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Essays on Negative Interest Rates and GDP Forecasting

Tese de Doutorado

Thesis presented to the Programa de Pós-graduação em Economia of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Economia.

Advisor : Prof. Tiago Couto Berriel
Co-advisor: Prof. Marcelo Cunha Medeiros

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To my family.

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Abstract

Guardado, Fernanda; Berriel, Tiago Couto (Advisor); Medeiros, Marcelo Cunha (Co-Advisor). **Essays on Negative Interest Rates and GDP Forecasting**. Rio de Janeiro, 2019. 95p. Tese de doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

The thesis is composed of three essays. The first designs a DSGE model based on Gertler and Karadi (2011) to study the effects of the adoption of negative interest rate policies along with liquidity intervention, in a scenario where the ZLB is transferred to private banks instead of central banks. We show that, during a recession, if banks do not pass along negative rates to depositors in an environment of heavy liquidity injection by the Central Bank, the main negative economic effects of the original ZLB are maintained and the recovery is slower. The second essay uses the same model in a simpler setting to study how the adoption of central bank digital currencies (CBDCs) might reestablish the traditional monetary policy transmission under negative interest rates, and analyses the responses of the economy under such a regime to monetary policy shocks. We show that while the adoption of a CBDC might improve the monetary policy toolkit, the wealth effects involved with changes exclusively in its interest rates make it a less reliable counter-cyclical tool. The third essay tries different models for the forecast of medium-term output growth. We use new methods such as adaLASSO and Random Forest, along with a very large data set of regressors, in order to improve accuracy over traditional model long term forecasting such as autoregressions and DSGE models, which have a very good track record. We show that Random Forest is able to better predict output growth over the two year horizon, but has mixed results in forecasting trend GDP growth and the output gap.

Keywords

Negative interest rates; Central Banks; Excess reserves; Central Bank Digital currencies; Monetary policy; Forecasting; Random Forests; adaLASSO;

Resumo

Guardado, Fernanda; Berriel, Tiago Couto; Medeiros, Marcelo Cunha. **Ensaios sobre Taxas de Juros Negativas e Projeção do PIB**. Rio de Janeiro, 2019. 95p. Tese de Doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Esta tese é composta por três artigos. O primeiro monta um modelo DSGE baseado em Gertler e Karadi (2011), para estudar os efeitos da adoção de políticas de taxas de juros negativas concomitantes à intervenções de liquidez por parte do Banco Central, em um cenário em que o zero lower bound (ZLB) é transferido dos bancos centrais para os bancos privados. Mostramos que, durante uma recessão, se os bancos privados não repassam as taxas negativas para seus depositantes em um ambiente de elevadas injeções de liquidez por parte do banco central, as consequências negativas do ZLB original se mantêm e a recuperação é mais lenta. O segundo artigo usa uma versão mais simplificada do mesmo modelo para estudar a adoção de moedas digitais por parte do banco central, que poderia reestabelecer a transmissão de política monetária sob taxas de juros negativas, e analisa as respostas da economia a choques de política monetária sob este regime. Mostramos que, apesar de se mostrar um ferramenta adicional interessante para o banco central, o efeito riqueza envolvido com mudanças exclusivamente da taxa de juros da moeda digital tornam-a um instrumento contra-cíclico menos confiável. O terceiro artigo testa diferentes modelos de projeção para o crescimento do PIB americano de médio prazo. Utilizamos novos métodos, como adaLASSO e Random Forest, em conjunto com um conjunto grande de regressores, para elevar a acurácia sobre modelos tradicionais de projeção, como auto-regressões e modelos DSGE. O artigo aponta que Random Forest é capaz de projeções superiores ao longo de um horizonte de dois anos, mas não tem performance consistemente superior para projeção de crescimento do produto potencial ou do hiato do produto.

Palavras-chave

Taxas de juros negativas; Bancos Centrais; Reservas excedentes; Moedas digitais de Bancos Centrais; Política monetária; Projeção; Random Forests; adaLASSO;

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Chapter 1
How Low Can Central Banks Go? The Banking Limits to Negative Interest Rates

1.1
Introduction

The Zero Lower Bound (ZLB) does not seem to be binding for some central banks anymore. Negative interest rates, usually depicted in economic textbooks as an impossibility due to the prospect of infinite demand for money, are now a reality in several countries due to different reasons, but mostly as a result of the continued effort to restart economic activity since the Great Recession (or contain currency depreciation). Since 2014, Denmark, Switzerland, Japan, Sweden and the Euro Area have experimented with negative interest rate policies (NIRP) of different flavors: while in Sweden and Switzerland the monetary policy benchmark interest rates are outright negative (-0.5% and -0.75% aa, respectively), in Denmark, Japan and the Euro Area benchmark interest rates are set at zero but the monetary authorities have steadily deepened rates on deposits at the Central Bank into negative territory, effectively setting a negative rate for these countries' interbank markets (deposit rates at -0.65%, -0.10% and -0.40% aa respectively at the time of writing).

Notwithstanding, these countries have not, so far, witnessed an explosion in money demand. Instead, while there has been some recovery in lending as Central Bank (CB) interest rates have transcended the ZLB (see Arteta et al. (2016)), two surprising facts have been observed: first, commercial banks have not passed on to their deposit rates the fall into negativity of CB rates, and secondly, banks have maintained large amounts of excessive reserves - specially in the Euro Area - even in the face of the growing cost of such assets in a scenario of negative returns of deposits at the CB.

