savings $a_{i,t}$ to maximize (2-1) subject to (2-2), (2-3), and (2-4).

As shown in Appendix A.1, the division of consumption between home and foreign goods implies the following allocation for household i

$$c_{i,H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_{i,t} \quad \text{and} \quad c_{i,F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} c_{i,t}. \tag{2-5}$$

By aggregating the equations across consumers, the demand for domestic goods and the demand for foreign goods are, respectively,

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t. \tag{2-6}$$

2.2
Foreign Households

Given that all foreign countries have identical preferences, technologies, and complete markets, we follow the literature and consider only one representative agent for the rest of the world. The variables representing the world economy are denoted with a star superscript and are not influenced by the domestic economy, as the latter is infinitesimally small.

Since the representative foreign agent faces the same problem as domestic households, then the external demand for domestic goods is

$$C^*_{H,t} = \alpha \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\gamma} C^*_t, \tag{2-7}$$

where $P^*_{H,t}$ is the foreign-currency price of home goods, P^*_t is the foreign price index, and $\gamma > 0$ is the elasticity of substitution across countries.

2.3 Real Exchange Rate

We assume that the world monetary authority maintains the foreign-currency price of the foreign good at a constant level, $P^*_{F,t} = P^*_t = 1$, and that the law of one price holds for both foreign and domestic goods, i.e.

$$P_{F,t} = \mathcal{E}_t P^*_{F,t} = \mathcal{E}_t \quad \text{and} \quad P^*_{H,t} = \frac{P_{H,t}}{\mathcal{E}_t},$$

where $\mathcal{E}_t$ represents the nominal exchange rate. Consequently, the real exchange rate is

$$Q_t = \frac{\mathcal{E}_t}{P_t}. \tag{2-8}$$

Using the equations for the domestic price index and the real exchange rate, we obtain the real price of home goods and foreign goods in domestic currency units, as well as the real price of home goods in foreign currency units

$$\frac{P_{H,t}}{P_t} = \left(\frac{1 - \alpha Q^{1-\eta}}{1-\alpha} \right)^{\frac{1}{1-\eta}}, \quad \frac{P_{F,t}}{P_t} = Q_t \quad \text{and} \quad \frac{P^*_{H,t}}{P^*_t} = \left(\frac{Q^{\eta-1} - \alpha}{1-\alpha} \right)^{\frac{1}{1-\eta}}. \tag{2-9}$$

As shown in Appendix A.2, the elasticity of imports with respect to the relative price of imports $P_{F,t}/P_{H,t}$, while holding aggregate consumption C_t constant, is $\eta(1-\alpha)$. Similarly, holding foreign consumption C^*_t fixed, the elasticity of exports with respect to the relative price faced by foreign consumers $P^*_{H,t}/P^*_{F,t}$ is γ. Therefore, the trade elasticity – the sum of these two elasticities – is

$$\chi = \eta(1-\alpha) + \gamma. \tag{2-10}$$

2.4 Mutual fund

The economy has free movement of capital flows between countries. In the domestic country, a representative mutual fund raises a real value A_t from households and invests it in three types of assets:

- domestic nominal bonds with an interest rate i_t.
- foreign nominal bonds with an interest rate i_t^*.
- domestic firm equity with return $\dfrac{p_{t+1} + d_{t+1}}{p_t}$ (p_t is the end-of-period price of the shares).

The objective of the mutual fund is to maximize the expected real return on its liabilities r_{t+1}^p. In equilibrium, this results in all assets offering an equal rate of return. As shown in Appendix A.3, this equality leads to the uncovered interest parity (UIP) condition

$$1 + r_t = (1 + r_t^*)\frac{Q_{t+1}}{Q_t}, \tag{2-11}$$

and the following ex-post return at $t + 1$ on the mutual fund

$$1 + r_{t+1}^p = 1 + r_t = \frac{p_{t+1} + d_{t+1}}{p_t}, \text{ for } t \geq 0. \tag{2-12}$$

The ex-post return at $t = 0$ depends on the initial portfolio of the mutual fund. To resolve the portfolio indeterminacy, we assume that the mutual fund is initially entirely invested in domestic stocks. Therefore,

$$1 + r_0^p = \frac{p_0 + d_0}{p}, \tag{2-13}$$

where p is the steady-state price of the firm equity.

Since there is a unit-mass of shares, the value of the firm is equal to the end-of-period price of the shares. As stated in Section 2.7, government bonds are in zero net supply. Therefore, the net foreign position – the difference between the value of domestic households' real assets and the total value of assets in net supply domestically – is given by

$$\text{nfa}_t = A_t - p_t. \tag{2-14}$$

2.5 Firms

A unit-mass of firms j act in monopolistic competition, using domestic labor as the only production factor

$$y_{j,t} = n_{j,t}. \tag{2-15}$$

Given that prices are fully flexible and ε represents the elasticity of substitution between domestic varieties, the price of home goods is defined as

$$p_{j,H,t} = P_{H,t} = \mu_p W_t \tag{2-16}$$

where $\mu_p = \varepsilon/(\varepsilon - 1)$ is a constant markup over marginal costs. In this context, the aggregate real dividend of firms is given by

$$d_t = \frac{P_{H,t}}{P_t} Y_t - \frac{W_t}{P_t} N_t \tag{2-17}$$

Considering that the law of one price holds, and using equations (2-15), (2-16), and (2-17), we can respectively express the real wage income and the real dividends as

$$Z_t = \frac{W_t}{P_t} N_t = \frac{1}{\mu_p} \frac{P_{H,t}}{P_t} Y_t \quad \text{and} \quad d_t = \left(1 - \frac{1}{\mu_p}\right) \frac{P_{H,t}}{P_t} Y_t. \tag{2-18}$$

2.6 Labor Unions

The hours worked by household i, $n_{i,k,t}$, are determined by its labor union k. There are a continuum of unions, and each aggregates the efficient units of work from its members into a specific task $N_{k,t} = \int e_{i,t} n_{i,k,t} di$. These tasks are packaged using a constant elasticity of substitution function and sold to firms at a price W_t by competitive labor intermediaries

$$N_t = \left(\int_k N_{k,t}^{\frac{\epsilon-1}{\varepsilon}} dk\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2-19}$$

To determine the demand for task k from labor intermediaries, we solve

$$\max_{N_{k,t}} \; W_t \left(\int_k N_{k,t}^{\frac{\epsilon-1}{\epsilon}} dk\right)^{\frac{\epsilon}{\epsilon-1}} - W_{k,t} N_{k,t},$$

where $W_t = \left(\int_k W_{k,t}^{1-\epsilon} dk\right)^{\frac{1}{1-\varepsilon}}$ represents the price index for aggregate employment services. Thus, the demand for the specific task of union k is

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t}\right)^{-\epsilon} N_t. \tag{2-20}$$

Each union sets its wage $W_{k,t}$ to maximize the utility of its average worker subject to Rotemberg (1982) utility adjustment costs

$$\zeta_{w,t}(W_{k,t}, W_{k,t-1}) = \frac{\mu_w}{\mu_w - 1}\frac{1}{2\kappa_w}\left[\log\left(\frac{W_{k,t}}{W_{k,t-1}}\right)\right]^2,$$

where $\mu_w = \epsilon/(\epsilon - 1)$ is a wage markup, and k_w governs the wage adjustment cost curvature. Union k faces the following problem at time t

$$\max_{W_{k,t}} \sum_{\tau \geq 0} \beta^{t+\tau}\Big[u(C_{t+\tau}) - v(N_{t+\tau}) - \zeta_{w,t}(W_{k,t}, W_{k,t-1})\Big]. \tag{2-21}$$

As shown in Appendix A.4, all unions set the same wage $W_{k,t} = W_t$ and, consequently, $N_{k,t} = N_t$ in equilibrium. This, in turn, results in the following wage Phillips curve

$$\log(1 + \pi_{w,t}) = \kappa_w \left[N_t v'(N_t) - \frac{1}{\mu_w}\frac{N_t W_t}{P_t} u'(C_t)\right] + \beta \log(1 + \pi_{w,t+1}), \tag{2-22}$$

where $\pi_{w,t} = \frac{W_t}{W_{t-1}} - 1$ is the nominal wage inflation.

2.7 Government

The fiscal authority does not spend or tax, and domestic bonds are always in zero net supply. The monetary authority follows a CPI-based Taylor rule with a coefficient of one on expected inflation. As a result, the real interest rate remains constant over time

$$i_t = r_{ss} + \pi_{t+1}, \tag{2-23}$$

where r_{ss} is the steady-state real interest rate.

2.8 Equilibrium

Definition 2.1 *Given sequences of foreign demand shocks $\{C_t^*\}$, initial values for the nominal wage and price levels, an initial portfolio allocation for the mutual fund, and an initial distribution of households i over their state* *variables such that the economy is initially at a steady state, a* ***competitive equilibrium*** *is defined as a path of*

- *policies* $\{c_{i,H,t}, c_{i,F,t}, c_{i,t}, a_{i,t+1}\}$ *and distribution* $\mathcal{D}_{i,t}$ *for households,*
- *prices* $\{\mathcal{E}_t, Q_t, P_t, P_{H,t}, P_{F,t}, W_t, i_t, r_t, r_t^p\}$,
- *and aggregate quantities* $\{C_t, C_{H,t}, C_{F,t}, Y_t, N_t, A_t, p_t, d_t, nfa_t\}$,

such that households optimize, the mutual fund optimizes, firms optimize, unions optimize, monetary and fiscal policy follow their respective rules, and both the domestic goods and asset markets clear

$$Y_t = C_{H,t} + C^*_{H,t} \quad \textit{and} \quad A_t = p_t + nfa_t.$$

As detailed in Appendix A.5, the current account identity holds in the equilibrium

$$\text{nfa}_t - \text{nfa}_{t-1} = \frac{P_{H,t}}{P_t}Y_t - C_t + r_{t-1}\text{nfa}_{t-1} + (r_t^p - r_{t-1})A_{t-1}, \tag{2-24}$$

where

$$NX_t = \frac{P_{H,t}}{P_t}Y_t - C_t = \frac{P_{H,t}}{P_t}C^*_{H,t} - \frac{P_{F,t}}{P_t}C_{F,t} \tag{2-25}$$

is the value of net exports in units of the consumer price index, and the valuation effect term $(r_t^p - r_{t-1})A_{t-1} = 0$ for all $t \geq 1$.

As detailed in Appendix A.6, we examine a steady state with no inflation and no external financial positions. This implies that the domestic mutual fund owns all shares of domestic firms. Additionally, we set $Q = C^* = 1$, which consequently leads to the normalization of the prices $P_H, P_F, P, P^*_H, \mathcal{E}$ to 1. As a result, the steady state satisfies $C = Y = 1$ and it is unique, meaning the economy returns to this state after transitory shocks. In other words, the model is stationary.

3
Results

This chapter presents the results of our basic model for the transmission of foreign demand shocks. The analysis is inspired by the work of Auclert et al. (2024) on the transmission of foreign interest rate shocks.

3.1
Analytical Results

We focus on the response of output, consumption, and trade balance (net exports). For the latter, equation (2-25) is employed. From equations (2-6) and (2-7), we obtain the home goods market clearing condition used to decompose the output response

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\gamma} C^*_t. \tag{3-1}$$

As a benchmark, we first consider the seminal representative-agent model of Galí and Monacelli (2005). In this model, the consumption is determined by a Euler equation (see Appendix B.1 for further details).

Proposition 3.1 *In response to a shock to foreign demand $d\boldsymbol{C}^*$, the impulse response of consumption in the complete-market representative-agent model is given by*

$$d\boldsymbol{C} = 0,$$

the impulse response of output $d\boldsymbol{Y}$ is determined by

$$d\boldsymbol{Y} = \underbrace{\alpha d\boldsymbol{C}^*}_{\substack{\text{Direct}\\ \text{channel}}},$$

and the impulse response of trade balance is given by

$$d\boldsymbol{NX} = \underbrace{\alpha d\boldsymbol{C}^*}_{\substack{\text{Direct}\\ \text{channel}}}.$$

Proof. See Appendix C. ∎

Proposition 3.1 presents the analytical impulse responses to a foreign demand shock in the complete-market representative-agent model. As the shock does not affect the interest rate, domestic consumption remains at its steady-state value. Regarding output and net exports, only the direct effect is observed, and its impact is amplified by the degree of economic openness.

In the heterogeneous-agent model, the existence of a continuum of households i, each with a level of savings $a_{i,s}$ and a productivity $e_{i,s}$, implies different consumption paths in the economy. From the budget constraint (2-3), we see that the consumption policy function for household i depends on real labor income $Z_s = \frac{W_s}{P_s} N_s$ and the return on the mutual fund r^p_s. As demonstrated in Auclert, Rognlie and Straub (2024), the individual consumption responses give rise to an aggregate consumption function. Using equations (2-12) for the return on the mutual fund and (2-18) for real wage income, the aggregate consumption function can be expressed as

$$C_t = \mathcal{C}_t\left(\{r^p_s\}_{s=0}^{\infty}, \left\{\frac{W_s}{P_s}N_s\right\}_{s=0}^{\infty}\right) = \mathcal{C}_t\left(r^p_0, \{r_s\}_{s=0}^{\infty}, \left\{\frac{1}{\mu}\frac{P_{H,s}}{P_s}Y_s\right\}_{s=0}^{\infty}\right), \tag{3-2}$$

Since the model assumes a constant real interest rate, the analysis depends on two linear maps:

$$\boldsymbol{M} \equiv \frac{\partial \mathcal{C}}{\partial Z} \quad \text{and} \quad \boldsymbol{M_v} \equiv \frac{\partial \mathcal{C}}{\partial r^p_0}.$$

The (t, s)-th entry of $\boldsymbol{M}$ is the equilibrium response of aggregate consumption at time t to a real income change at time s

$$\begin{array}{cc} & \begin{array}{cccc} \text{change in Z} & \text{change in Z} & \text{change in Z} & \\ \text{at time 0} & \text{at time 1} & \text{at time 2} & \end{array} \\ \boldsymbol{M} = & \begin{pmatrix} \partial\mathcal{C}_{0,0}/\partial Z_0 & \partial\mathcal{C}_{0,1}/\partial Z_1 & \partial\mathcal{C}_{0,2}/\partial Z_2 & \cdots \\ \partial\mathcal{C}_{1,0}/\partial Z_0 & \partial\mathcal{C}_{1,1}/\partial Z_1 & \partial\mathcal{C}_{1,2}/\partial Z_2 & \cdots \\ \partial\mathcal{C}_{2,0}/\partial Z_0 & \partial\mathcal{C}_{2,1}/\partial Z_1 & \partial\mathcal{C}_{2,2}/\partial Z_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{array}{l} \text{response of C at time 0} \\ \text{response of C at time 1} \\ \text{response of C at time 2} \\ \\ \end{array} \end{array}$$

The t-th entry of $\boldsymbol{M_v}$ is the equilibrium response of aggregate consumption at time t to a change in the return on the mutual fund at $s = 0$

$$\begin{array}{cc} & \begin{array}{c} \text{change in } r^p \\ \text{at time 0} \end{array} \\ \boldsymbol{M_v} = & \begin{pmatrix} \partial\mathcal{C}_{0,0}/\partial r^p_0 \\ \partial\mathcal{C}_{1,0}/\partial r^p_0 \\ \partial\mathcal{C}_{2,0}/\partial r^p_0 \\ \vdots \end{pmatrix} \begin{array}{l} \text{response of C at time 0} \\ \text{response of C at time 1} \\ \text{response of C at time 2} \\ \\ \end{array} \end{array}$$

Proposition 3.2 *In response to a shock to foreign demand $d\boldsymbol{C}^*$, the impulse response of consumption in the heterogeneous-agent model is given by*

$$d\boldsymbol{C} = \underbrace{\boldsymbol{M}d\boldsymbol{Y}}_{\substack{\textit{Multiplier}\\ \textit{channel}}} + \underbrace{\boldsymbol{M_v}dr_0^p}_{\substack{\textit{Revaluation}\\ \textit{channel}}},$$

the impulse response of output $d\boldsymbol{Y}$ is determined by

$$d\boldsymbol{Y} = \underbrace{\alpha d\boldsymbol{C}^*}_{\substack{\textit{Direct}\\ \textit{channel}}} + \underbrace{(1-\alpha)\boldsymbol{M}d\boldsymbol{Y}}_{\substack{\textit{Multiplier}\\ \textit{channel}}} + \underbrace{(1-\alpha)\boldsymbol{M_v}dr_0^p}_{\substack{\textit{Revaluation}\\ \textit{channel}}},$$

and the impulse response of trade balance is given by

$$d\boldsymbol{NX} = \underbrace{\alpha d\boldsymbol{C}^*}_{\substack{\textit{Direct}\\ \textit{channel}}} - \underbrace{\alpha\boldsymbol{M}d\boldsymbol{Y}}_{\substack{\textit{Multiplier}\\ \textit{channel}}} - \underbrace{\alpha\boldsymbol{M_v}dr_0^p}_{\substack{\textit{Revaluation}\\ \textit{channel}}}.$$

Proof. See Appendix C. ■

Proposition 3.2 shows that now the responses to the shock also depend on two indirect effects: (i) the multiplier channel, and (ii) the revaluation channel. The multiplier channel operates through households' real labor income, as the expansion of output leads to higher employment, which, in turn, raises the level of spending by agents. The revaluation channel reflects unexpected capital gains during the shock period, arising from changes in the present discounted value of dividends.