On the first fact, Jobst and Lin (2016) document the different paths of policy rates and bank's lending and deposit rates for different NIRP countries, while Eggertsson et al. (2017) not only present the fact but also document a breakdown in the pass-through of monetary policy to lending rates in Sweden and other countries, as noticed previously in Heider et al. (2016). In a few countries such as France, negative interest rates on savings deposits are actually forbidden by law (as the bank must repay “at least” the sum deposit by the client), automatically capping the pass-through, but competition and regulation issues as well as costs to intermediation in an environment of falling spreads have been suggested as causes for this phenomenon (see ESBG (2016)). A possible alternative explanation comes from the competition of foreign banks in open economies with advanced financial systems, where money can be shifted abroad. Figure 1.1 presents the paths of average deposit rates at credit institutions for non-financial corporations and households along with the relevant interest rates of national Central Banks - the “benchmark” interest rate and the deposit facility (used for deposit of bank’s excess reserves) interest rate. It is evident how deposit rates, especially for households, have not fallen into negative territory - especially in places where the Central Bank has been most aggressive, such as Switzerland (a notable exception is the corporate deposit rate in Denmark). As excess reserves reached the substantial volumes of late even the historically normal negative spread paid by the deposit facility means that they are now a rising source of costs for banks, and that in the Euro Area, bank’s profitability is also declining through a compression of spreads.

Meanwhile, excess reserves (reserves held by banks with the CB above the legal minimum required level), which were virtually zero up to the 2008 crisis in most developed countries, have seen a huge rise in both the US and in Europe, as a reflection of liquidity injections in both places as well as the beginning of a remuneration policy for excess reserves that was put in place in the US - where they are in a downward path, while still on the rise in the Euro Area, as can be seen in Figure 1.2 (Figure 1.3 in the Appendix plots the path of deposits at the Swiss National Bank). Excess reserves are always a cost from the point of view of banks - since they are usually funded by liabilities that pay interest and deposited at the CB at a lower rate - and therefore not a reasonable destiny of asset allocation, except when banks are faced with liquidity concerns or very high uncertainty. This fact is compatible with the pre-crisis almost nil levels. But when banks build up large amounts of excess reserves in an environment of NIRP in conjunction with a lower bound on the cost of bank funding (i.e. the ZLB on the deposit rate), further declines of the rates on CB deposits become a growing burden for banks. It has been noticed that some of the credit lines offered by Central Banks, especially in the EU, have worked more to support bank funding rather than lending in the economy[1] (Ratings (2014)). The asset purchase programs undertaken

[1]As also noted by Baldo et al. (2017): “The investment of excess liquidity might be constrained also by risk-based capital requirements for secured and unsecured funding, since, whenever a bank lends money on the interbank market, it builds up an exposure towards its

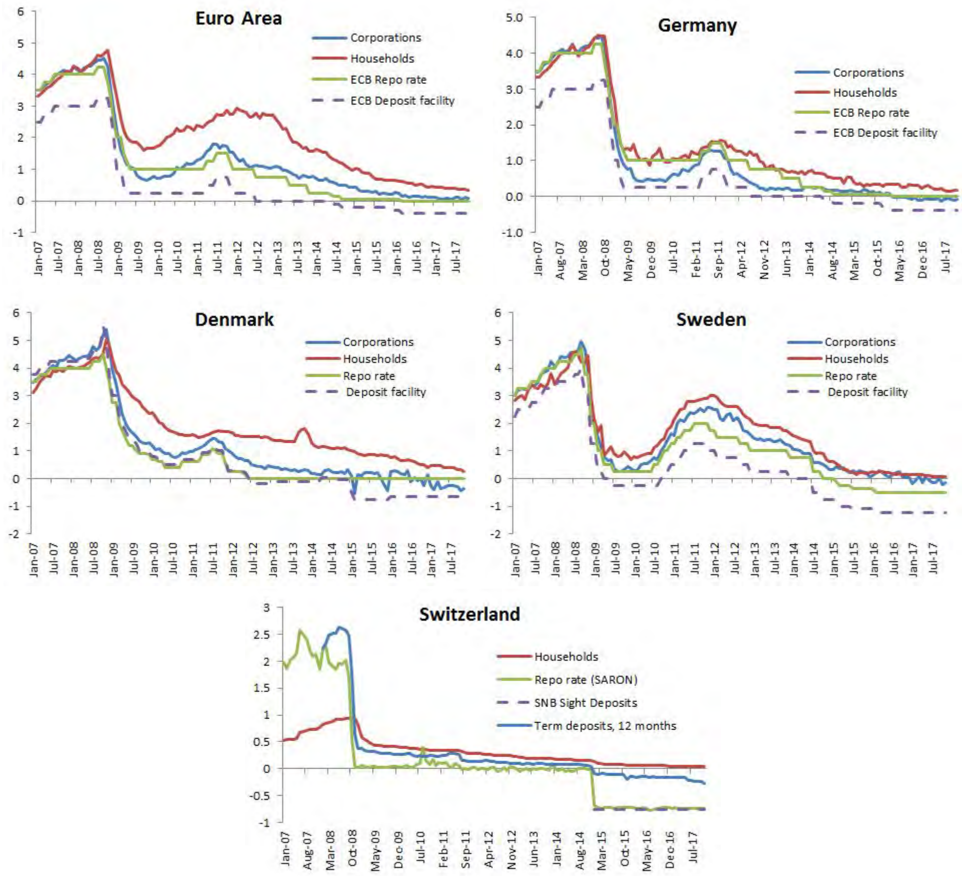
in the Eurozone have been suggested as the main culprit behind this rise in excess reserves, as banks have used the extra liquidity granted by the ECB as inssurance rather than funding for extra lending.