The mechanism of the revaluation channel can be understood through equation (2-12). By iterating this equation forward, we can derive the return on the mutual fund at $t = 0$

$$1 + r_0^p = \frac{p_0 + d_0}{p} = \frac{1}{p}\sum_{t\geq 0}\left(\prod_{s<t}\frac{1}{1+r_s}\right)d_t. \tag{3-3}$$

Since the foreign demand shock induces positive changes in the sequence $\{d_t\}_{t=0}^{\infty}$, the present discounted value increases. Consequently, there is an unexpected gain in the return on savings at $t = 0$.

Finally, comparing our analytical results for foreign demand shocks with those obtained by Auclert et al. (2024) for foreign interest rate shocks, the key difference lies in the behavior of the real exchange rate. In the case of the foreign demand shock, the real exchange rate remains constant. However, in the case of the foreign interest rate shock, the transmission to the domestic economy occurs through a depreciation of the real exchange rate. Since the domestic real interest rate remains unchanged, both behaviors are consistent with the UIP condition.

3.2 Quantitative Results

Given that our analysis is inspired by the work of Auclert et al. (2024) on the transmission of foreign interest rate shocks, we adopt the same calibration to represent the dynamics of the basic model. The parameterization is presented in Table 3.1. In this section, our aim is not to examine the specific implications for the Brazilian economy, but rather to assess the relative importance of some transmission channels.

As is common in the literature, the idiosyncratic productivity e follows a Markov process. Here, there are 7 states, a standard deviation σ_e of 0.833, and a persistence coefficient ρ_e of 0.912. Additionally, the model incorporates discount factor heterogeneity, with a three-point distribution at $\{\beta - \Delta/2, \beta, \beta + \Delta/2\}$ and a third of agents in each.

The foreign discount factor implies an annualized real interest rate of 4 percent in the steady state. In the simulation of the representative-agent model, the domestic discount factor β is set equal to the foreign discount factor β^*. Lastly, the elasticities of substitution across goods η and γ are calibrated such that the trade elasticity is $\chi = \eta(1-\alpha) + \gamma = 0.5$.

Table 3.1: Calibration of the Basic Model

Parameter		Value
Households		
Elasticity of intertemporal substitution	$1/\sigma$	1
Frisch elasticity	$1/\varphi$	0.5
Disutility of labor	ψ	1
Elasticity of substitution between home and foreign good	η	$0.5/(2-\alpha)$
Elasticity of substitution across countries	γ	$0.5/(2-\alpha)$
Foreign discount factor	β^*	0.99
Average discount factor	β	0.965
Dispersion of the average discount factor	Δ	0.05
Cross-sectional std of earnings	σ_e	0.833
Persistence of earnings/idiosyncratic skills	ρ_e	0.912
Business		
Price markup	μ_p	1.043
Wage markup	μ_w	$1/\mu_p$
Calvo parameter for wage	θ_ω	0.938
Degree of economic openness	α	0.40
Steady-state net foreign asset position	nfa	0

3.2.1 Impulse Response Functions

We simulate a foreign demand shock under the assumption that C_t^* follows a first-order autoregressive process $AR(1)$ with a quarterly persistence of $p_{C^*} = 0.75$. The shock is normalized to have an initial positive effect of 1% on foreign demand.

Figure 3.1 presents the impulse response functions (IRFs) for the shock. As previously discussed, the real exchange rate remains unchanged in both models. In the heterogeneous-agent model, the return on savings initially increases as a result of the change in the present discounted value of dividends, whereas it stays constant in the representative-agent model. As anticipated from Propositions 3.1 and 3.2, the HANK model exhibits a larger output response, a positive consumption response, and a smaller trade balance response. The latter turns negative after the seventh quarter due to the weakening of the external demand and the strengthening of domestic consumption.

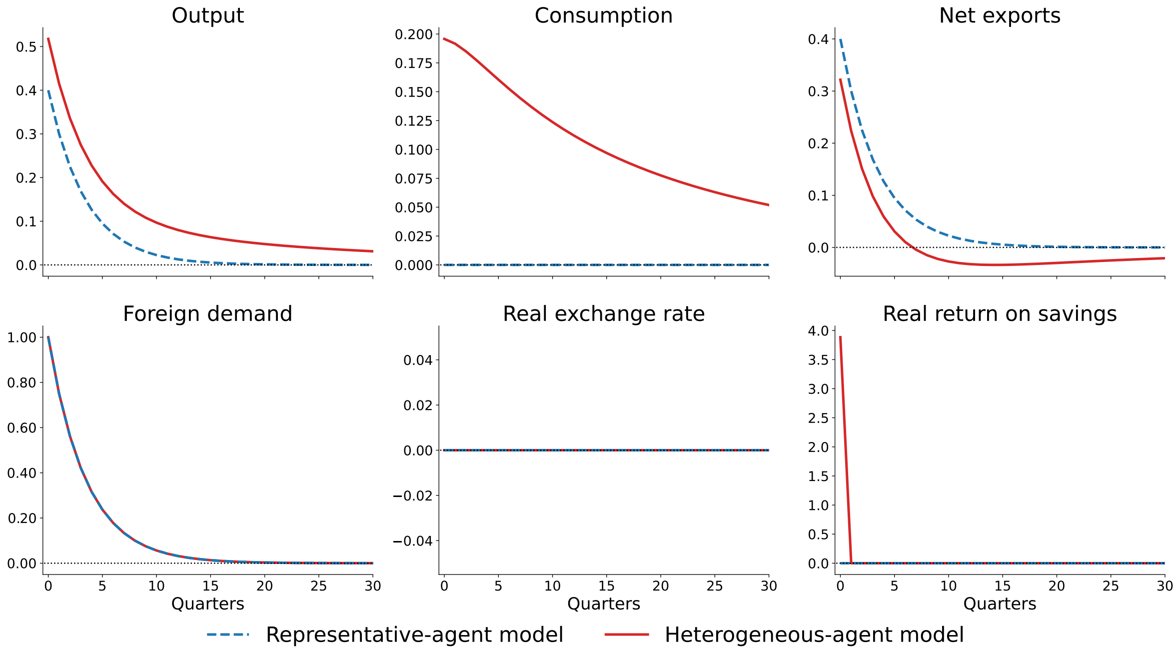
Figure 3.2 illustrates the decomposition of the shock across the channels. We observe that the multiplier channel plays the most significant role in amplifying the response, while the revaluation effect contributes minimally to the overall impact of the shock on the economy. To evaluate the influence of each channel over the entire period, Table 3.2 presents the relative contributions to the present value of the responses. Panel A illustrates that the multiplier channel predominates in the transmission of consumption and output, while net exports are equally influenced by the direct channel and the sum of the indirect channels. Panel B reaffirms that the transmission in the representative-agent model occurs solely through the direct channel.

Table 3.2: Transmission Channels in the Basic Model

	Direct	Multiplier	Revaluation
(A) Heterogeneous-agent model			
Output	40%	58%	2%
Consumption	0%	96%	4%
Net Exports	50%	48%	2%
(B) Representative-agent model			
Output	100%	0%	0%
Consumption	0%	0%	0%
Net Exports	100%	0%	0%

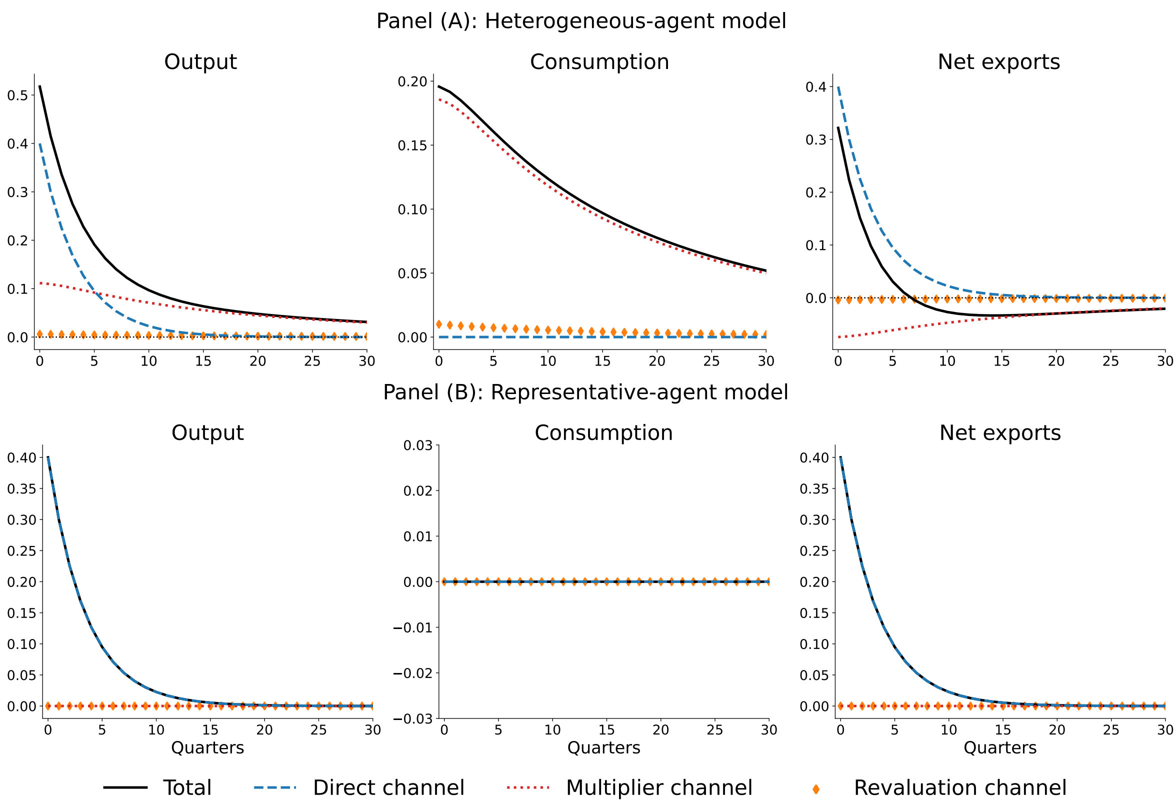
Note: The values denote the relative contribution of each channel to the present value of the response of the aggregate variables. Mathematically, the present values are calculated as $\sum_{t=0}^{\infty} dV_t/(1+r)^t$, where $dV_t \in \{dY_t, dC_t, dNX_t\}$ is the deviation from steady state of variable V at time t.

Figure 3.1: IRFs to a Foreign Demand Shock



Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output because its steady-state value is zero, and the real return on savings, which is in basis points.

Figure 3.2: Decomposing the Effects



Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output.

4
Extended Model

This chapter presents the extended version of our heterogeneous-agent model. It incorporates the following modifications: (i) investment as a component of aggregate demand; (ii) price rigidity to model imperfect exchange rate pass-through; (iii) a standard Taylor rule; and (iv) a segmented international financial market. The first and fourth items are new compared to the extended model employed by Auclert et al. (2024).

Investment is included because we want to understand how the transmission of the shock differs between output and its components. The incorporation of imperfect exchange rate pass-through introduces the expenditure switching and real income channels to the foreign demand shock. This happens because price rigidity breaks the law of one price. These two modifications lead to a more complete supply side, which now includes domestic retailers, exporters, importers, and capital producers.

The inclusion of the Taylor rule generates the real interest rate channel. The segmented international financial market is introduced to break the UIP condition. Foreign intermediaries have exclusive access to the external bond market, where they employ a carry trade strategy. Empirically, this is supported by the extensive use of the Brazilian currency in such strategies. Theoretically, by breaking the UIP, there is a larger adjustment in the real exchange rate, which helps to dampen interest rate responses. This is important because investment decisions are highly sensitive to the real interest rate.

4.1
Production Sector

The production sector comprises four types of agents: domestic retailers, exporters, importers, and capital producers.

4.1.1
Domestic Retailers

A unit-mass of firms j act in monopolistic competition, utilizing labor and capital as inputs in a Cobb-Douglas production function

$$y_{j,t} = f(k_{j,t}, n_{j,t}) = \Theta k_{j,t}^{a} n_{j,t}^{1-a}, \tag{4-1}$$

where Θ denotes the constant total factor productivity (TFP), and a is the capital share. Firms hire capital $k_{j,t}$ at a price of R_t^k/P_t and labor $n_{j,t}$ at a wage rate of W_t/P_t. Since ε represents the elasticity of substitution between home goods, the demand for the good j is given by

$$y_{j,t} = \left(\frac{P_{j,H,t}}{P_{H,t}}\right)^{-\varepsilon} Y_t, \tag{4-2}$$

where $\mu_H = \varepsilon/(\varepsilon - 1)$ is the home goods price markup. Firm j is also subject to Rotemberg (1982) adjustment costs

$$\zeta_{H,t}(P_{j,H,t}, P_{j,H,t-1}) = \frac{\mu_H}{\mu_H - 1}\frac{1}{2\kappa_H}\left[\log\left(\frac{P_{j,H,t}}{P_{j,H,t-1}}\right)\right]^2, \tag{4-3}$$

where κ_H is the slope of the Phillips curve for home goods in domestic currency.

In this context, the firm j chooses its price $P_{j,H,t}$, labor $n_{j,t}$ and capital $k_{j,t}$ to maximize the present value of its future profits

$$\begin{aligned} J_t^{DR}(P_{j,H,t-1}) = \max_{P_{j,H,t},\, k_{j,t},\, n_{j,t}} \Bigg\{ & \frac{P_{j,H,t}}{P_t} f(k_{j,t}, n_{j,t}) - \frac{W_t}{P_t} n_{j,t} - \frac{R_t^k}{P_t} k_{j,t} \\ & - \zeta_{H,t}(P_{j,H,t}, P_{j,H,t-1}) \frac{P_{H,t}}{P_t} Y_t + \frac{1}{1+r_t} J_{t+1}^{DR}(P_{j,H,t}) \Bigg\} \\ & \text{s.t. (4-1), (4-2), and (4-3)} \end{aligned}$$

As shown in Appendix D.1.1, all firms set the same price $P_{j,H,t} = P_{H,t}$ in equilibrium. This behavior results in the following Phillips curve for home goods in domestic currency

$$\log(1 + \pi_{H,t}) = \kappa_H \left(mc_{H,t} - \frac{1}{\mu_H}\right) + \frac{1}{1+r_t}\frac{Y_{t+1}}{Y_t}\log(1 + \pi_{H,t+1}), \tag{4-4}$$

where $\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} - 1$ is the home goods inflation, and the marginal cost is

$$mc_{H,t} = \frac{1}{1-a}\frac{W_t}{P_{H,t}}\frac{N_t}{Y_t}.$$

Finally, the aggregate real dividend of the domestic retailer firms is

$$d_t^{DR} = \frac{P_{H,t}}{P_t} Y_t - \frac{W_t}{P_t} N_t - \frac{R_t^k}{P_t} K_{t-1} - \frac{P_{H,t}}{P_t}\zeta_{H,t}(P_{H,t}, P_{H,t-1}) Y_t. \tag{4-5}$$

4.1.2 Exporters

To model imperfect pass-through to export prices, we assume that each home good j has an associated exporter. This exporter purchases the variety j at the domestic-currency price $P_{j,H,t}$ and sells it to foreign agents at the domestic-currency price $\mathcal{E}_t P^*_{j,H,t}$.

The production function of the exporter firm j and the demand function for its good are, respectively,

$$x^*_{j,H,t} = g(x_{j,t}) = x_{j,t} \quad \text{and} \quad x^*_{j,H,t} = \left(\frac{P^*_{j,H,t}}{P^*_{H,t}}\right)^{-\varepsilon^*} X^*_{H,t}, \tag{4-6}$$

where $x_{j,t}$ is the quantity purchased by the exporter, ε^* is the elasticity of substitution between exports, and $\mu^*_H = \varepsilon^*/(\varepsilon^* - 1)$ is the export price markup. Given the foreign demand X^*_t, the external demand for domestic goods is

$$X^*_{H,t} = \alpha \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\gamma} X^*_t. \tag{4-7}$$

The exporter firm j is also subject to Rotemberg (1982) adjustment cost

$$\zeta^*_{H,t}(P^*_{j,H,t}, P^*_{j,H,t-1}) = \frac{\mu^*_H}{\mu^*_H - 1} \frac{1}{2\kappa^*_H} \left[\log\left(\frac{P^*_{j,H,t}}{P^*_{j,H,t-1}}\right)\right]^2, \tag{4-8}$$

where κ^*_H is the slope of the Phillips curve for home goods in foreign currency.

In this scenario, the exporter j chooses its foreign-currency price $P^*_{j,H,t}$, and the quantity to be exported $x_{j,t}$ to maximize the present value of its profits

$$\begin{aligned} J^E_t(P^*_{j,H,t-1}) = \max_{P^*_{j,H,t},\, x_{j,t}} \Bigg\{ & \frac{P^*_{j,H,t}}{P_t} \mathcal{E}_t g(x_{j,t}) - \frac{P_{H,t}}{P_t} x_{j,t} \\ & - \zeta^*_{H,t}(P^*_{j,H,t}, P^*_{j,H,t-1}) \frac{P^*_{H,t}}{P_t} \mathcal{E}_t X^*_{H,t} + \frac{1}{1+r_t} J^E_{t+1}(P^*_{j,H,t}) \Bigg\} \\ & \text{s.t. (4-6), and (4-8)} \end{aligned}$$

Following the same steps shown in Appendix D.1.1, we obtain the Phillips curve for home goods in foreign currency

$$\log(1+\pi^*_{H,t}) = \kappa^*_H \left(\frac{P_{H,t}}{\mathcal{E}_t P^*_{H,t}} - \frac{1}{\mu^*_H}\right) + \frac{1}{1+r_t} \frac{X^*_{H,t+1}}{X^*_{H,t}} \log(1+\pi^*_{H,t+1}). \tag{4-9}$$

Finally, the aggregate real dividend of the exporter firms is

$$d^E_t = \left(\frac{P^*_{H,t}}{P_t}\mathcal{E}_t - \frac{P_{H,t}}{P_t}\right) X^*_{H,t} - \frac{P^*_{H,t}}{P_t} \mathcal{E}_t \zeta^*_{H,t}(P^*_{H,t}, P^*_{H,t-1}) X^*_{H,t}. \tag{4-10}$$

4.1.3 Importers

We model the imperfect pass-through to import prices by assuming that foreign importers purchase the foreign good s at the domestic-currency price[1] $\mathcal{E}_t P^*_{s,F,t} = \mathcal{E}_t$ and sell it in the domestic economy at the domestic-currency price $P_{s,F,t}$. Following the same steps used in the exporters' problem, we can derive the Phillips curve for foreign goods in domestic currency

$$\log(1+\pi_{F,t}) = \kappa_F \left(\frac{\mathcal{E}_t}{P_{F,t}} - \frac{1}{\mu_F} \right) + \frac{1}{1+r_t^*} \frac{M_{F,t+1}}{M_{F,t}} \log(1+\pi_{F,t+1}), \qquad (4\text{-}11)$$

where μ_F is the import price markup, and $M_{F,t}$ is the domestic demand for foreign goods.

4.1.4 Capital Producer

A representative capital producer builds the capital stock and rents it to domestic retailers at a rate R_t^k/P_t. The capital stock depreciates at a rate δ, and its production is subject to an investment adjustment cost given by

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\Psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

The capital producer chooses the capital stock for the next period[2] K_t and the investment I_t to maximize the present value of its future profits

$$J_t^K(K_{t-1}, I_{t-1}) = \max_{K_t,\, I_t} \left\{ \frac{R_t^k}{P_t} K_{t-1} - I_t + \frac{1}{1+r_t} J_{t+1}^K(K_t, I_t) \right\}$$
$$\text{s.t. } K_t = (1-\delta)K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

As shown in Appendix D.1.2, defining Tobin's q as the marginal value of capital at the end of period t, the investment dynamics is characterized by

$$q_t = \frac{1}{1+r_t}\left[\frac{R_{t+1}^k}{P_{t+1}} + q_{t+1}(1-\delta)\right], \qquad (4\text{-}12)$$

$$1 = q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \frac{1}{1+r_t} q_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right). \qquad (4\text{-}13)$$

[1] As discussed in Section 2.3, the foreign monetary authority maintains the foreign-currency price of foreign goods at a constant level $P^*_{s,F,t} = P^*_{F,t} = 1$.

[2] $K_t = \int k_{j,t+1} dj$, where $k_{j,t+1}$ is the capital demand from firm j at time $t+1$.

The real dividend of the capital producer is

$$d_t^K = \frac{R_t^k}{P_t} K_{t-1} - I_t. \tag{4-14}$$

Investment, akin to consumption, is allocated between domestic goods $I_{H,t}$ and foreign goods $I_{F,t}$. We assume the same degree of economic openness and the same elasticity of substitution between home and foreign goods for both consumption and investment. Consequently, the price index – which is the same for both macroeconomic aggregates – and the investment basket are

$$I_t = \left[\alpha^{\frac{1}{\eta}} I_{F,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} I_{H,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \text{ and } P_t = \left[\alpha P_{F,t}^{1-\eta} + (1-\alpha) P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{4-15}$$

Using a procedure analogous to that in Appendix A.1, we find the following allocation of investment between domestic and foreign goods

$$I_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} I_t \quad \text{and} \quad I_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} I_t. \tag{4-16}$$

4.2 Financial Sector

The financial sector has two types of agents: the mutual fund and a representative foreign professional intermediary. The mutual fund continues to exhibit the same behavior as in the basic model, except that it can not directly buy foreign bonds. Despite this, the return on the mutual fund remains as

$$1 + r_{t+1}^p = \begin{cases} 1 + r_t = \dfrac{p_{t+1} + d_{t+1}}{p_t}, \text{ for } t \geq 0 \\ \dfrac{p_0 + d_0}{p}, \text{ for } t = -1, \end{cases} \tag{4-17}$$

where $d_t = d_t^{DR} + d_t^E + d_t^K$.

The foreign intermediary trades bonds in both currencies using a carry trade strategy. In each period, it takes a long position in the foreign bond and a short position in the domestic bond, or vice versa. These transactions are carried out by borrowing from the domestic mutual fund or foreign households.

In this scenario, the foreign intermediary maximizes the expected value of its firm at each period

$$V_t = \mathbb{E}_t \left[(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) - (1+i_t^*)\right] B_t^I, \tag{4-18}$$

where B_t^I is the size of the position. A positive (negative) position represents a long position in the home (foreign) bond and a short position in the foreign (home) bond.

Following Gabaix and Maggiori (2015), the foreign financier can divert a portion of the funds. Since creditors anticipate the incentives to divert funds, the intermediary is subject to a limited commitment constraint

$$\underbrace{V_t}_{\substack{\text{Intermediary}\\ \text{Value}}} \geq \underbrace{|B_t^I|}_{\substack{\text{Total}\\ \text{Claims}}} \underbrace{\Gamma|B_t^I|}_{\substack{\text{Diverted}\\ \text{Portion}}} = \underbrace{\Gamma\left(B_t^I\right)^2}_{\substack{\text{Divertable}\\ \text{Funds}}}. \tag{4-19}$$

Therefore, the foreign professional intermediary's problem is given by

$$\max_{B_t^I} V_t = \mathbb{E}_t \left[(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) - (1+i_t^*)\right] B_t^I \quad \text{s.t.} \quad V_t \geq \Gamma\left(B_t^I\right)^2$$

As detailed in Appendix D.2, the demand for domestic bonds from the intermediary is

$$B_t^I = \frac{1}{\Gamma}\left[(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) - (1+i_t^*)\right], \tag{4-20}$$

where Γ is the ability to bear risks. As Γ increases, the risk-bearing capacity of the financier decreases, and the bond market becomes more segmented.

In this segmented international financial market, the foreign intermediary hold the inverse of the net foreign asset position

$$\text{nfa}_t = A_t - p_t = -B_t^I. \tag{4-21}$$

Substituting (2-8), and (4-21) into (4-20), we obtain that the uncovered interest parity (UIP) condition is not satisfied

$$(1+r_t)\left(\frac{Q_t}{Q_{t+1}}\right) = (1+r_t^*) - \Gamma\text{nfa}_t. \tag{4-22}$$

Note that if the financier can take infinite positions whenever there is a nonzero expected return, $\Gamma \to 0$, then the UIP condition is recovered.

4.3 Monetary Policy

The constant real interest rate rule is replaced by a standard Taylor rule

$$i_t = r_{ss} + \phi\pi_t, \tag{4-23}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the domestic inflation, and r_{ss} is the steady-state real interest rate.

4.4 Equilibrium

Definition 4.1 *Given sequences of foreign demand shocks $\{X_t^*\}$, initial values for the nominal wage and price levels, an initial portfolio allocation for the mutual fund, an initial capital level, and an initial distribution of households i over their state variables such that the economy is initially at a steady state, a* ***competitive equilibrium*** *is defined as a path of*

- *policies $\{c_{i,H,t}, c_{i,F,t}, c_{i,t}, a_{i,t+1}\}$ and distribution $\mathcal{D}_{i,t}$ for households,*
- *prices $\{\mathcal{E}_t, Q_t, P_t, P_{H,t}, P_{F,t}, W_t, R_t^k, q_t, i_t, r_t, r_t^p\}$,*
- *and aggregates $\{C_t, C_{H,t}, C_{F,t}, I_t, I_{H,t}, I_{F,t}, M_{F,t}, Y_t, K_t, N_t, A_t, B_t^I, p_t, d_t^{DR}, d_t^E, d_t^K, d_t, nfa_t\}$,*

such that households optimize, the mutual fund optimizes, the professional intermediary optimizes, firms optimize, unions optimize, monetary and fiscal policy follow their respective rules, and all markets clear:

- *the goods market $Y_t = C_{H,t} + I_{H,t} + X_{H,t}^* + \zeta_{H,t} Y_t + \zeta_{H,t}^* X_{H,t}^*$,*
- *the factor market $N_t = \int n_{j,t} dj$ and $K_{t-1} = \int k_{j,t} dj$[3],*
- *and the asset market $A_t = p_t + nfa_t$.*

As in the basic model, the current account identity holds in the equilibrium

$$\mathrm{nfa}_t - \mathrm{nfa}_{t-1} = NX_t + r_{t-1}\mathrm{nfa}_{t-1} + (r_t^p - r_{t-1})A_{t-1}, \tag{4-24}$$

where

$$NX_t = \frac{P_{H,t}^*}{P_t}\mathcal{E}_t X_{H,t}^* - \frac{P_{F,t}}{P_t} M_{F,t} \tag{4-25}$$

is the value of net exports, $X_{H,t}^*$ is the external demand for home goods, and $M_{F,t} = I_{F,t} + C_{F,t}$ is the domestic demand for foreign goods.

As outlined in Appendix D.3, we focus on the steady state with no inflation and no external financial positions. The latter condition implies that the mutual fund is initially fully invested in domestic stocks. Furthermore, we normalize foreign demand X^*, output Y, employment N, and real exchange rate Q to 1. To further simplify the steady-state expressions, we also set the prices of P_H, P_F, P_H^*, and P to 1.

[3]This notation indicates that capital is predetermined from the perspective of capital producer, but not from the perspective of domestic retailers.

5
Foreign Demand Shock in the Brazilian Economy

This chapter presents the quantitative results of our extended model. Initially, we calibrate the model using Brazilian macroeconomic aggregates, as well as wealth and income distribution variables. Next, we analyze the impulse response functions (IRFs) to a foreign demand shock. We then decompose the transmission channels that affect output and its three components: consumption, investment, and net exports. Finally, we investigate the heterogeneous behavior of households by analyzing the responses of consumption, savings, and disposable income across different wealth percentiles.

5.1
Data and Calibration

We calibrate the model at a quarterly frequency to match two sets of moments: macroeconomic aggregates and distributional variables. The sample period spans from the first quarter of 1999 to the fourth quarter of 2022. The starting point of 1999 is chosen because it marks the year when the Central Bank of Brazil adopted the inflation targeting regime for monetary policy, while the ending point of 2022 is determined by the availability of data.

5.1.1
Data

Most of the macroeconomic aggregates come from the *Sistema de Contas Nacionais* (IBGE, 2024). The exceptions are: the capital stock time series, which is from the *Instituto de Pesquisa Econômica Aplicada* (IPEA, 2023); the wealth-income ratio, from the World Inequality Database (WID, 2024); and the long-term interest rate, which corresponds to the value used by the Central Bank in its DSGE model (Fasolo et al., 2023). Regarding the distributional variables, all data is sourced from the World Inequality Database (WID, 2024). Table 5.2 presents the average of each variable over the period 1999-2022.

5.1.2
Calibration

Table 5.1 presents our parameterization. Concerning the parameters of households, we set the elasticity of intertemporal substitution $1/\sigma$ to 0.5, following the world mean value estimated in the meta-analysis of Havranek et al. (2015, Table A2). The Frisch elasticity of labor supply $1/\varphi$ is fixed at

Table 5.1: Calibration of the Extended Model

Parameter		Value	Source / Target
Households			
Elasticity of intertemporal substitution	$1/\sigma$	0.5	Havranek et al. (2015)
Frisch elasticity	$1/\varphi$	0.5	Chetty et al. (2011)
Disutility of labor	ψ	0.834	Equation (D-9)
Elasticity of substitution home and foreign goods	η	1.022	Fasolo et al. (2023)
Elasticity of substitution across countries	γ	1.022	Fasolo et al. (2023)
Foreign discount factor	β^*	0.98	Data moments of Table 5.2
Average discount factor	β	0.85	Data moments of Table 5.2
Dispersion of the average discount factor	Δ	0.07	Data moments of Table 5.2
Cross-sectional std of earnings	σ_e	1.65	Data moments of Table 5.2
Persistence of earnings	ρ_e	0.985	Data moments of Table 5.2
Firms			
Total factor productivity	Θ	0.817	Equation (D-8)
Investment adjustment cost	Ψ	3	Standard
Depreciation rate	δ	0.073	Data moments of Table 5.2
Capital share	a	0.228	Data moments of Table 5.2
Wage markup	μ_ω	1.01	Data moments of Table 5.2
Price markup	μ_H	1.01	Data moments of Table 5.2
Exports markup	μ_{H^*}	1	Law of one price in the s.s.
Imports markup	μ_F	1	Law of one price in the s.s.
Economy			
Degree of economic openness	α	0.13	Data moments of Table 5.2
Steady-state net foreign asset position	nfa	0	Standard
Risk-bearing capacity of the financier	Γ	0.05	Standard
Prices			
Calvo parameter for wage	θ_ω	0.784	Fasolo et al. (2023)
Calvo parameter for domestic goods	θ_H	0.573	Fasolo et al. (2023)
Calvo parameter for imported goods	θ_F	0.632	Fasolo et al. (2023)
Calvo parameter for exported goods	θ_{H^*}	0.819	Fasolo et al. (2023)
Taylor rule coefficient	ϕ	1.5	Standard

the standard value of 0.5 (Chetty et al., 2011). The disutility of labor is fixed according to equation (D-9) to ensure a steady-state wage inflation of $\pi_w = 0$. The elasticities of substitutions between home and foreign goods, and across countries are set to $\eta = \gamma = 1.911/(2-\alpha)$ in order to achieve a trade elasticity of $\chi = \eta(1-\alpha) + \gamma = 1.911$, which corresponds to the value used by the Central Bank in its DSGE model (Fasolo et al., 2023).

To match the distributional moments presented in Table 5.2, we introduce heterogeneity in the discount factor. Specifically, we use a three-point distribution at $\{\beta - \Delta/2, \beta, \beta + \Delta/2\}$, with one-third of agents in each group,

Table 5.2: Target Moments

Variable		Data	Model
Macroeconomic aggregates			
Output, normalized	Y	1.00	1.00
Consumption	C	0.82	0.82
Investment	I	0.18	0.18
Exports	X	0.13	0.13
Imports	M	0.13	0.13
Capital	K	2.47	2.43
Wealth-income ratio	A/Z	3.81	3.82
Long-term interest rate	r	0.01	0.02
Distributional variables			
Bottom 50% wealth share		0.01	0.01
Middle 40% wealth share		0.23	0.23
Top 10% wealth share		0.76	0.76
Bottom 50% income share		0.10	0.06
Middle 40% income share		0.31	0.36
Top 10% income share		0.59	0.58

Note: The data are averages over the period 1999-2022. Macroeconomic aggregates are sourced from IBGE (2024), except for the capital (IPEA, 2023), the wealth-income ratio (WID, 2024), and the long-term interest rate (Fasolo et al., 2023). Distributional variables are calculated from WID (2024).

where $\beta = 0.85$ and $\Delta = 0.07$. The idiosyncratic productivity e follows a Markov process. Here, there are 30 states, a standard deviation σ_e of 1.65, and a persistence coefficient ρ_e of 0.985. These values are chosen to replicate the extreme wealth and income inequality observed in Brazil.

Regarding the parameters of firms, the TFP of $\Theta = 0.817$ is determined by equation (D-8), while the investment adjustment cost of $\Psi = 3$ is a standard value. We define the depreciation rate and capital share as $\delta = 0.073$ and $a = 0.228$, respectively, to match the capital stock presented in Table 5.2. The wage markup and price markup are both fixed at $\mu_w = \mu_H = 1.01$ to ensure the wealth-income ratio expressed in Table 5.2. Furthermore, we also have to set the foreign discount rate to $\beta^* = 0.98$ to achieve this moment. Consequently, both the foreign and domestic economy have annualized real interest rate of 8 percent in the steady state. Finally, the exports and imports markups are fixed at $\mu_H* = \mu_F = 1$ to guarantee the law of one price in the steady state.

In relation to the parameters of the economy, the degree of economic openness is fixed at $\alpha = 0.13$, based on the values of exports and imports presented in Table 5.2. On the one hand, a zero steady-state net foreign asset position is the standard value in the literature. On the other hand, $nfa = 0$ is the only value for which the current account identity holds in equilibrium when $NX = 0$ and $r \neq 0$. Following Itskhoki and Mukhin (2025), the risk-bearing capacity of the professional intermediary is fixed at a small value, $\Gamma = 0.05$.

Regarding the parameters of prices, the Taylor rule coefficient is fixed at $\phi = 1.5$, which is the standard value in the literature. To calibrate the slopes of the Phillips curves, we use the relationship between the nominal rigidity models of Calvo (1983) and Rotemberg (1982): $\kappa_p = (1 - \beta^* \theta_p)(1 - \theta_p)/\theta_p$ for prices $p \in \{H, F, H^*\}$, and $\kappa_w = (1 - \beta\theta_w)(1 - \theta_w)/\theta_w$ for wages. Thus, we can use the same Calvo parameters as Fasolo et al. (2023).