Both facts therefore suggest that the transmission mechanism of monetary policy partially "breaks down" once NIRP are introduced, effectively keeping in place the original ZLB problems and restrictions, despite the CB's bold dive into negative interest rates. It is possible that policy makers expect other channels than the return on savings to be at work under NIRP - such as the decline in long term interest rates, lower spreads on lending or expectations of the duration of the NIRP policy. But still, these two facts add an additional dimension of concern since NIRP under them also implies a growing fragility of banks profits and possibly their net capital, leaving them therefore more exposed to negative shocks.

In this paper, we aim to develop a framework that aims to replicate these facts and enables the analysis of the different NIRP now in place, as well as the effect of the large amount of excess reserves being held by banks, on the economy. We try to investigate whether the adoption of NIRP and liquidity injections might turn such policies hurtful from the point of view of banks (particularly through their net wealth), and whether other channels might be in place that will counterbalance them. To do so, we take a benchmark general equilibrium model with financial frictions presented in Gertler and Karadi (2011) and add to it the appearance of excess reserves by ways of limited interbank participation as described in Guntner (2015). We also allow for different interest rates and a different kind of CB intervention - which becomes a credit line to banks, much in the way the the FED and ECB have done throughout the past years. We view the Guntner (2015) framework as a way for modeling the incentives for CB intervention as well as for the appearance of excess reserves, but we acknowledge that the marked rise in excess reserves might also reflect other factors not modeled here, as we mentioned above. As we show, adopting NIRP in an environment of limited pass-through to deposit rates actually means that the hurdles of NIRP remain in place, and diving further into the negative side of interest rates can actually hurt the recovery in banking side rather than help, if undertaken for a long period of time. Despite the rich setup, the other channels of monetary policy in the model - such as the Q effect and lower spreads on loans - are not enough to counterbalance the

counterparty, which is subject to a capital charge with varying degrees of risk weights, while excess liquidity is not. Following this rationale, capital requirements might also be a reason for the concentration of excess liquidity at a country level as the environment of low interest rates makes the expected return from some kinds of investments (e.g. unsecured overnight lending) not worth the capital cost attached."

Figure 1.1: Deposit rates at credit institutions and Monetary policy relevant interest rates (Repo rates and deposit facilities (excess reserves) rates)



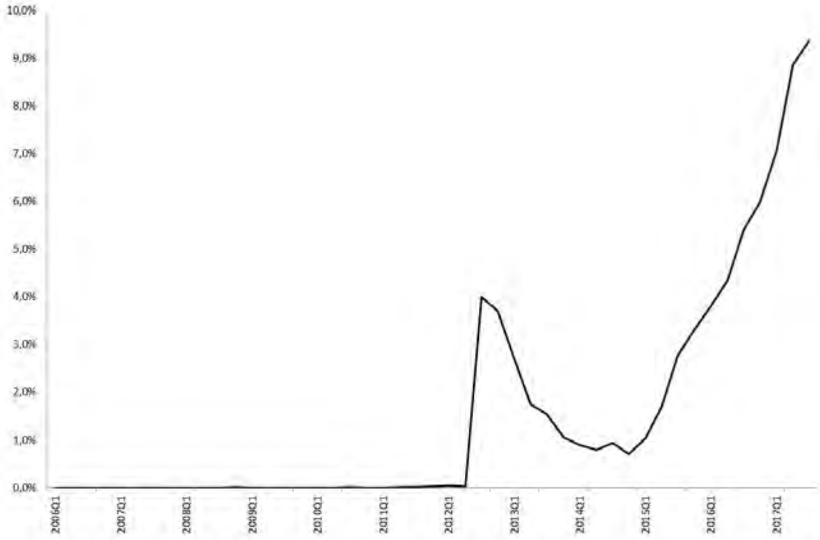
Sources: ECB,SNB and Riksbank

negative shock when banks are not passing the NIRP along to borrowers.

We believe that this work helps to fill the modeling gap that surrounds the current debate on NIRPs, while helping to study the levels and associated effects of such policies on the economy wide and on banks in particular, although we point out the recent contribution by Eggertsson et al. (2017), which tackles the break in monetary policy transmission represented by the ZLB on deposit rates and that we also use in our model.While theoretical in nature, we believe our model and exercises help to clarify some of the boundaries to NIRP from the point of view of the banking system, which has been one of the main concerns related to such policies. In our model we focus at the effects on the whole of the banking system's net capital, as a proxy for systemic risk.

Our paper builds mainly on the literature of DSGE models with financial frictions and the ZLB, with main contributions from Kiyotaki and Moore (1997); Bernanke et al. (1999); Eggertsson and Woodford (2003); Woodford (2010). We derive specially from Gertler and Karadi (2011), from where most

Figure 1.2: Excess Reserves of Euro Area Commercial Banks, % of deposits from non-MFIs.