As shown in Table 5.2, the calibration fits the macroeconomics aggregates, and the income and wealth distribution data well. The only notable discrepancy is in the annualized long-term interest rate, which we set at 8 percent, rather than 4 percent. This adjustment was necessary to achieve the desired wealth-income ratio. Besides, to match this moment, we also have to set very small wage and price markups. Thus, as a robustness exercise, Appendix E.1 presents the IRFs for the model calibrated with $r = 0.01$ and the same markup values employed by the Central Bank in its DSGE model.

5.2 Impulse Responses

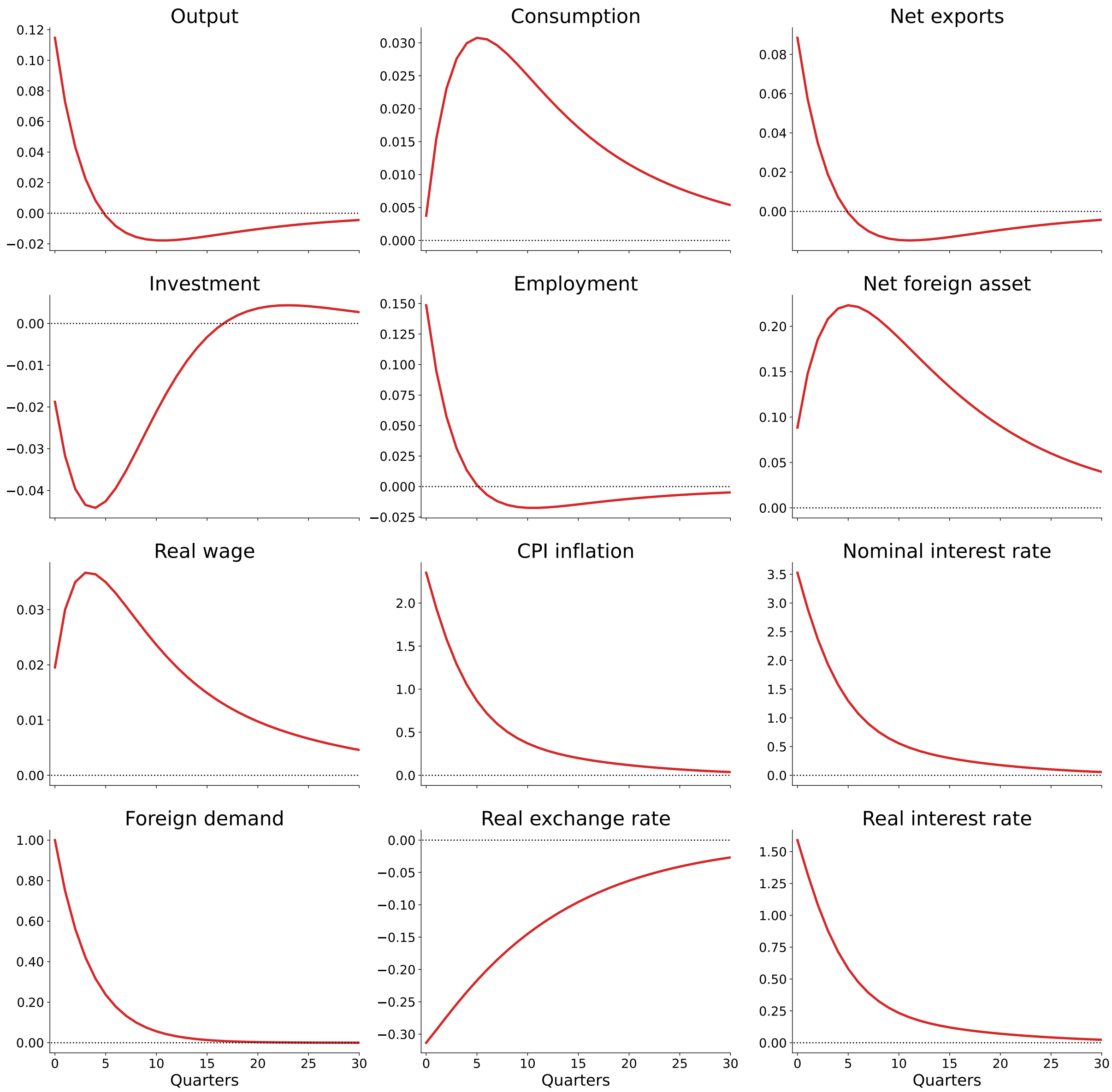
Figure 5.1 presents the IRFs to a foreign demand shock, under the assumption that $X^*_{H,t}$ follows a first-order autoregressive process $AR(1)$ with a quarterly persistence of $p_{X^*_H} = 0.75$. The shock is normalized to have an initial positive effect of 1% on foreign demand. These assumptions are consistent with those used in the quantitative results of the basic model.

The expansion of foreign demand increases net exports, which, in turn, boosts production. As the economy strengthens, both domestic employment and prices rise. Higher demand for labor drives up real wages, and the simultaneous growth in employment and real wages stimulates consumption.

The increase in domestic prices leads to a rise in the consumer price index. In response, the central bank raises the nominal interest rate, which causes an increase in the real interest rate. This results in the appreciation of the real exchange rate, which, in turn, promotes an expansion in imports and a reduction in exports. Imports also increase as a result of the higher consumption level. Consequently, the improvement in the trade balance weakens.

Over time, the decline in consumption and the deterioration of the trade balance lead to falls in both employment and output. This, in turn, slows inflation, prompting a reduction in both nominal and real interest rates. As a

Figure 5.1: IRFs to a Foreign Demand Shock in the Brazilian Economy



Note: Most of the variables are expressed in percentage deviation from steady state. There are two groups of exceptions: (i) net exports, and the net foreign asset position; (ii) CPI inflation, the nominal interest rate, and the real interest rate. The first group is in percentage of steady-state output, as their steady-state values are zero. The second group is in basis points.

consequence, the real exchange rate begins to revert to its steady-state value. The diminished appreciation of the exchange rate stimulates exports, leading to an increase in both output and net exports, which eventually return to their steady-state values.

With respect to investment, the initial rise in the real interest rate causes a contraction. However, as output surpasses its trough and the real interest rate declines, investment begins to recover. Due to its sensitivity to the interest rate, the initial decline in investment after a demand shock is a commonly observed feature in New Keynesian models. Nonetheless, the dynamics of output (growth, decline, and recovery), net exports (improvement, deterioration, and recovery), and investment (initial decline followed by growth) are consistent with the results observed in the previous and current version of the Brazilian DSGE model (Castro et al., 2011; Fasolo et al., 2023).

5.3 Transmission

Given that the monetary authority follows a Taylor rule and the law of one price is no longer valid, the transmission of foreign demand shocks now occur through six distinct channels: (i) the direct effect, (ii) the expenditure switching effect, (iii) the multiplier effect, (iv) the real income effect, (v) the interest rate effect, and (vi) the revaluation effect. The first and second effects can be classified as trade channels, the third and fourth as labor channels, and the fifth and sixth as wealth channels.

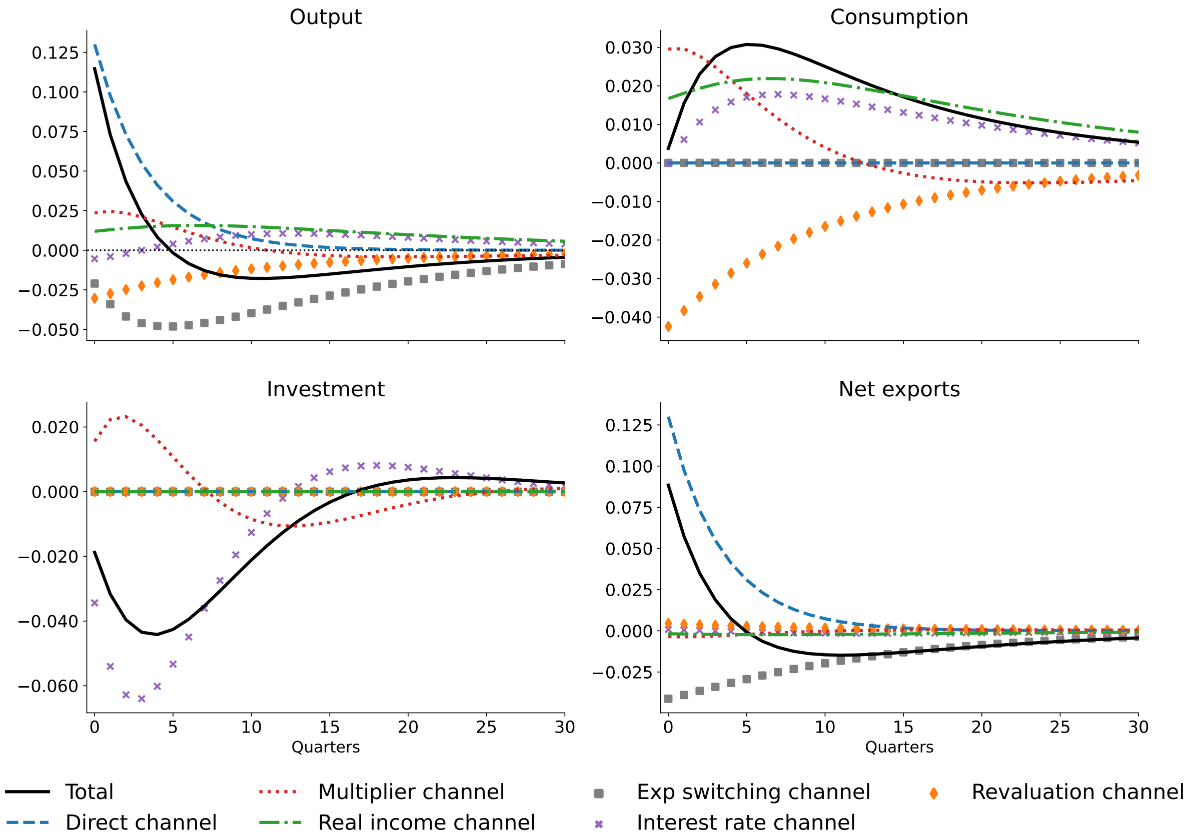
Beginning with the two trade channels, the direct effect reflects the increase in production driven by a positive change in external demand. The expenditure switching effect captures the expansion in foreign demand for domestic goods due to a decrease in their relative prices.

With respect to labor channels, the real income effect captures the fact that an increase in the real wage leads to a rise in real labor income. This, in turn, results in an increase in aggregate consumption and output. The multiplier effect also operates through income. An expansion in production generates an increase in employment, which subsequently raises the level of spending by agents. Additionally, this channel also influences investment, as the rise in output and employment creates incentives for capital accumulation.

Regarding wealth channels, the interest rate effect reflects changes in monetary policy. For firms, an increase in the real interest rate results in a reduction in investment. For households, the interest rate channel involves both the income and substitution effects. The income effect causes households to feel richer after an increase in r_t, prompting them to consume more. The substitution effect encourages households to save more in order to enable greater consumption in the future. The response of consumption through the interest rate channel is determined by the trade-off between these two effects.

The revaluation channel captures unexpected capital gains or losses during the shock period, arising from changes in the present discounted value of dividends. As indicated in Equation (3-3), this present value is determined by the sequences of dividends and real interest rates. In the event of unexpected capital gains, household disposable income and consumption increase.

Figure 5.2: Decomposing the Effects for the Brazilian Economy



Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output.

Figure 5.2 presents the decomposition of the transmission channels for output and its three components: consumption, investment, and net exports. To assess the influence of each channel over the entire period, Table 5.3 displays the relative contributions to the present value of the responses.

For output, Figure 5.2 illustrates that, initially, the direct channel is predominating. The expenditure switching and revaluation channels contribute to a reduction in output, while the labor channels work to enhance it. However, the indirect channels nearly offset one another. Over time, the influence of the direct effect diminishes, and the indirect effects become the dominant drivers of the output response. In this context, the expenditure switching and revaluation channels act as negative forces, while the real labor income and interest rate channels serve as positive ones.

Table 5.3 shows that, throughout the entire period, the trade channels account for 55% of the response (with emphasis on the expenditure switching effect), the labor channels contribute 24% (with emphasis on the real income effect), and the wealth channels represent 21%. In total, the direct channel is responsible for only 22% of the variation in output.

Table 5.3: Transmission Channels for the Brazilian Economy

	Trade Channels		Labor Channels		Wealth Channels	
	Direct	Expenditure Switching	Multiplier	Real Income	Interest Rate	Revaluation
Output	22%	33%	10%	14%	8%	13%
Consumption	0%	0%	19%	31%	21%	28%
Investment	0%	0%	28%	0%	72%	0%
Net Exports	45%	41%	3%	4%	3%	4%

Note: The values denote the relative contribution of each channel to the present value of the response of aggregate variables. Mathematically, the present values are calculated as $\sum_{t=0}^{\infty} dV_t/(1+r)^t$, where $dV_t \in \{dY_t, dC_t, dI_t, dNX_t\}$ is the deviation from steady state of variable V at time t.

Regarding consumption, after the shock, the multiplier is the most important positive force, while the revaluation is the negative force. Over time, the real income and interest rate channel strengthen the consumption, but the revaluation continues to diminish it. Table 5.3 indicates that, over the entire period, the labor and financial channels have equal significance, with emphasis on the real income and revaluation channels. It is worth noting that the interest rate channel is positive; that is, the income effect dominates the substitution effect. The next section offers an explanation for this.

The last two components of aggregate demand have a simpler transmission structure. In the case of investment, the interest rate channel is the primary driver throughout the entire period. For net exports, the direct channel serves as the main positive force, while the expenditure switching channel (resulting from the real exchange rate appreciation) acts as the negative force. The other channels make only a small contribution to the net exports response.

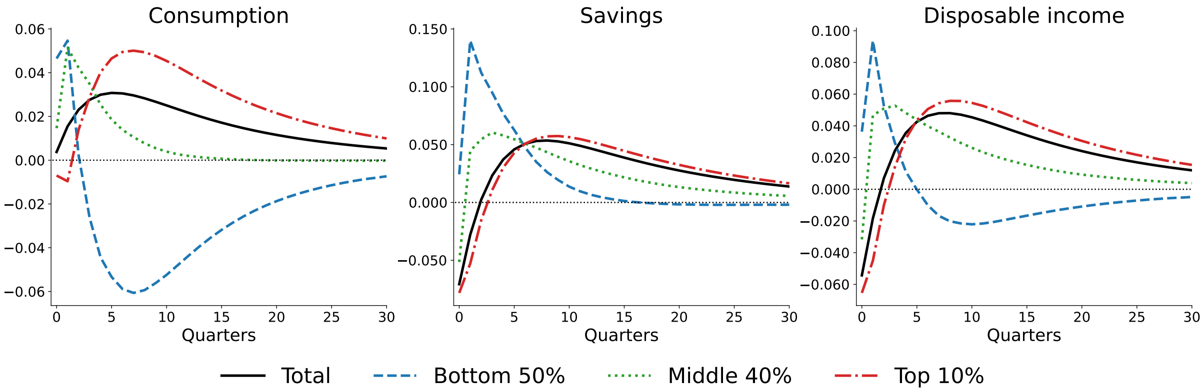
5.4 Inequality

In this section, we examine how the foreign demand shock affects three different wealth groups: (i) the bottom 50%, which represents households within the 0th to 50th percentiles of the wealth distribution; (ii) the middle 40%, which includes households from the 50th to the 90th percentiles; and (iii) the top 10%, which includes households within the 90th to 100th percentiles. These are the same percentiles used to calibrate the model.

Figure 5.3 illustrates the responses of consumption, savings, and disposable income for each wealth group. Initially, the bottom 50% observe an increase in their disposable income[1], which is accompanied by an increase in

[1]The disposable income of household i is given by $(1 + r_t^p)a_{i,t-1} + e_{i,t}\frac{W_t}{P_t}n_{i,t}$.

Figure 5.3: IRFs by Wealth Percentiles



Note: All variables are expressed in percentage deviation from steady state.

both savings and consumption. In contrast, the other two groups experience a decrease in disposable income due to a reduction in the value of their assets. However, the expansion in labor income (see real wage and employment in Figure 5.1) leads to an increase in consumption for the middle 40% and only a slight decrease in consumption for the top 10%. The labor component of income also contributes to the increase in savings for the bottom 50%.

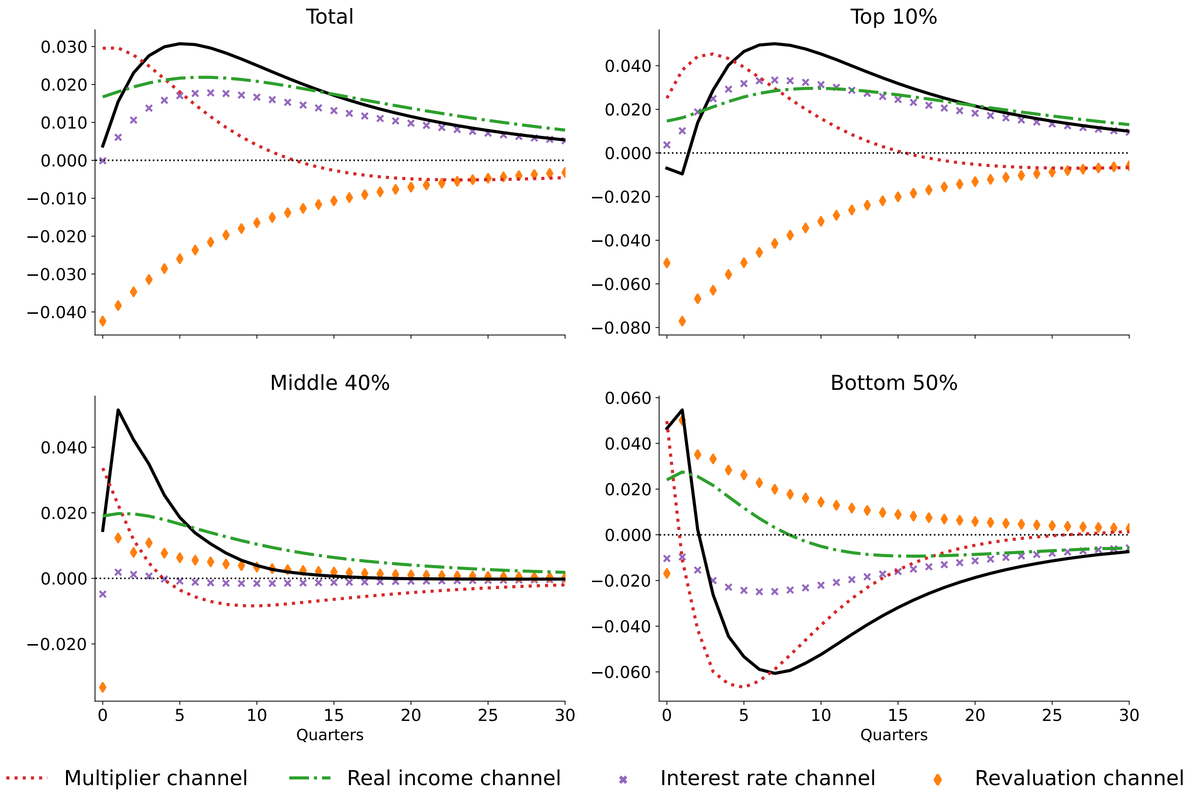
Over time, the response of disposable income varies across wealth groups. As a result, the poor have a decline in consumption, and their savings quickly return to their steady-state value. The middle class sees an increase primarily in savings, but also in consumption. The rich have increases in both consumption and savings. In aggregate terms, the response is largely driven by the top 10%, reflecting the extreme income and wealth inequality in the Brazilian economy. When considering all periods after the shock, Table 5.4 reveals that the poor are the most negatively affected in terms of consumption and disposable income, while the rich emerge as the main beneficiaries.

Table 5.4: Present Value of IRFs by Wealth Percentiles

	Consumption	Savings	Disposable Income
Total	0.44%	0.70%	0.65%
Bottom 50%	-0.60%	0.60%	-0.05%
Middle 40%	0.22%	0.63%	0.51%
Top 10%	0.68%	0.73%	0.72%

Note: The values denote the present value of the percentage deviation of each variable from its respective steady state. Mathematically, the present values are calculated as: $\sum_{t=0}^{\infty} dx_t^j/(1+r)^t$, where dx_t^j is the percentage deviation from steady state of variable x for the wealth group j at time t.

Figure 5.4: Decomposing the Consumption Effects by Wealth Percentiles



Note: The consumption is expressed in percentage deviation from steady state.

In Figure 5.4, we present the decomposition of the transmission channels for the component of output that is directly determined by households. To assess the influence of each channel throughout the entire period, Table 5.5 displays the relative contributions to the present value of consumption for the aggregate response and the responses of each wealth percentile.

Starting with the multiplier channel, Figure 5.4 shows that this effect is initially positive for all wealth groups. However, over time, it becomes negative, first for the poor, then for the middle class, and finally for the rich. The magnitude of its influence also follows this order. Therefore, the decline in employment (see Figure 5.1) initially and most significantly affects the bottom 50%. The other labor effect is positive for all groups, except the poor after some time. This suggests that the decrease in real wages (see Figure 5.1) also has the most substantial impact on the bottom 50%. The negative impact of labor channels on the less well-off is attributed to the fact that these workers are the most affected by negative productivity shocks. As a result, labor demand and wages decline more rapidly for them than for other groups.

Regarding the wealth effects, the aggregate interest rate channel is positive because the transmission to the top 10% is driven by the income effect. The increase in the interest rate makes wealthy households feel even

Table 5.5: Transmission Channels of Consumption by Wealth Percentiles

	Labor Channels		Wealth Channels	
	Multiplier	Real Income	Interest Rate	Revaluation
Total	19%	31%	21%	28%
Bottom 50%	38%	17%	23%	22%
Middle 40%	32%	42%	5%	20%
Top 10%	19%	26%	24%	31%

Note: The values denote the relative contribution of each channel to the present value of the consumption response. Mathematically, the present values are calculated as $\sum_{t=0}^{\infty} dc_t^j/(1+r)^t$, where dc_t^j is the deviation from steady-state consumption for the wealth group j at time t.

richer, which encourages their consumption. For the revaluation channel, we observe that it is initially negative for all percentiles. However, after the first quarter, it becomes positive for the poor and middle class. This shift occurs because these groups hold a relatively small portion of total wealth, meaning that unexpected capital losses have a limited impact on them. Additionally, the decline in asset values leads to a drop in the stock market, which discourages households from saving significantly. In other words, the negative revaluation makes these households prefer increased consumption.

As previously discussed and reiterated in Table 5.5, aggregate consumption is equally influenced by labor and wealth channels. For the bottom 50%, labor channels account for 55% of the transmission. For the middle 40%, this figure rises to 74%, as the interest rate channel contributes minimally. While the substitution effect clearly dominates for the poor and the income effect clearly dominates for the rich, these effects nearly offset each other for the middle class, which leads to a small contribution from the real interest rate channel. Finally, for the top 10%, the importance of labor channels is 45%. Therefore, as expected, the poor are more influenced by labor dynamics, while the rich are more influenced by financial dynamics.

5.5 Robustness

Appendix E presents two robustness exercises. The first exercise involves calibrating the model using the markup values and the long-term interest rate from SAMBA (Fasolo et al., 2023), which is the Brazilian DSGE model. The second exercise compares the IRFs of our extended model with those obtained from four alternative specifications. Both exercises demonstrate that the dynamics of output (growth, decline, and recovery), consumption (inverted

U-shaped growth), and net exports (improvement, deterioration, and recovery) remain consistent across different variations of the model.

Tables E.1 and E.2 present the differences between the original calibration and the one based on SAMBA. Although the new calibration incorporates the Brazilian long-term interest rate, the wealth-income ratio is more than four times greater than the empirical value. Regarding IFRs, Figure E.1 illustrates that the key difference lies in the investment, which experiences a slight decline but soon begins to increase. Regarding transmission, Table E.3 demonstrates that, owing to the higher steady-state wealth level, the two wealth channels play a more significant role in influencing output, consumption, and net exports. However, this pattern does not apply to investment. The smaller drop in output after the fifth quarter and the negative deviation of the real interest rate after the tenth quarter relatively strengthen the multiplier channel.

For the specification without investment, Table E.1 presents the parameters whose values were modified to match the Brazilian moments in Table E.2. Figure E.2 indicates that the main difference is that consumption initially decreases after the shock; that is, it partially absorbs the response of the missing component of aggregate demand. This underscores the importance of including investment, even if its IRF is not empirically accurate.

Figure E.3 shows that when the UIP condition holds, the appreciation of the real exchange rate is smaller, resulting in a more pronounced reaction of the real interest rate. Consequently, the decline in investment is both deeper and more persistent. In turn, Figure E.4 highlights the importance of the real interest rate in the investment response. With a real interest rate rule, this macroeconomic aggregate follows the output trajectory. Finally, Figure E.5 demonstrates that a lower risk-bearing capacity of the representative financier, $\Gamma = 0.20$ instead of $\Gamma = 0.05$, leads to a greater appreciation of the real exchange rate. As a result, the real interest rate reacts less, and investment becomes positive. Thus, depending on the calibration, our model is able to generate a positive comovement between consumption and investment.

6
Conclusion

In this thesis, we study how foreign demand shocks transmit to the Brazilian economy. To do so, we first incorporate heterogeneous agents into the seminal open economy model of Galí and Monacelli (2005). We assume sticky wages, flexible prices, and a constant real interest rate to derive analytical results. The results show that, while the seminal model is influenced only by the direct channel, our model captures two indirect channels, which generate a positive response in consumption. Consequently, there is a larger expansion in output and a smaller increase in net exports.

Second, we extend our heterogeneous-agent model to incorporate additional transmission channels. Four modifications are introduced: (a) investment as a component of aggregate demand, (b) price rigidity to model imperfect exchange rate pass-through, (c) a standard Taylor rule, and (d) a segmented international financial market. As a result, foreign demand shocks are now transmitted through the direct channel and five indirect channels.

We calibrate the model to match a set of Brazilian aggregate and distributional moments, then simulate a foreign demand shock. Our results show that only 22% of the output variation is explained by the direct channel. Breaking down the transmission, we have that trade channels contribute 55% to the output response, labor channels account for 24%, and wealth channels represent 21%. For consumption, labor and wealth channels are equally important. In the case of investment, the interest rate channel is the primary driver. Regarding net exports, the direct channel acts as the positive force, while the expenditure switching channel serves as the negative force.

Finally, the responses across different wealth percentiles indicate that the poor experience the largest decline in consumption and disposable income, while the rich emerge as the main beneficiaries of the shock. When decomposing the effects for consumption, we find that the poor are more influenced by labor dynamics, while the rich are more influenced by financial dynamics.

7
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A
Basic Heterogeneous-Agent Model

A.1
Domestic Households

Household i's problem can be divided into five steps. The first one determines the consumption vector of home goods for a given level of consumption expenditures on home goods. The second one does the same for the consumption of goods from each foreign country. The third one determines how much to import from each foreign country for a given level of consumption expenditures on imported goods. The fourth one determines the relative consumption of home goods versus imported goods for a given level of consumption expenditures. The final step addresses the consumption/saving decision and is solved by using the endogenous grid method of Carroll (2006), as implemented in Auclert et al. (2021).

In this section, we present the analytical solution for the first four steps of the household i's problem. The first one determines the consumption vector of home goods ($c_{j,i,H,t}$ for each good j) for a given level of consumption expenditures on home goods ($T_{i,H,t}$)

$$\max_{c_{j,i,H,t}} \left(\int_0^1 c_{j,i,H,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ subject to } \int_0^1 P_{j,H,t} c_{j,i,H,t} dj = T_{i,H,t}.$$

Recalling that $c_{i,H,t} = \left(\int_0^1 c_{j,i,H,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, we find the following first-order condition (FOC)

$$c_{i,H,t}^{\frac{1}{\varepsilon}} c_{j,i,H,t}^{-\frac{1}{\varepsilon}} = \lambda_{H,t} P_{j,H,t} \quad \forall j \in [0,1],$$

where $\lambda_{H,t}$ is the Lagrange multiplier. Using the FOC, we have the following relationship between goods j and a

$$c_{a,i,H,t} P_{a,H,t}^{\varepsilon} = c_{j,i,H,t} P_{j,H,t}^{\varepsilon}.$$

Using $P_{H,t} = \left(\int_0^1 P_{j,H,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$, $c_{i,H,t} = \left(\int_0^1 c_{j,i,H,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, and the last equation, we get the demand function for home good j

$$c_{j,i,H,t} = \left(\frac{P_{j,H,t}}{P_{H,t}}\right)^{-\varepsilon} c_{i,H,t}. \tag{A-1}$$

The second step of the household problem determines the consumption vector of country k goods ($c_{j,i,k,t}$ for each good j) for a given level of consumption expenditures on country k goods $(T_{i,k,t})$

$$\max_{c_{j,i,k,t}} \left(\int_0^1 c_{j,i,k,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ subject to } \int_0^1 P_{j,k,t} c_{j,i,k,t} dj = T_{i,k,t}.$$

In a similar way to the previous step, we find the demand function for good j imported from country k

$$c_{j,i,k,t} = \left(\frac{P_{j,k,t}}{P_{k,t}}\right)^{-\varepsilon} c_{i,k,t}. \tag{A-2}$$

The third step of the household problem determines how much to import from each foreign country ($c_{i,k,t}$ for each k) for a given level of consumption expenditures on imported goods $(T_{i,F,t})$.

$$\max_{c_{i,k,t}} \left(\int_0^1 c_{i,k,t}^{\frac{\gamma-1}{\gamma}} dk\right)^{\frac{\gamma}{\gamma-1}} \text{ subject to } \int_0^1 P_{k,t} c_{i,k,t} dk = T_{i,F,t}.$$

Similarly to the previous steps, we get the demand function for goods imported from country k

$$c_{i,k,t} = \left(\frac{P_{k,t}}{P_{F,t}}\right)^{-\gamma} c_{i,F,t}. \tag{A-3}$$

The fourth step of the household problem determines the allocation of consumption between home goods $(c_{i,H,t})$ and imported goods $(c_{i,F,t})$ for a given level of consumption expenditures $(T_{i,t})$

$$\max_{c_{i,H,t}, c_{i,F,t}} \left[(1-\alpha)^{\frac{1}{\eta}} c_{i,H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{i,F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \text{ subject to } P_{H,t} c_{i,H,t} + P_{F,t} c_{i,F,t} = T_{i,t}.$$

The first-order conditions are

$$\{c_{i,H,t}\} \left[(1-\alpha)^{\frac{1}{\eta}} c_{i,H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{i,F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} (1-\alpha)^{\frac{1}{\eta}} c_{i,H,t}^{-\frac{1}{\eta}} = \lambda_{HF} P_{H,t},$$
$$\{c_{i,F,t}\} \left[(1-\alpha)^{\frac{1}{\eta}} c_{i,H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{i,F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} \alpha^{\frac{1}{\eta}} c_{i,F,t}^{-\frac{1}{\eta}} = \lambda_{HF} P_{F,t},$$

where $\lambda_{H,F}$ is the Lagrange multiplier.

Using $P_t = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$, $T_{i,t} = P_t c_{i,t}$, and the result of dividing the first FOC by the second, we obtain the demand functions for home and foreign goods

$$c_{i,H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_{i,t} \quad \text{and} \quad c_{i,F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} c_{i,t}. \tag{A-4}$$

A.2 Trade elasticities

From the law of one price and equation (2-9), we have

$$\frac{P_{H,t}}{P_{F,t}} = \left(\frac{Q^{\eta-1}-\alpha}{1-\alpha}\right)^{\frac{1}{1-\eta}} \iff \frac{P_{F,t}}{P_t} = \left((1-\alpha)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\eta-1} + \alpha\right)^{\frac{1}{\eta-1}}.$$

Using the previous equation and (2-6), we derive the elasticity of imports with respect to the relative price of imports

$$-\left.\frac{\partial \ln C_{F,t}}{\partial \ln P_{F,t}/P_{H,t}}\right|_{\frac{P_{F,t}}{P_{H,t}}=1} = \eta \left.\frac{\partial \ln P_{F,t}/P_t}{\partial \ln P_{F,t}/P_{H,t}}\right|_{\frac{P_{F,t}}{P_{H,t}}=1} = \eta(1-\alpha). \tag{A-5}$$

Similarly, using equation (2-7) and the fact that $P^*_{F,t} = P^*_t = 1$, the elasticity of exports with respect to the relative price faced by foreign consumers is

$$-\left.\frac{\partial \ln C^*_{H,t}}{\partial \ln P^*_{H,t}/P^*_{F,t}}\right|_{\frac{P^*_{H,t}}{P^*_{F,t}}=1} = \gamma \left.\frac{\partial \ln P^*_{H,t}/P^*_t}{\partial \ln P^*_{H,t}/P^*_{F,t}}\right|_{\frac{P^*_{H,t}}{P^*_{F,t}}=1} = \gamma. \tag{A-6}$$

A.3 Mutual fund

Let $\mathcal{A}_t$ be the amount of shares the domestic mutual fund sold to households, i^p_t the nominal return of the mutual fund, s^H_t the mutual fund's holdings of domestic shares, and B^H_t (B^F_t) the mutual fund's holdings of domestic (foreign) bonds.

At the beginning of period t, the value of the liabilities is equal to the value of the assets

$$(1+i^p_t)\mathcal{A}_{t-1} = (d_t + p_t)P_t s^H_{t-1} + (1+i_{t-1})B^H_{t-1} + (1+i^*_{t-1})\mathcal{E}_t B^F_{t-1}. \tag{A-7}$$

At the end of period t, the value of newly purchased assets is

$$\mathcal{A}_t = p_t P_t s^H_t + B^H_t + \mathcal{E}_t B^F_t. \tag{A-8}$$

Given that the ex-post real return to the mutual fund is defined as

$$1 + r_t^p = (1 + i_t^p)\frac{P_{t-1}}{P_t}, \tag{A-9}$$

maximizing the expected real return on the fund's liabilities r_{t+1}^p is equivalent to maximizing the expected nominal return i_{t+1}^p.