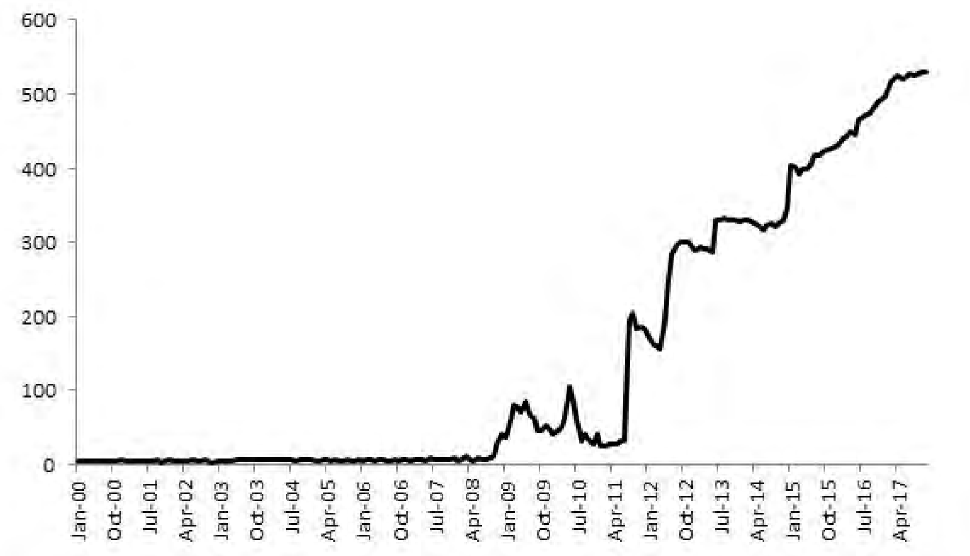
Source: ECB

of our model comes, as well as Gertler and Kiyotaki (2010), and Christiano et al. (2005). Another important strand to which we relate has to do with the growing literature surrounding NIRP, with early contributions from Buiter (2009), and Kimball and Agarwal (2013). But most of these papers discuss strategies for dealing with a heightened demand for paper currency in an environment of NIRP, a subject we avoid in our model through a constant semi-elasticity of demand, in line with the so far behaved response seen in NIRP countries. Rognlie (2016) shows that negative interest rates can be welfare enhancing, although not touching on the subject of the limited pass-through in the banking system. We point out that recently Eggertsson et al. (2017) also try to model the effects of NIRP on the economy and banking system in a simpler framework, with results similar to ours, but not tackling CB intervention on top of NIRP. Brunnermeier and Koby estimates a "reversal" interest rates which turns accommodative monetary policy contractionary, although such reversal rate might be either negative or positive. On the subject of the rise in excess reserves after the 2008 crisis, apart from Guntner (2015), we also note the contributions from Chang et al. (2014) and Ennis (2014), which also develops a general equilibrium model with excess reserves to study the determination of the price level. Recently, a very detailed study on the potential causes for the accumulation of excess reserves in the EU and its marked heterogeneity is present in Baldo et al. (2017), which help to underscore some of the effects we try to model in this paper.

This paper is organized as follows: section 1.2 presents the model; section 1.3

turns to the model's calibration, while section 1.4 reports several scenarios and the model's results; section 1.5 concludes.

Figure 1.3: Total Sight deposits at the SNB (CHF Billions)



1.2 The Model

The model closely follows Gertler and Karadi (2011), but we make three important distinctions: First, we introduce money; secondly, we transfer the ZLB from CB interest rates to the banking sector, creating a non-linearity in interest rates paid by the banking sector on time and saving deposits; and third, we introduce a participation shock in the interbank market, such as described in Guntner (2015).

1.2.1 Households

There is a continuum of identical households of measure unity. Each household is at any moment in time composed of two types of agents: a percentage f of bankers, which manage financial intermediaries and transfer their profits back to the household, who owns the banks; and a (1-f) percentage of workers, which supply labor (L_t) to firms and kick back to the household any wages it earns. Each household holds its savings in banks not owned by it. In each period, bankers face a probability θ, independent of history, of remaining bankers, and a $(1-\theta)$ chance of becoming workers, in which case they transfer the financial intermediary's net worth back to the household. As pointed in Gertler and Karadi (2011), $(1-\theta)\,f$ bankers become workers every period,

being replaced by a similar amount of randomly chosen workers - therefore, proportions remain constant throughout time.
The household has an additive utility function for real money $v(\frac{M_t}{P_t})$, reflecting, as is usual in the literature, the services provided by money in goods's transactions (see Christiano et al. (2005)). Money will serve as a substitute for time deposits when nominal interest rates are negative. This is a concave function, and the household therefore has diminishing utility gains from real cash balances, with utility from cash balances converging to zero as cash holding go to infinity.

$$v\left(\frac{M_t}{P_t}\right) = \psi_q \frac{m_t^{1-\sigma_q}}{1-\sigma_q} \tag{1-1}$$

The household can only choose to save through real time/savings deposits D_t in the financial intermediary. Time and savings deposits are paid the gross real interest rate R^d_{t+1}, which equal the risk free interest rate set by the Central Bank R_{t+1} *as long as the benchmark nominal interest rate* i_t *is non-negative.* When i_{t+1} dips into negative territory, the bank's nominal deposit rates (i^d_{t+1}) remains at zero, as a strategy by the financial intermediary to contain losses of deposits when rates dip below zero - banks fear that they might be out of funding should households divert savings massively towards money. Therefore, real deposit interest rates are:

$$R^d_{t+1} = \begin{cases} R_{t+1} & if\ i_t \geq 0 \\ -\pi_t & if\ i_t < 0 \end{cases} \tag{1-2}$$

Where π_t is the rate of prices change, and the household problem is:

$$\max_{C_t, D_{t+1}, M_{t+1}, L_t} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} + \psi_q \frac{m_t^{1-\sigma_q}}{1-\sigma_q} \right] \right\} \tag{1-3}$$

subject to the budgetary restriction[2]:

$$C_t = W_t L_t + \frac{m_{t-1}}{\pi_t} + R^d_t D_t - m_t - D_{t+1} - T_t + \Upsilon_t$$

where $\Upsilon_t$are the net transfers to new bankers and T_t are taxes net of transfers, and all variables are in real terms. From the FOCs, we get:

$$\frac{1}{C_t - hC_{t-1}} + \frac{\beta h}{C_{t+1} - hC_t} = \lambda_t \tag{1-4}$$

$$\frac{\chi}{\lambda_t} L_t^{\varphi} = W_t \tag{1-5}$$

[2]We denote D_t as real deposits received at time t, so that D_{t+1} are nominal deposits determined at time t divided by the price level (P_t) at t.

$$E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \beta R^d_{t+1} \right\} = 1 \tag{1-6}$$

$$\psi_q m_t^{-\sigma_q} = \lambda_t \left[1 - E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \beta \right\} \right] = \lambda_t \left(1 - \frac{1}{i^d_{t+1}} \right) \tag{1-7}$$

Where λ_t is the Lagrange multiplier of the maximization problem, and we use equation 1-51 in the FOC for real cash balances. Define:

$$\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \tag{1-8}$$

We will follow Christiano et al. (2005) and rewrite 1-7 by log-linearizing it around the steady state value $m*$, where the marginal utility of real cash balances equates the right-hand side of equation 1-7 at the steady state interest rate $\bar{i} = \bar{R} = 1/\beta$. We get:

$$\ln \left(\frac{m_t}{m*} \right) = - \frac{1}{\sigma_q} \left(\hat{\lambda}_t + \frac{\hat{i^d}_{t+1}}{\bar{R} - 1} \right) \tag{1-9}$$

where all variables with a hat denote deviations from steady state values. From 1-9, we have that the semi-elasticity of money demand to changes in the interest rate is:

$$\frac{\partial ln \left(m_t \right)}{\partial \hat{i^d}_{t+1}} = - \frac{1}{\sigma_q \left(\bar{R} - 1 \right)} \tag{1-10}$$

Equation 1-10 tells us that a 1 percentage point change in R_{t+1} will lead to a $-\frac{1}{\sigma_q \left(\bar{R} - 1 \right)}$% change in money demand relative to the optimum level $m*$, as noted in Rognlie (2016). This demand curve for money will have no material economic effect in the model, aside from serving as the "outside option" for households in the decision of its savings portfolio, as described in the work of Lagos and Zhang (2019). This "threat" from money, despite its low volume, implies a different response than assumed under a cashless economy according to the authors, and could be a reason for banks not to charge negative rates on deposits.

1.2.2 Financial Intermediaries

The financial intermediaries are a unit mass continuum of bankers, each running an individual banks. The financial intermediaries are the agents in the model responsible for the financing of productive capital to firms, without which production is impossible. It channels "long term savings" from the households to these firms, earning a spread that is converted into profits and net capital.

Let N_{jt} be the net worth of intermediary j at the end of period t; S_{jt} its quantity of financial claims on non-financial firms (or assets on its balance sheet); and Q_t the relative price of each claim. D_{jt+1} is the long term deposits from households in intermediary j (since checking deposits are equivalent to money in the model), which earn the non-contingent gross real rate R^d_{t+1} at t+1, and make up the liabilities side of the intermediary's balance sheet. Remember that, from 1-2, this rate is equal to R_{t+1} if $i_{t+1}$is non-negative. Banks participate in an interbank market for reserves, and can therefore even out differences in financing needs that might eventually arise. Bank j's borrowing in this interbank market is given by b_{jt}, which takes a negative value if the bank is on the lending side of the market. Financing in the interbank market is cleared at the gross real rate of R^I_{t+1}. As is traditional, R^I_{t+1} is bounded above by the discount window lending rate of the Central Bank, and below by R^E_{t+1}, the excess reserves rate paid by the Central Bank. In times of excessive liquidity, R^I_{t+1} will usually converge to R^E_{t+1}., but during normal times it will fluctuate in this range.

The following equality must hold in the intermediary´s balance sheet at each period:

$$Q_t S_{jt} = N_{jt} + D_{jt+1} - Ex_{jt+1} + \psi_{jt+1} + b_{jt+1} \tag{1-11}$$