As the optimization condition implies that the nominal return of all assets is equal, we have

$$\frac{(d_{t+1} + p_{t+1})P_{t+1}}{p_t P_t} = 1 + i_t = \frac{(1 + i_t^*)\mathcal{E}_{t+1}}{\mathcal{E}_t}. \tag{A-10}$$

Substituting (A-8) and (A-10) into (A-7), we get

$$1 + i_{t+1}^p = 1 + i_t, \text{ for } t \geq 0.$$

Since $P^* = 1$ and the real interest rate is given by $1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$, we can use equations (2-8), (A-9) and (A-10) to obtain

$$1 + r_{t+1}^p = 1 + r_t = \frac{d_{t+1} + p_{t+1}}{p_t} = \frac{(1 + r_t^*)Q_{t+1}}{Q_t}, \text{ for } t \geq 0. \tag{A-11}$$

A.4
Labor Unions

At time t, each union sets its wage $W_{k,t}$ to maximize the utility of its average worker subject to Rotemberg (1982) utility adjustment costs

$$\max_{W_{k,t}} \sum_{\tau \geq 0} \beta^{t+\tau} \Big[u(C_{t+\tau}) - v(N_{t+\tau}) - \zeta_{w,t}(W_{k,t}, W_{k,t-1})\Big]. \tag{A-12}$$

The unions aggregate the efficient units of work from its members into a specific task $N_{k,t} = \int e_{i,t} n_{i,k,t} di$. This task has the following demand from labor intermediaries

$$N_{k,t} = \left(\frac{W_{k,t}}{W_t}\right)^{-\epsilon} N_t. \tag{A-13}$$

The real wage income of the average worker is given by

$$Z_t = \frac{1}{P_t}\int_0^1 W_{k,t}\left(\frac{W_{k,t}}{W_t}\right)^{-\epsilon} N_t dk.$$

On the one hand, the envelope theorem applied to the household problem implies that all income from the union wage change is consumed. Therefore,

$$\frac{\partial C_t}{\partial W_{k,t}} = \frac{\partial Z_t}{\partial W_{k,t}} = \frac{1}{P_t}(1-\epsilon)N_{k,t}. \tag{A-14}$$

On the other hand, since the hours worked by the average worker are $N_{k,t}$, we have

$$\frac{\partial N_{k,t}}{\partial W_{k,t}} = -\epsilon\frac{N_{k,t}}{W_{k,t}}. \tag{A-15}$$

Using equations (A-14) and (A-15), we can derive the first-order condition (FOC) for the labor union problem

$$\begin{aligned} u'(C_t)\frac{1}{P_t}(1-\epsilon)N_{k,t} + v'(N_t)\epsilon\frac{N_{k,t}}{W_{k,t}} - \frac{\mu_w}{\kappa_w(\mu_w-1)}\log(1+\pi_{w,k,t})\frac{1}{1+\pi_{w,k,t}}\frac{1}{W_{k,t-1}} \\ + \beta\frac{\mu_w}{\kappa_w(\mu_w-1)}\log(1+\pi_{w,k,t+1})\frac{1}{\pi_{w,k,t+1}}\frac{W_{k,t+1}}{W_{k,t}^2} = 0 \end{aligned}$$

Since $\mu_w = \epsilon/(\epsilon-1)$, we can rewrite the FOC as

$$\begin{aligned} -\kappa_w\frac{1}{\mu_w}u'(C_t)\frac{1}{P_t}N_{k,t} + \kappa_w v'(N_t)\frac{N_{k,t}}{W_{k,t}} - \log(1+\pi_{w,k,t})\frac{1}{1+\pi_{w,k,t}}\frac{1}{W_{k,t-1}} \\ + \beta\log(1+\pi_{w,k,t+1})\frac{1}{\pi_{w,k,t+1}}\frac{W_{k,t+1}}{W_{k,t}^2} = 0 \end{aligned}$$

In equilibrium, all unions set the same wage $W_{k,t} = W_t$, and so $N_{k,t} = N_t$. Therefore,

$$\log(1+\pi_{w,t}) = \kappa_w\left[N_t v'(N_t) - \frac{1}{\mu_w}\frac{N_t W_t}{P_t}u'(C_t)\right] + \beta\log(1+\pi_{w,t+1}). \tag{A-16}$$

A.5 Current Account Identity

Aggregating the budget constraints of households, equation (2-3), we obtain:

$$\frac{P_{F,t}}{P_t}C_{F,t} + \frac{P_{H,t}}{P_t}C_{H,t} + A_t = (1+r_t^p)A_{t-1} + \frac{W_t}{P_t}N_t. \tag{A-17}$$

Using equations (2-12) and (2-14), we can rewrite the aggregate ex-post return as

$$\begin{aligned} (1+r_t^p)A_{t-1} &= (1+r_{t-1})A_{t-1} + (r_t^p - r_{t-1})A_{t-1} \\ &= d_t + p_t + (1+r_{t-1})\mathsf{nfa}_{t-1} + (r_t^p - r_{t-1})A_{t-1}. \end{aligned} \tag{A-18}$$

Substituting (2-14) and (A-18) into (A-17), we have

$$\frac{P_{F,t}}{P_t}C_{F,t}+\frac{P_{H,t}}{P_t}C_{H,t}+p_t+\mathsf{nfa}_t = d_t+p_t+(1+r_{t-1})\mathsf{nfa}_{t-1}+(r^p_t-r_{t-1})A_{t-1}+\frac{W_t}{P_t}N_t.$$

Incorporating real dividends, equation (2-17), into the last equation, we obtain the currency account identity

$$\mathsf{nfa}_t - \mathsf{nfa}_{t-1} = \frac{P_{H,t}}{P_t}Y_t - C_t + r_{t-1}\mathsf{nfa}_{t-1} + (r^p_t - r_{t-1})A_{t-1}, \tag{A-19}$$

where the value of net exports in units of the consumer price index is defined as

$$NX_t = \frac{P_{H,t}}{P_t}Y_t - C_t = \frac{P_{H,t}}{P_t}C^*_{H,t} - \frac{P_{F,t}}{P_t}C_{F,t}. \tag{A-20}$$

A.6 Steady state

Substituting equations (2-6) and (2-7) into the domestic goods market clearing condition, we obtain

$$\begin{aligned} Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\gamma} C^*_t \\ &= (1-\alpha)(p_H(Q_t))^{-\eta}C_t + \alpha(p^*_H(Q_t))^{-\gamma}C^*_t, \end{aligned} \tag{A-21}$$

where $p_H(Q_t)$ and $p^*_H(Q_t)$ come from equation (2-9).

Substituting the same equations into (2-25), we get

$$\begin{aligned} NX_t &= \alpha\frac{P_{H,t}}{P_t}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\gamma} C^*_t - \alpha\frac{P_{F,t}}{P_t}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \\ &= \alpha p_H(Q_t)(p^*_H(Q_t))^{-\gamma}C^*_t - \alpha(Q_t)^{1-\eta}C_t. \end{aligned} \tag{A-22}$$

A steady state is characterized by a constant level for all aggregates $\{C, C_H, C_F, Y, A, p, D, \mathsf{nfa}\}$ and relative prices $\{Q, P_H/P, P^*_H/P^*, W/P, r, r^*\}$. From the UIP condition, equation (2-11), $r = r^*$. Equation (2-18) determines the long-run real wage and dividends

$$\frac{W}{P} = \frac{1}{\mu_p}p_H(Q) \;\text{ and }\; D = \left(1 - \frac{1}{\mu_p}\right)p_H(Q).$$

From equation (2-12), we have the long-run domestic stock price

$$p = \frac{1}{r}\left(1 - \frac{1}{\mu_p}\right)p_H(Q).$$

Equation (2-14) implies the long-run net foreign asset position

$$\mathsf{nfa} = A - p,$$

where $A = A(r, p_H(Q))$ is a function of the steady-state real interest rate r and real income $p_H(Q)$.

The aggregate consumption function C must satisfy

$$C = C(r, p_H(Q)) \tag{A-23}$$

and also equation (2-24)

$$r \cdot \mathsf{nfa} = -NX = C - p_H(Q)Y. \tag{A-24}$$

From (A-21), we get the long-run output

$$Y = (1-\alpha)(p_H(Q))^{-\eta}C + \alpha(p_H^*(Q))^{-\gamma}C^*. \tag{A-25}$$

Substituting (A-25) into (A-24), we obtain the long-run net export equation. Finally, from (2-22) and $Y = N$, we have the long-run wage inflation rate

$$\log(1+\pi_w) = \frac{1}{1-\beta}\kappa_w \left[Yv'(Y) - \frac{1}{\mu_w}\frac{YW}{P}u'(C) \right]. \tag{A-26}$$

The equations presented above allow us to conclude that a steady state is characterized by a 4-tuple (Y, Q, C, nfa) for output, real exchange rate, consumption, and net foreign position, which satisfies equations (A-23), (A-24), and (A-25). Moreover, a zero-inflation steady state requires that this 4-tuple also satisfies equation (A-26) with $\pi_w = 0$. Since we have four equations and four variables, the zero-inflation steady state is unique.

Without loss of generality, we can normalize $C^* = Q = 1$ so that all relative prices are 1. Then, by solving the system derived from equations (A-24) and (A-25), we have:

$$Y = 1 + \frac{1-\alpha}{\alpha} \cdot r \cdot \mathsf{nfa} \quad \text{and} \quad C = 1 + \frac{1}{\alpha} \cdot r \cdot \mathsf{nfa}.$$

Setting nfa = 0, we obtain $Y = C = 1$. Since $\psi = 1$ and $\mu_w = 1/\mu_p$, equation (A-26) implies that $\pi_w = 0$.

B
Representative-Agent Model

Here, we present the complete-market representative-agent model.

B.1
Representative Domestic Household

The representative domestic agent can buy and sell Arrow-Debreu securities for each aggregate state s^t of the world in order to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} Pr(s^t)\beta^t \{u(C_t) - v(N_t)\} \tag{B-1}$$

subject to the sequence of budget constraints at each t:

$$P_t C_t + \sum_{s_{t+1}} M_{t,t+1}(s_{t+1}) B_{t+1}(s_{t+1}) + \mathcal{E}_t \sum_{s_{t+1}} M^*_{t,t+1}(s_{t+1}) B^*_{t+1}(s_{t+1}) \leq W_t N_t + D_t + B_t + \mathcal{E}_t B^*_t,$$

where $\mathcal{E}_t$ is the nominal exchange rate, $B_{t+1}(s_{t+1})$ ($B^*_{t+1}(s_{t+1})$) denotes holdings of domestic (foreign) Arrow securities that pay one unit of domestic (foreign) currency in state s_{t+1}, $M_{t,t+1}(s_{t+1})$ ($M^*_{t,t+1}(s_{t+1})$) denotes the domestic-currency (foreign-currency) price at time t of these securities, and D_t is the sum of dividends from domestic firms, which are now exclusively owned by the representative agent.

The first-order conditions for the choices of B_{t+1} are

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = M_{t,t+1}, \tag{B-2}$$

and for the choices of B^*_{t+1} are

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = M^*_{t,t+1} \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right). \tag{B-3}$$

Combining these two equations and using the fact that $\frac{1}{1+i_t} = \sum_{s_{t+1}} M_{t,t+1}(s_{t+1})$ and $\frac{1}{1+i^*_t} = \sum_{s_{t+1}} M^*_{t,t+1}(s_{t+1})$, we are able to recover the UIP condition (equation 2-11).

Finally, by summing equation (B-2) across all states and using the Fisher equation, we recover the Euler equation

$$C_t^{-\sigma} = \beta(1 + r_t) C_{t+1}^{-\sigma}. \tag{B-4}$$

B.2
Representative Foreign Household

The representative foreign agent can buy and sell Arrow-Debreu securities for each aggregate state s^t of the world in order to maximize

$$\sum_{t=0}^{\infty}\sum_{s^t} Pr(s^t)(\beta^*)^t \mathcal{B}_t \left\{u(C_t^*) - v(N_t^*)\right\} \tag{B-5}$$

subject to the sequence of budget constraints at each t:

$$\begin{aligned} P_t^* C_t^* &+ \sum_{s_{t+1}} M_{t,t+1}^*(s_{t+1}) B_{t+1}^*(s_{t+1}) \\ &+ \frac{1}{\mathcal{E}_t}\sum_{s_{t+1}} M_{t,t+1}(s_{t+1}) B_{t+1}(s_{t+1}) \le W_t^* N_t^* + D_t^* + \frac{B_t}{\mathcal{E}_t} + B_t^*, \end{aligned}$$

where $\mathcal{B}_t$ represents time-varying patience for the representative foreign household. By assumption, $\mathcal{B}_t \in (0, \overline{\mathcal{B}})$ has initial value $\mathcal{B}_{-1} = 1$ and reverts to 1 in the long run, that is, $\lim_{t\to\infty} \mathcal{B}_t = 1$.

Combining the first-order conditions for C_t^* and N_t^*, we obtain:

$$\frac{v'(N_t^*)}{u'(C_t^*)} = \frac{W_t^*}{P_t^*}. \tag{B-6}$$

The first-order conditions for the choices of B_{t+1}^* are:

$$\beta^* \frac{\mathcal{B}_{t+1}}{\mathcal{B}_t}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right) = M_{t,t+1}^*, \tag{B-7}$$

and for the choices of B_{t+1} are:

$$\beta^* \frac{\mathcal{B}_{t+1}}{\mathcal{B}_t}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right) = M_{t,t+1}\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right). \tag{B-8}$$

Applying the same steps as those used for the representative domestic household, we recover the following Euler equation

$$\mathcal{B}_t (C_t^*)^{-\sigma} = \beta^*(1 + r_t^*)\mathcal{B}_{t+1}(C_{t+1}^*)^{-\sigma}. \tag{B-9}$$

B.3
Foreign Demand Shock

All foreign countries produce their own goods under constant returns to scale, with flexible prices and wages

$$Y_t^* = Z_t^* N_t^* \quad \text{and} \quad P_t^* = \mu \frac{W_t^*}{Z_t^*}.$$

Given that the home country is small, the market clearing for the composite foreign good is

$$C_t^* = Y_t^*.$$

Our foreign demand shock is induced by a foreign productivity shock. To prevent fluctuations in the foreign interest rate and to reconcile the computational modeling of external consumption as an $AR(1)$ process with the foreign representative household problem, the foreign productivity shock must be accompanied by a preference shock.

Using the definitions of N_t^* and W_t^*/P_t^* in equation (B-6), we obtain

$$(C_t^*)^{\sigma+\varphi} = \frac{1}{\mu}(Z_t^*)^{1+\varphi}. \tag{B-10}$$

Substituting (B-10) into (B-9):

$$1 + r_t^* = \frac{1}{\beta^*}\frac{\mathcal{B}_t}{\mathcal{B}_{t+1}}\left(\frac{Z_{t+1}^*}{Z_t^*}\right)^{\frac{\sigma(1+\varphi)}{\sigma+\varphi}}. \tag{B-11}$$

To maintain the foreign interest rate at its steady-state value, $r_t^* = 1/\beta^* - 1$, it is necessary that:

$$\frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} = \left(\frac{Z_t^*}{Z_{t+1}^*}\right)^{\frac{\sigma(1+\varphi)}{\sigma+\varphi}}.$$

Given that $\mathcal{B}_{-1} = 1$ and $\lim_{t\to\infty}\mathcal{B}_t = 1$, we get the following sequence of preference shocks

$$\mathcal{B}_t = \prod_{s\geq t}\left(Z_s^*\right)^{\frac{\sigma(1+\varphi)}{\sigma+\varphi}}.$$

C
Proofs of Propositions

Proof of Proposition 3.1: We divide the proof into three steps. First, we show the impulse response of consumption $d\boldsymbol{C}$. Second, we show the impulse response of output $d\boldsymbol{Y}$. Third, we show the impulse response of trade balance $d\boldsymbol{NX}$.

(i) Consumption:

Combining equations (B-2) and (B-8), we get

$$\frac{\mathcal{B}_t}{\mathcal{B}_{t+1}}\left(\frac{Q_{t+1}}{Q_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}.$$

Substituting (B-9) into the previous expression

$$\beta^*(1+r^*_t)\left(\frac{Q_{t+1}}{Q_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = 1.$$

Given that $(1+r^*_t) = (1+r^*_{ss}) = 1/\beta^*$ and $Q_t = 1$ for all t, then $C_{t+1} = C_t = C$ for all t.

(ii) Output:

Since $d\boldsymbol{Q} = \boldsymbol{0}$, equation (2-9) implies

$$d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) = -\frac{\alpha}{1-\alpha}d\boldsymbol{Q} = \boldsymbol{0},\, d\left(\frac{\boldsymbol{P_F}}{\boldsymbol{P}}\right) = d\boldsymbol{Q} = \boldsymbol{0},\text{ and } d\left(\frac{\boldsymbol{P^*_H}}{\boldsymbol{P^*}}\right) = -\frac{1}{1-\alpha}d\boldsymbol{Q} = \boldsymbol{0}.$$

We differentiate equation (3-1) around the steady state, $P_H/P = P^*_H/P^* = 1$ and $C = C^* = Y = 1$, and use $d\boldsymbol{Q} = d\boldsymbol{C} = \boldsymbol{0}$ to find

$$\begin{aligned} d\boldsymbol{Y} &= (1-\alpha)\left(-\eta d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) + d\boldsymbol{C}\right) + \alpha\left(-\gamma d\left(\frac{\boldsymbol{P^*_H}}{\boldsymbol{P^*}}\right) + d\boldsymbol{C^*}\right) \\ &= \alpha d\boldsymbol{C^*}. \end{aligned}$$

(iii) Net exports:

Following the same steps as for the output, the differentiation of equation (2-25) around the steady state implies

$$\begin{aligned} d\boldsymbol{NX} &= \alpha\left(d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) - \gamma d\left(\frac{\boldsymbol{P^*_H}}{\boldsymbol{P^*}}\right) + d\boldsymbol{C^*}\right) - \alpha\left((1-\eta)d\left(\frac{\boldsymbol{P_F}}{\boldsymbol{P}}\right) + d\boldsymbol{C}\right) \\ &= \alpha d\boldsymbol{C^*}. \end{aligned}$$

■

Proof of Proposition 3.2: We divide the proof into three steps. First, we show the impulse response of consumption $d\boldsymbol{C}$. Second, we show the impulse response of output $d\boldsymbol{Y}$. Third, we show the impulse response of trade balance $d\boldsymbol{NX}$.