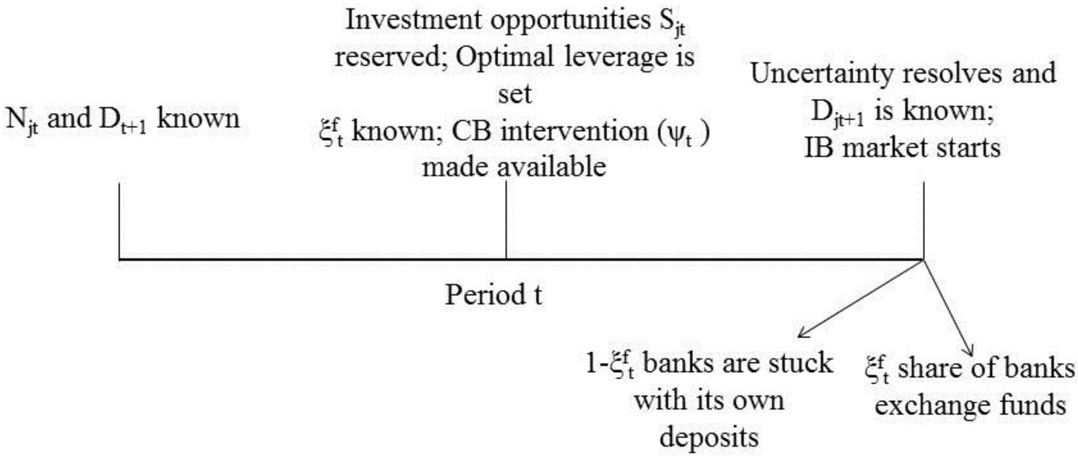
Where ψ_{jt} is the amount of central bank lending the bank takes on (which will only differ from zero in periods of stress - more below). Ex_{jt} is the amount of excessive reserves , which are determined endogenously at each period t. A financial intermediary might choose to hold excessive reserves at times when the return on assets is momentarily below the rate paid by the Central Bank on reserves, or for precautionary reasons. In our model, we follow the set up outlined by Guntner (2015), of individual deposit uncertainty and limited interbank market participation, in which banks will hold excess reserves because a share of them is left out of the interbank market and cannot lend whatever excess funds it's got. The volume of deposits made available to each individual bank at each period is a random draw from a time-varying normal distribution with mean D_{t+1} and standard deviation σ^2_t, so that $D_{jt+1} \sim N(D_{t+1}, \sigma^2_t)$, and will only be known at the end of each period t. This uncertainty allows for many sources of uncertainty in balance sheet management, but it serves especially well to model the uncertainty for banks during the 2008 crisis in the US as well as the one surrounding banks of some European countries during 2010 and 2011 in Europe. We acknowledge that it might not effectively reflect the situation from 2014 onward in the EU, as rising excess reserves seemed to actually result from rising volume of asset purchases by the ECB[3], but works as a source of both CB liquidity intervention

[3]see Baldo et al. (2017) for a discussion of the different periods in the EU.

as well as the incentive for rising reserves. Denote by $f(D_{jt+1})$ and $F(D_{jt+1})$ the probability density function and the cumulative distribution function of each bank´s deposits.

The set up is divided in 3 stages for financial intermediaries (Figure 1.4): At the beginning of each period t, each bank knows its own net capital and the aggregate amount of long term deposits made available by the representative household (D_{t+1}), but not its individual amount D_{jt+1}. The amount of net capital, for the sake of tractability, is known and identical to all banks at this point.

Figure 1.4: Financial Intermediaries decision process



In the middle of the period, banks will maximize their profit function under the uncertainty posed by their effective level of deposits, and being all equal ex-ante, determine their leverage ratio. The amount of investment projects in the economy will be unveiled and banks will "reserve" the projects which are able to financed in a competitive market - contrary to Gertler and Kiyotaki (2010), where opportunities are made available only to certain "islands" and therefore projects might have different rates of return, in our setup all projects have the same average rate of return of R_{kt+1}. The intermediary's assets will earn the stochastic rate of return R_{kt+1} at t+1, which will be determined endogenously. Intermediaries also learn whether the interbank market is functioning well, which will mean that ξ_t^f, the share of participating banks, equals 1 in the benchmark scenario of 100% participation of banks in this market. Meanwhile, the Central Bank becomes aware of the possibility of malfunctioning in the interbank market $\left(\xi_t^f < 1\right)$ and could offer a line of credit to banks, in the total amount of Ψ_t, to overcome the constraint imposed on banks who might receive lower deposits and not be able to finance themselves in the interbank market, being forced to "give up" on some of the previously reserved "investment opportunities" of the economy. The credit line comes with the real gross cost R^x_{t+1}, where:

$$R^x_{t+1} = R_{t+1} + \alpha_t \tag{1-12}$$

Where α_t will the the penalty rate that the CB may or may not impose on its credit line, and will be time dependent. The above equation means that, eventually, the credit line could even have a negative spread (as was implemented by the ECB in March 2017).

At the end of the period, banks discover how much deposits each of them gets, and whether this volume is enough or not to finance the optimal lending decision at each period - which we will clarify below and relates to a optimum leverage ratio. After the uncertainty is resolved, each bank will find themselves in one of the two situations: 1) either the realized deposits equal or exceeds the financing needs ($Q_t S_{jt} \leq N_{jt} + D_{jt+1}$), and therefore it will have available funds to lend to other banks in the interbank market; or 2) $Q_t S_{jt} > N_{jt} + D_{jt+1}$, and therefore the bank is short in funds and will need to either borrow the missing amount in the interbank market or settle with a lower lending level, commensurate with its level of deposits. In the event that $\xi^f_t < 1$, a fraction $1 - \xi^f_t$ of banks is eventually cut out of the interbank market and therefore not able to reach their desired leverage ratio due to this limitation. In this situation, excess reserves will arise as a result of limited interbank market participation for those banks which have been allotted with higher deposits.