(i) Consumption:

Using equations (2-6) and (3-2), the aggregate consumption function can be expressed as

$$C_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(r_0^p, \{r_s\}_{s=0}^{\infty}, \left\{\frac{1}{\mu}\frac{P_{H,s}}{P_s}Y_s\right\}_{s=0}^{\infty}\right) + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(r_0^p, \{r_s\}_{s=0}^{\infty}, \left\{\frac{1}{\mu}\frac{P_{H,s}}{P_s}Y_s\right\}_{s=0}^{\infty}\right).$$

We differentiate the aggregate consumption function around the steady state, $P_H/P = P_H^*/P^* = 1$ and $C = C^* = Y = 1$, to find

$$\begin{aligned} d\boldsymbol{C} =& (1-\alpha)\left[-\eta d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) + \boldsymbol{M_v} dr_0^p + \boldsymbol{M_r} d\boldsymbol{r} + \boldsymbol{M_Y}\frac{1}{\mu_p}\left(d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) + d\boldsymbol{Y}\right)\right] \\ &+ \alpha\left[-\eta d\left(\frac{\boldsymbol{P_F}}{\boldsymbol{P}}\right) + \boldsymbol{M_v} dr_0^p + \boldsymbol{M_r} d\boldsymbol{r} + \boldsymbol{M_Y}\frac{1}{\mu_p}\left(d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) + d\boldsymbol{Y}\right)\right], \end{aligned}$$

where $\boldsymbol{M_v}$ is a vector representing the increased spending from revaluation r_0^p, $\boldsymbol{M_r}$ is the Jacobian of aggregate consumption with respect to the path of real interest rate r_s, and $\boldsymbol{M_Y}$ is the Jacobian of aggregate consumption with respect to the path of output Y_s.

To simply the notation, we define the Jacobian of aggregate consumption with respect to the real labor income Z_s as

$$\boldsymbol{M} = \frac{1}{\mu_p}\boldsymbol{M_Y}.$$

Since $d\boldsymbol{Q} = \boldsymbol{0}$, equation (2-9) implies

$$d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) = -\frac{\alpha}{1-\alpha} d\boldsymbol{Q} = \boldsymbol{0} \text{ and } d\left(\frac{\boldsymbol{P_F}}{\boldsymbol{P}}\right) = d\boldsymbol{Q} = \boldsymbol{0}.$$

Therefore, by utilizing $\boldsymbol{M}$, $d\boldsymbol{Q} = \boldsymbol{0}$ and $d\boldsymbol{r} = \boldsymbol{0}$, we obtain

$$d\boldsymbol{C} = \boldsymbol{M} d\boldsymbol{Y} + \boldsymbol{M_v} dr_0^p. \tag{C-1}$$

(ii) Output:

Since $d\boldsymbol{Q} = \mathbf{0}$, equation (2-9) implies

$$d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) = -\frac{\alpha}{1-\alpha}d\boldsymbol{Q} = \mathbf{0} \text{ and } d\left(\frac{\boldsymbol{P_H^*}}{\boldsymbol{P^*}}\right) = -\frac{1}{1-\alpha}d\boldsymbol{Q} = \mathbf{0}.$$

We differentiate equation (3-1) around the steady state, $P_H/P = P_H^*/P^* = 1$ and $C = C^* = Y = 1$, and use equation (C-1) to find

$$\begin{aligned} d\boldsymbol{Y} &= (1-\alpha)\left(-\eta d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) + d\boldsymbol{C}\right) + \alpha\left(-\gamma d\left(\frac{\boldsymbol{P_H^*}}{\boldsymbol{P^*}}\right) + d\boldsymbol{C^*}\right) \\ &= (1-\alpha)\boldsymbol{M}d\boldsymbol{Y} + (1-\alpha)\boldsymbol{M_v}dr_0^p + \alpha d\boldsymbol{C^*}. \end{aligned} \tag{C-2}$$

(iii) Net exports:

Following the same steps as for the output, the differentiation of equation (2-25) around the steady state implies

$$\begin{aligned} d\boldsymbol{NX} &= \alpha\left(d\left(\frac{\boldsymbol{P_H}}{\boldsymbol{P}}\right) - \gamma d\left(\frac{\boldsymbol{P_H^*}}{\boldsymbol{P^*}}\right) + d\boldsymbol{C^*}\right) - \alpha\left((1-\eta)d\left(\frac{\boldsymbol{P_F}}{\boldsymbol{P}}\right) + d\boldsymbol{C}\right) \\ &= \alpha d\boldsymbol{C^*} - \alpha\boldsymbol{M}d\boldsymbol{Y} - \alpha\boldsymbol{M_v}dr_0^p. \end{aligned} \tag{C-3}$$

■

D
Extended Heterogeneous-Agent Model

D.1
Production Sector

In this section, we present the solutions to the domestic retailers' problem and the capital producer's problem. The problems of exporters and importers are solved in a manner similar to that of domestic retailers.

D.1.1
Domestic Retailers

Using $\mu_H = \varepsilon/(\varepsilon - 1)$, (4-1), (4-2), and (4-3), we can rewrite the firm j's problem as

$$J_t^{DR}(P_{j,H,t-1}) = \max_{P_{j,H,t},\, k_{j,t},\, n_{j,t}} \left\{ \frac{P_{j,H,t}}{P_t} f(k_{j,t}, n_{j,t}) - \frac{W_t}{P_t} n_{j,t} - \frac{R_t^k}{P_t} k_{j,t} \right.$$
$$\left. - \frac{\mu_H}{\mu_H - 1} \frac{1}{2\kappa_H} \left[\log\left(\frac{P_{j,H,t}}{P_{j,H,t-1}} \right) \right]^2 \frac{P_{H,t}}{P_t} Y_t + \frac{1}{1+r_t} J_{t+1}^{DR}(P_{j,H,t}) \right\}$$
$$\text{s.t.} \left(\frac{f(k_{j,t}, n_{j,t})}{Y_t} \right)^{\frac{1-\mu_H}{\mu_H}} \frac{P_{H,t}}{P_t} Y_t = \frac{P_{j,H,t}}{P_t} Y_t$$

Let $\lambda_{j,t}$ denote the Lagrange multiplier on the production constraint. The first-order condition (FOC) for labor $n_{j,t}$ is

$$\left[\frac{P_{j,H,t}}{P_t} + \lambda_{j,t} \left(\frac{1-\mu_H}{\mu_H} \right) \left(\frac{f(k_{j,t}, n_{j,t})}{Y_t} \right)^{\frac{1-2\mu_H}{\mu_H}} \frac{P_{H,t}}{P_t} \right] f_n(k_{j,t}, n_{j,t}) = \frac{W_t}{P_t},$$

In equilibrium, all firms select the same price $P_{j,H,t} = P_{H,t}$. Consequently, $f(k_{j,t}, n_t) = F(K_{t-1}, N_t) = Y_t$, and the FOC can be rewritten as

$$\underbrace{\left[1 + \lambda_t \left(\frac{1-\mu_H}{\mu_H} \right) \right]}_{mc_{H,t}} \frac{P_{H,t}}{P_t} F_n(k_{t-1}, N_t) = \frac{W_t}{P_t},$$

where $mc_{H,t}$ denotes the marginal cost. From this FOC, we find the following relationship between marginal cost and the Lagrange multiplier

$$mc_{H,t} = 1 + \lambda_t \left(\frac{1-\mu_H}{\mu_H} \right), \tag{D-1}$$

and we also obtain the labor demand of domestic retailers

$$\frac{W_t}{P_t} = mc_{H,t}(1-a)\frac{P_{H,t}}{P_t}\frac{Y_t}{N_t}. \tag{D-2}$$

The FOC for capital $k_{j,t}$ is

$$\left[\frac{P_{j,H,t}}{P_t} + \lambda_{j,t}\left(\frac{1-\mu_H}{\mu_H}\right)\left(\frac{f(k_{j,t},n_{j,t})}{Y_t}\right)^{\frac{1-2\mu_H}{\mu_H}}\frac{P_{H,t}}{P_t}\right] f_k(k_{j,t},n_{j,t}) = \frac{R_t^k}{P_t},$$

Following the same logic applied to the labor FOC, we can derive the capital demand of domestic retailers as

$$\frac{R_t^k}{P_t} = mc_{H,t}a\frac{P_{H,t}}{P_t}\frac{Y_t}{K_{t-1}}. \tag{D-3}$$

The FOC for price $P_{j,H,t}$ is

$$\begin{aligned}
&\frac{1}{P_t}f(k_{j,t},n_{j,t}) - \frac{\mu_H}{\kappa_H(\mu_H-1)}\log\left(1+\pi_{j,H,t}\right)\frac{1}{1+\pi_{j,H,t}}\frac{1}{P_{j,H,t-1}}\frac{P_{H,t}}{P_t}Y_t \\
&\quad + \frac{1}{1+r_t}\frac{\mu_H}{\kappa_H(\mu_H-1)}\log\left(1+\pi_{j,H,t+1}\right)\frac{1}{1+\pi_{j,H,t+1}}\frac{P_{j,H,t+1}}{P_{j,H,t}^2}\frac{P_{H,t+1}}{P_{t+1}}Y_{t+1} - \lambda_{j,t}\frac{1}{P_t}Y_t = 0.
\end{aligned}$$

The previous equation shows that all firms indeed choose the same price $P_{j,H,t} = P_{H,t}$. Thus, multiplying the FOC by P_t, we can rewrite it as

$$(1-\lambda_t)\frac{\kappa_H(\mu_H-1)}{\mu_H}Y_t - \log(1+\pi_{H,t})Y_t + \frac{1}{1+r_t}\log(1+\pi_{H,t+1})Y_{t+1} = 0.$$

Incorporating (D-1) into the FOC, we derive the Phillips curve for home goods in domestic currency

$$\log(1+\pi_{H,t}) = \kappa_H\left(mc_{H,t} - \frac{1}{\mu_H}\right) + \frac{1}{1+r_t}\frac{Y_{t+1}}{Y_t}\log(1+\pi_{H,t+1}), \tag{D-4}$$

where $mc_{H,t} = \dfrac{1}{1-a}\dfrac{W_t}{P_{H,t}}\dfrac{N_t}{Y_t}$ is the marginal cost, which is derived from (D-2).

D.1.2 Capital Producer

The capital producer's problem can be written as

$$J_t^K(K_{t-1},I_{t-1}) = \max_{K_t,\, I_t}\left\{\frac{R_t^k}{P_t}K_{t-1} - I_t + \frac{1}{1+r_t}J_{t+1}^K\left((1-\delta)K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t, I_t\right)\right\}$$

Let's define Tobin's q as the marginal value of capital at the end of period t

$$q_t = \frac{1}{1+r_t}\frac{\partial J^K_{t+1}(K_t, I_t)}{\partial K_t}.$$

The envelope condition for K_{t-1} is

$$\frac{\partial J^K_t(K_{t-1}, I_{t-1})}{\partial K_{t-1}} = \frac{R^K_t}{P_t} + \frac{1}{1+r_t}\frac{\partial J^K_{t+1}(K_t, I_t)}{\partial K_t}(1-\delta).$$

Using the definition of Tobin's q, we have

$$q_t(1+r_t) = \frac{R^K_{t+1}}{P_{t+1}} + q_{t+1}(1-\delta). \tag{D-5}$$

The envelope condition for I_{t-1} is

$$\frac{\partial J^K_t(K_{t-1}, I_{t-1})}{\partial I_{t-1}} = \frac{1}{1+r_t}\frac{\partial J^K_{t+1}(K_t, I_t)}{\partial K_t} S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2.$$

The FOC with respect to I_t is

$$-1+\frac{1}{1+r_t}\frac{\partial J^K_{t+1}(K_t, I_t)}{\partial K_t}\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right]+\frac{1}{1+r_t}\frac{\partial J^K_{t+1}(K_t, I_t)}{\partial I_t} = 0$$

Substituting the definition of Tobin's q and the envelpe condition for K_{t-1} into the FOC, we obtain

$$1 = q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \frac{1}{1+r_t}q_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right). \tag{D-6}$$

D.2 Financial Sector

The problem of the representative foreign professional intermediary is

$$\max_{B^I_t}\ V_t = \mathbb{E}_t\left[(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) - (1+i^*_t)\right]B^I_t \quad \text{s.t.} \quad V_t \geq \Gamma\left(B^I_t\right)^2$$

The constraint always binds because the objective function is linear in B^I_t and the right-hand side of the constraint is convex in B^I_t. Thus, the solution is

$$B^I_t = \frac{1}{\Gamma}\left[(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) - (1+i^*_t)\right]. \tag{D-7}$$

D.3 Steady State

We focus on the steady state with no inflation, $\pi_w = \pi_H = \pi_H^* = \pi_F = 0$, and no external financial positions, $nfa = 0$. These conditions imply that $A = p$ by the market-clearing condition for the asset market, and $i = r_{ss}$ by the Taylor rule. The foreign demand X^*, output Y, employment N, and real exchange rate Q are normalized to 1. Additionally, the prices P_H, P_F, P_H^*, P are each set to 1.

In this steady state, the domestic demand for foreign goods and the external demand for home goods are, respectively,

$$M_F = C_F + I_F = \alpha(C + I) \quad \text{and} \quad X_H^* = \alpha.$$

Since $S = 0$ in the steady state, the law of motion of capital implies that $I = \delta K$. Furthermore, the investment conditions (4-12) and (4-13) imply $q = 1$ and a capital rate of

$$q = \frac{1}{1+r}\left[\frac{R^k}{P} + q(1-\delta)\right] \iff R^k = r + \delta.$$

The Phillips curve for home goods, equation (4-4), implies $mc_H = 1/\mu_H$. Consequently, the labor demand, equation (D-2), leads to the following steady-state wage

$$mc_H = \frac{1}{1-a}\frac{N}{Y}\frac{W}{P_H} \iff W = \frac{1-a}{\mu_H}.$$

The capital demand, equation (D-3), results in the steady-state capital stock

$$\frac{R^k}{P} = \frac{a}{\mu_H}\frac{P_H}{P}\frac{Y}{K} \iff K = \frac{1}{\mu_H}\frac{a}{(r+\delta)}.$$

From the production function of domestic retailers, we determine the TFP level that is consistent with our steady state

$$Y = \Theta K^a N^{1-a} \iff \Theta = \frac{1}{K^a} \iff \Theta = \left(\frac{\mu_H(r+\delta)}{a}\right)^a. \tag{D-8}$$

The steady-state real dividend of the capital producer, equation (4-14), is

$$d^K = \frac{R^k}{P}K - I \iff d^K = \frac{r}{\mu_H}\frac{a}{(\delta + r)}.$$

Since $\zeta_H = 0$, the steady-state dividend of the domestic retailers, equation (4-5), is

$$d^{DR} = \frac{P_H}{P}Y - \frac{W}{P}N - \frac{R^k}{P}K \iff d^{DR} = 1 - \frac{1}{\mu_H}.$$

We define the export and import price markups such that $\pi_H^* = \pi_F = 0$ and the law of one price holds in the steady state. Consequently, from equations (4-9) and (4-11), we obtain $\mu_H^* = \mu_F = 1$. Furthermore, given the law of one price and $\zeta_H^* = 0$, the steady-state dividend of the exporters is $d^E = 0$.