The Central Bank pays R^E_{t+1} on the excess reserves banks choose to keep at the CB account. The definition of R^E_{t+1} will follow the objectives of current monetary policy such that:

$$R^E_{t+1} = R_{t+1} - \tau_t \tag{1-13}$$

Where is τ_t the spread of the deposit facility. The intermediary's net worth will change over time as a result of the different paths on its assets and liabilities sides, as a result of the different returns on them:

$$N_{jt+1} = R_{kt+1} Q_t S_{jt} - R^d_{t+1} D_{jt+1} - R^I_{t+1} b_{jt+1} + R^E_{t+1} Ex_{jt} - R^x_{t+1} \psi_{jt} \tag{1-14}$$

Notice that, in the usual situations where the return on assets is higher than the cost of liabilities, banks will wish to lend as much as they can, or most of their deposits - borrowing in the interbank markets allows banks short on deposits to do so (up to the point allowed by the leverage ratio), while for banks with excessive cash that participate in the interbank market, lending will eliminate the cost represented by those reserves. As in Gertler and Karadi (2011), we define: $\beta^i \Lambda_{t,t+i}$ as the stochastic discount of the banker at time t over earnings in the future t+i. Given the non-contingent cost R_{t+1} that if faces on its liabilities, banks will not fund asset that earn less than said rate on a discounted basis. Therefore, the bank's condition to operate in period i is:

$$E_t\left\{\beta^i \Lambda_{t,t+1+i}(R_{kt+1+i} - R^d_{t+1+i})\right\} \geq 0 \tag{1-15}$$

While the risk adjusted return on assets is larger than the cost of its funding, it pays for the intermediary to build assets, as long as it remains in the industry. The banker's objective is to maximize it expected terminal wealth, given by:

$$V_{jt}\left(S_{jt-1}, b_{jt-1}, Ex_{jt}, D_{jt}\right) = \underset{N_{jt+1+i}}{max}\, E_{t-1}\left\{\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\beta^{i+1}\Lambda_{t,t+1+i}N_{jt+i}\right\} =$$
$$= \max E_{t-1}\Lambda_{t,t-1}\beta E_t\left\{\left(1-\theta\right)N_{jt} + \theta\max_{\psi_{jt}}\left[\max_{s_{jt},b_{jt}} V_{jt}\left(S_{jt}, b_{jt}, Ex_{jt}, D_{jt+1}\right)\right]\right\} \tag{1-16}$$

As in Gertler and Karadi (2011), we assume that the intermediary can divert a fraction λ of its assets, but not the funds borrowed from the interbank market. Such a possibility will limit the amount of net capital that each financial intermediary can extract from the representative household. The intermediary will refrain from diverting assets as long as:

$$V_{jt} \geq \lambda\left(Q_t S_{jt} - b_{jt}\right) \tag{1-17}$$

As we show in the Appendix, from the bank's optimal behavior, we can rewrite equation 1-16 as:

$$V_{jt} = \upsilon_{st}Q_tS_{jt} + \upsilon_{xt}Ex_{jt} - \upsilon_tD_{jt+1} - \upsilon_{bt}b_{jt} \tag{1-18}$$

where:

$$\upsilon_t = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}R^d_{t+1} + \theta\beta\Lambda_{t,t+1}\left(1+\lambda^i_{t+1}\right)R^d_{t+1}\upsilon_{t+1}\right\} \tag{1-19}$$

$$\upsilon_{st} = \frac{R_{kt+1}}{R^d_{t+1}}\upsilon_t \tag{1-20}$$

$$\upsilon_{xt} = E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R^E_{t+1} + \Lambda_{t,t+1}\theta\beta\Lambda_{t,t+1}\left(1+\lambda^i_{t+1}\right)\left(1-\xi^f_t\right)R^E_{t+1}\upsilon_{t+1}\right\} \tag{1-21}$$

$$\upsilon_{bt} = E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R^I_{t+1}\right\} + E_t\left\{\Lambda_{t,t+1}\theta\beta\Lambda_{t,t+1}\left(1+\lambda^i_{t+1}\right)\left(1-\xi^f_t\right)R^I_{t+1}\upsilon_{t+1}\right\} \tag{1-22}$$

And:

$$\Omega_{t+1} = \beta\left[1-\theta+\theta\left(1+\lambda^i_{t+1}\right)\upsilon_{t+1}\right] \tag{1-23}$$

Where λ_{t+1}^{i} is the Lagrange multiplier from the bank's optimization problem (the same for all banks). As we show in the Appendix, the bank's asset allocation will respect::

$$Q_t S_{jt} = \phi_{jt} N_{jt} + b_{jt} \tag{1-24}$$

where:

$$\phi_{jt} = \frac{v_t}{\lambda - v_{st} + v_t} \tag{1-25}$$

Notice that all banks will choose in the middle of period t the same ex-ante leverage ratioϕ_{jt}. Since is not made up of any bank specific parameter, and supposing that the interbank market clears (more below), we can sum up over the whole industry to find the optimal ex-ante relationship between assets and net worth:

$$Q_t S_t = \phi_t N_t \tag{1-26}$$

Where $\phi_t = \phi_{jt}$ is the aggregate banking sector leverage ratio and S_t is the total amount of banking assets. This will be the ex-ante desired level of leverage that banks will be targeting. Equation 1-26 implies that, in the benchmark scenario of full interbank market participation:

$$\phi_t N_t + b_{jt} = N_t + D_{jt+1} + b_{jt} \Rightarrow (\phi_t - 1)\, N_t = D_{jt+1}$$