Given that $nfa = 0$, equation (4-22) implies that the UIP condition is satisfied. Consequently, $B^I = 0$ and $r_{ss} = r = r^*$. Moreover, we obtain the steady-state price of the firm equity from equation (4-17)

$$p = \frac{d}{r}, \quad \text{where} \quad d = d^{DR} + d^K + d^E = \left(1 - \frac{1}{\mu_H}\right) + \frac{r}{\mu_H}\frac{a}{(\delta + r)}.$$

We define the labor disutility parameter ψ such that the wage Phillips curve (2-22) implies $\pi_w = 0$

$$\psi N^{1+\varphi} = \frac{1}{\mu_w}\frac{NW}{P}C^{-\sigma} \iff \psi = \frac{1}{\mu_w}WC^{-\sigma}. \tag{D-9}$$

Finally, since $Y = 1$, $I = \delta K$, $X_H^* = \alpha$, and $\zeta_H = \zeta_H^* = 0$, the market-clearing condition for home goods determines the steady-state consumption

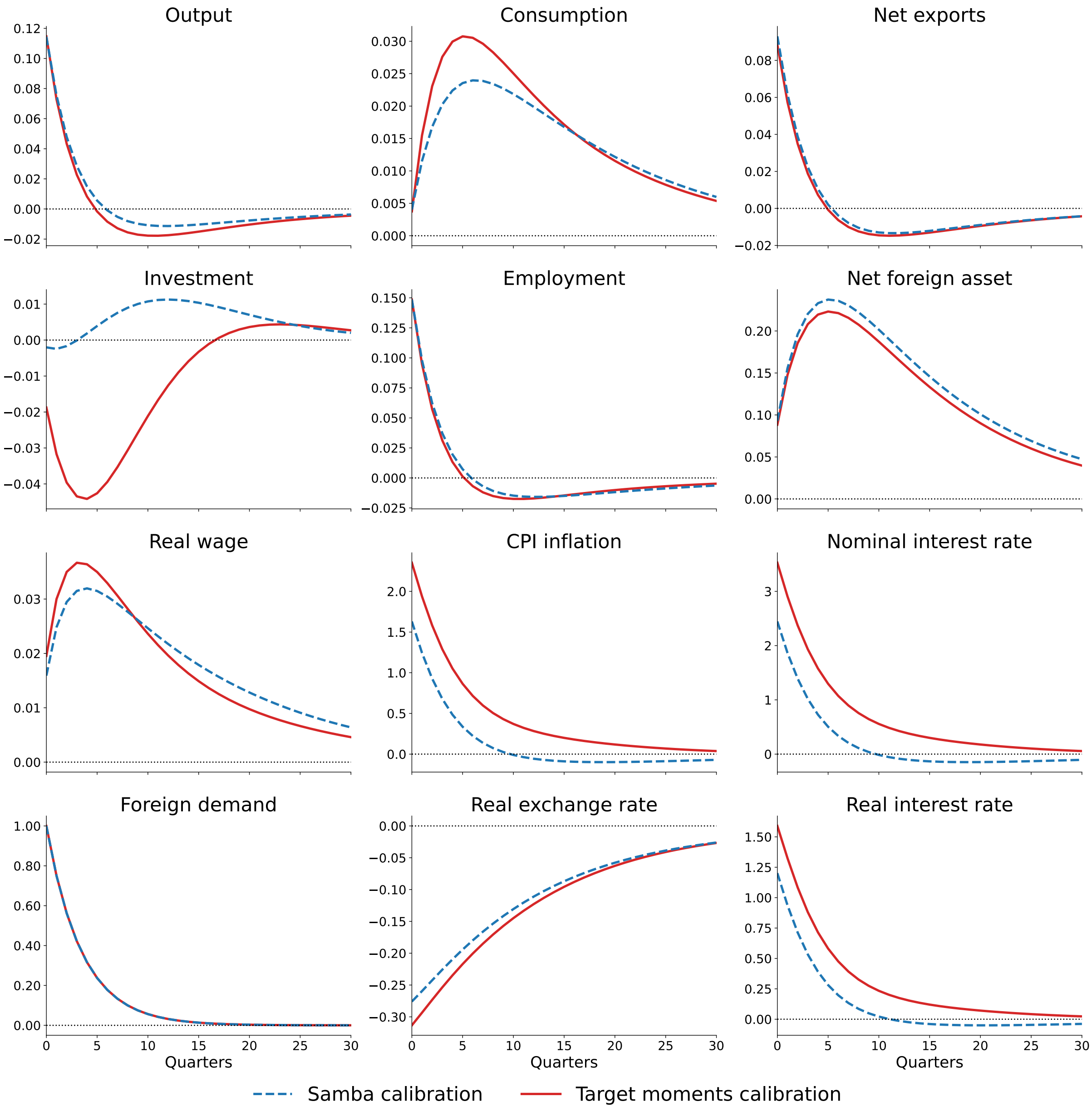
$$Y = C_H + I_H + X_H^* \iff Y = (1-\alpha)(C+I) + \alpha \iff C = 1 - \frac{\delta a}{\mu_H(r+\delta)}.$$

Since $nfa = 0$, it follows that $NX = 0$ by equation (4-24). Thus, equation (4-25) and $X_H^* = \alpha$ imply $M_F = \alpha$.

E
Robustness Exercises

E.1
Alternative Calibration

Figure E.1: IRFs for the Target Moments and Samba Calibrations



Note: Most of the variables are expressed in percentage deviation from steady state. There are two groups of exceptions: (i) net exports, and the net foreign asset position; (ii) CPI inflation, the nominal interest rate, and the real interest rate. The first group is in percentage of steady-state output, as their steady-state values are zero. The second group is in basis points.

Table E.1: Comparing the Calibrations

Parameter		Target Moments	Samba	Without Investment
Households				
Foreign discount factor	β^*	0.98	0.99	0.99
Average discount factor	β	0.85	0.93	0.85
Cross-sectional std of earnings	σ_e	1.65	1.90	1.67
Persistence of earnings	ρ_e	0.985	0.987	0.984
Firms				
Depreciation	δ	0.073	0.075	
Capital share	a	0.228	0.230	
Wage markup	μ_ω	1.01	1.1	$1/\mu_H$
Price markup	μ_H	1.01	1.1	1.038

Note: This table illustrates the differences between our original calibration – target moments calibration – and the Samba calibration. Furthermore, it presents the parameters with different values in the specification without investment.

Table E.2: Target Moments in Each Calibration

Variable		Data	Target Moments	Samba	Without Investment
Macroeconomic aggregates					
Output, normalized	Y	1.00	1.00	1.00	1.00
Consumption	C	0.82	0.82	0.82	1.00
Investment	I	0.18	0.18	0.18	
Exports	X	0.13	0.13	0.13	0.13
Imports	M	0.13	0.13	0.13	0.13
Capital	K	2.47	2.43	2.46	
Wealth-income ratio	A/Z	3.81	3.82	16.50	3.80
Long-term interest rate	r	0.01	0.02	0.01	0.01
Distributional variables					
Bottom 50% wealth share		0.01	0.01	0.01	0.01
Middle 40% wealth share		0.23	0.23	0.23	0.23
Top 10% wealth share		0.76	0.76	0.76	0.76
Bottom 50% income share		0.10	0.06	0.06	0.06
Middle 40% income share		0.31	0.36	0.35	0.35
Top 10% income share		0.59	0.58	0.59	0.59

Note: This table shows the values of the target moments in each calibration.

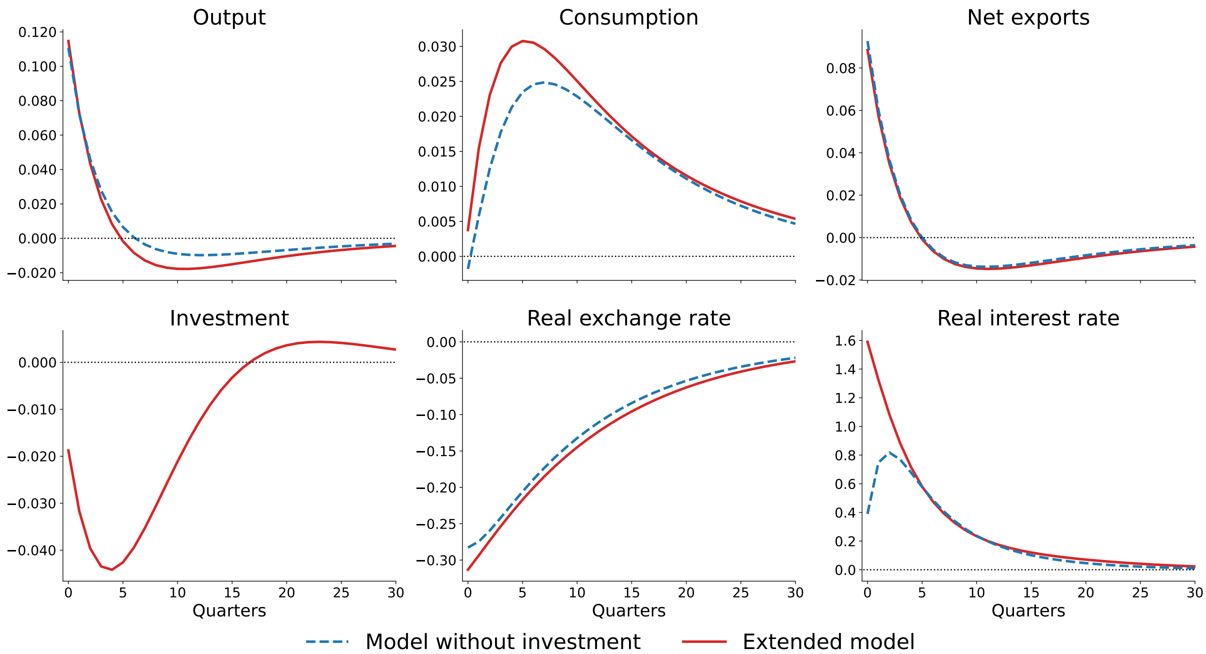
Table E.3: Transmission Channels in Each Calibration

	Trade Channels		Labor Channels		Wealth Channels	
	Direct	Expenditure Switching	Multiplier	Real Income	Interest Rate	Revaluation
	(A) Target Moments Calibration					
Output	22%	33%	10%	14%	8%	13%
Consumption	0%	0%	19%	31%	21%	28%
Investment	0%	0%	28%	0%	72%	0%
Net Exports	45%	41%	3%	4%	3%	4%
	(B) Samba Calibration					
Output	20%	30%	7%	14%	13%	16%
Consumption	0%	0%	11%	28%	28%	33%
Investment	0%	0%	41%	0%	59%	0%
Net Exports	44%	40%	2%	5%	4%	5%

Note: The values denote the relative contribution of each channel to the present value of the response of aggregate variables. Mathematically, the present values are calculated as $\sum_{t=0}^{\infty} dV_t/(1+r)^t$, where $dV_t \in \{dY_t, dC_t, dI_t, dNX_t\}$ is the deviation from steady state of variable V at time t.

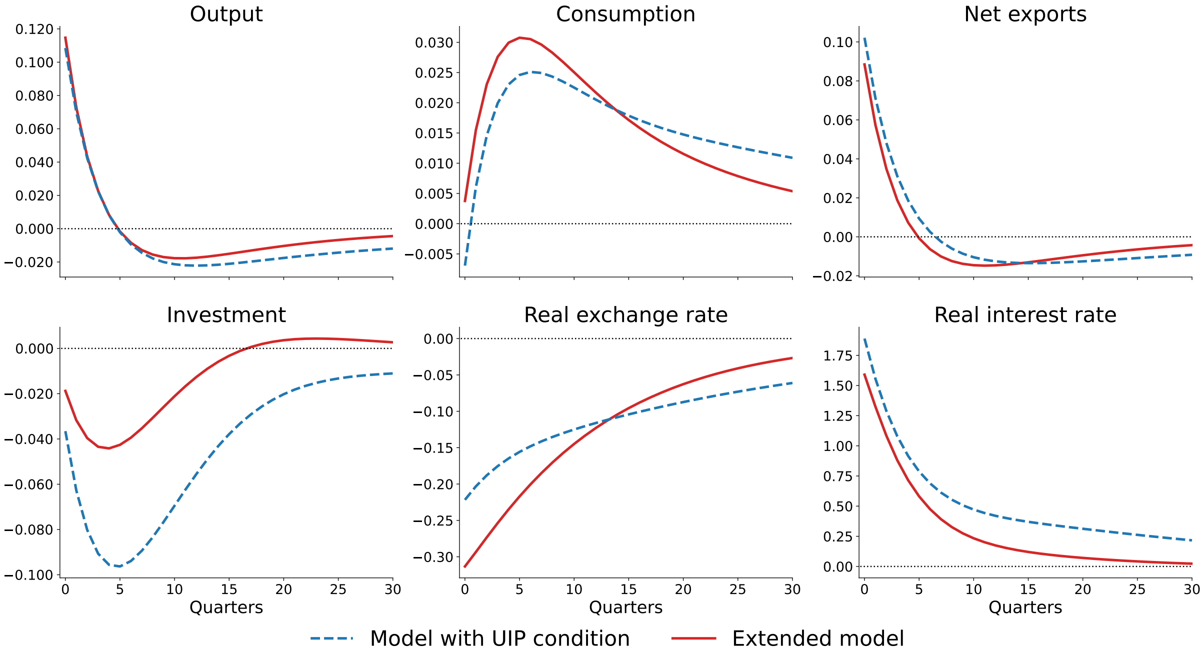
E.2
Alternative Specifications of the Model

Figure E.2: IRFs for the Specification without Investment



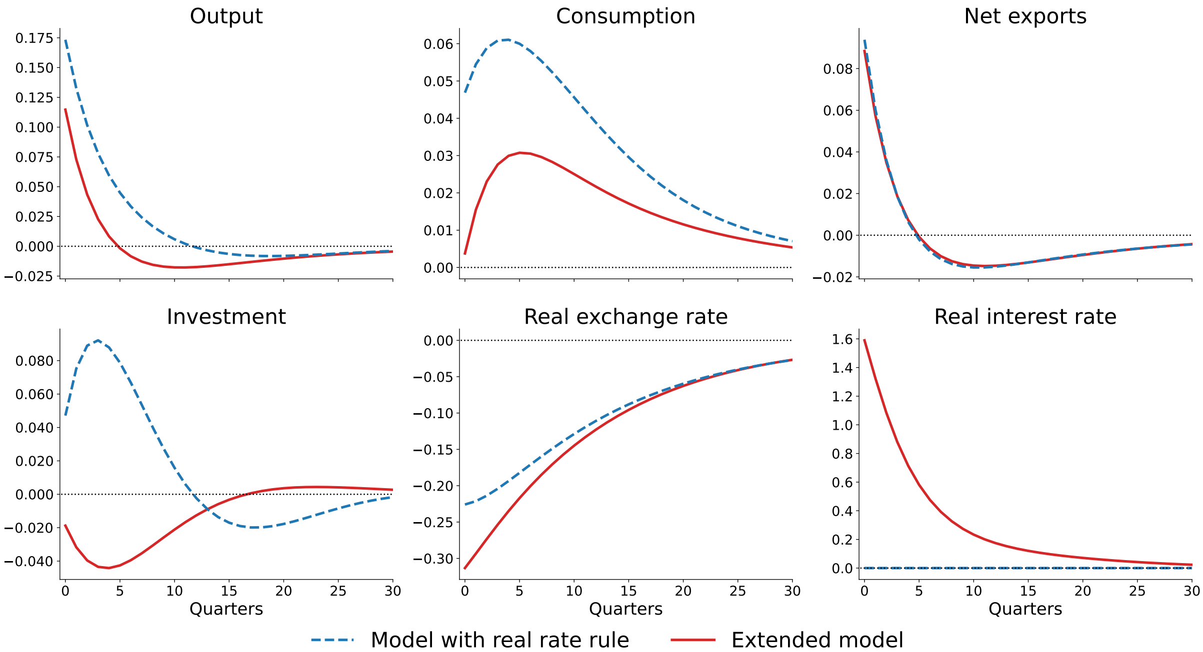
Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output, and the real interest rate, which is in basis points.

Figure E.3: IRFs for the Specification in which the UIP Condition Holds



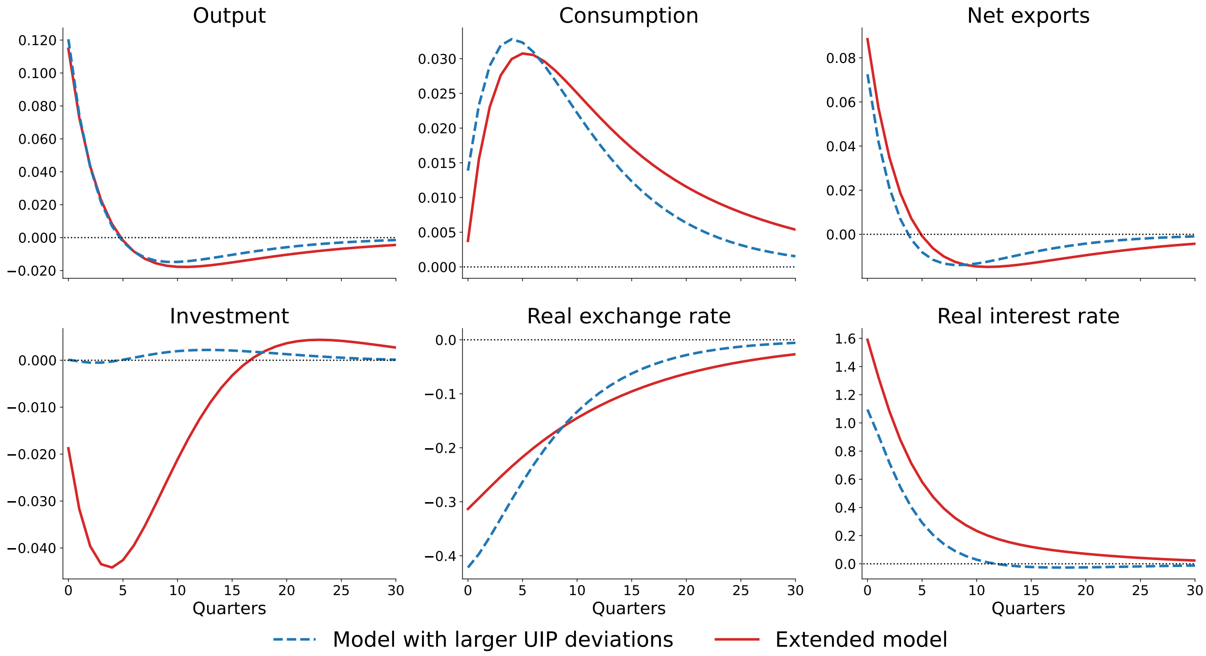
Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output, and the real interest rate, which is in basis points.

Figure E.4: IRFs for the Specification with Real Rate Rule



Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output, and the real interest rate, which is in basis points.

Figure E.5: IRFs for the Specification with Larger UIP Deviations



Note: All variables are expressed in percentage deviation from steady state, except net exports, which is in percentage of steady-state output, and the real interest rate, which is in basis points.