For the interbank market to clear, the amount borrowed must equal the amount lent in each period, at the interest rate R_{t+1}^{I}. Therefore:

$$E_t \left\{ b_{jt+1}/D_{jt+1} > (\phi_{jt} - 1) N_{jt} \right\} = E_t \left\{ b_{jt+1}/D_{jt+1} < (\phi_{jt} - 1) N_{jt} \right\}$$

In the appendix, we show that this implies:

$$D_{t+1} = (\phi_t - 1) N_t \tag{1-27}$$

After the uncertainty is resolved (at the end of the period) and individual deposits are unveiled, banks will find themselves in one of the four following situations:

1. Receives "high" amount of deposits but doesn't participate in the IB market

2. Receives "high" amount of deposits and participates

3. Receives "low" amount of deposits and participates

4. Receives "low" amount of deposits but doesn't participate

Now, as we noted above, a share $(1-\xi_t^f)$ cut out of the market and stuck with whatever deposits they were sorted. Banks will be targeting the optimum ex-ante leverage ratio of the industry. For banks who receive a high level of deposits $(D_{jt+1} > (\phi_t - 1)N_t)$, they can comfortably finance the best leverage ratio and lend in the interbank market or maintain excess reserves. Banks with low allotment of deposits $(D_{jt+1} < (\phi_t - 1)N_t)$ will borrow in the interbank market in order to reach their desired financing level or be stuck with a lower level of lending, in case they are cut out of the interbank market. So:

$$Q_t S_{jt} = \begin{cases} \phi_t N_t & for\ \xi_t^f\ participating\ banks \\ \phi_t N_t & with\ probability\ (1-\xi_t^f), \quad if\ D_{jt+1} > (\phi_t - 1)N_t \\ N_t + D_{jt+1} & with\ probability\ (1-\xi_t^f), \quad if\ D_{jt+1} < (\phi_t - 1)N_t \end{cases} \tag{1-28}$$

And therefore:

$$Q_t S_t = \phi_t N_t - (1-\xi_t^f)\left\{\left[(\phi_t - 1)N_t - D_{t+1}\right] F\left((\phi_t - 1)N_t\right) + \sigma_t^2 f\left((\phi_t - 1)N_t\right)\right\} \tag{1-29}$$

We will define interbank unmet demand by ID, so that:

$$ID_t = (1-\xi_t^f)\left\{\left[(\phi_t - 1)N_t - D_{t+1}\right] F\left((\phi_t - 1)N_t\right) + \sigma_t^2 f\left((\phi_t - 1)N_t\right)\right\} \tag{1-30}$$

Similarly, the amount of excess reserves will be determined by:

$$Ex_{jt} = \begin{cases} 0 & if\ D_{jt+1} \leqslant (\phi_t - 1)N_t \\ 0 & with\ probability\ \xi_t^f, \quad if\ D_{jt+1} > (\phi_t - 1)N_t \\ D_{jt+1} - (\phi_t - 1)N_t & with\ probability\ (1-\xi_t^f), \quad if\ D_{jt+1} > (\phi_t - 1)N_t \end{cases}$$

But during periods of stress in the interbank market, banks might be limited in the amount of capital they can finance, since some banks with low deposits won't be able to get the necessary funding to close the gap relative to their desired amount of lending. In this situation, there will be less capital financed in the economy than demanded, and activity will be below potential. In this situation, the CB might choose to make available to the financial intermediaries its credit line in volume Ψ_t, to guarantee that the available financing projects are undertaken and productive capacity reaches its steady-state value. The amount of assets financed and excess reserves, therefore, will also be a product of the amount of excess liquidity in the banking system, once the uncertainty is resolved.

Since all banks face the same risk of falling in situation 4 described above after the uncertainty is resolved, all of the banks will apply for the credit line in order to guarantee sufficient funding for all its projects. Ex-ante equal banks will receive an equal amount of credit, since at this point they are subject to the same uncertainty, that will equal the expected gap in financing, so that:

$$Q_t S_{jt} = \begin{cases} \phi_t N_t & for\ \xi_t^f\ participating\ banks \\ \phi_t N_t & with\ probability\ \xi_t^f, \quad if\ D_{jt+1} > (\phi_t - 1)N_t \\ N_t + \Psi_t + D_{jt+1} & with\ probability\ (1-\xi_t^f), \quad if\ D_{jt+1} < (\phi_t - 1)N_t \end{cases} \tag{1-31}$$

$$Ex_{jt} = \begin{cases} 0 & if\ D_{jt+1} \leqslant (\phi_t - 1)N_t \\ \Psi_t & with\ probability\ \xi_t^f, \quad if\ D_{jt+1} > (\phi_t - 1)N_t \\ \Psi_t + D_{jt+1} - (\phi_t - 1)N_t & with\ probability\ (1-\xi_t^f), \quad if\ D_{jt+1} > (\phi_t - 1)N_t \end{cases} \tag{1-32}$$

Notice that we limit the quantity of assets that may be acquired by banks to the supply of projects by the economy known in the middle of the period, when the leveraging decision was made and the uncertainty remained. Therefore, we do not allow the credit intervention to imply in more assets than would be available under a perfect foresight scenario. The total volume of assets financed and excess reserves are given respectively by:

$$Q_t S_t = \phi_t N_t - (1-\xi_t^f)\left\{\left[(\phi_t - 1)N_t - D_{t+1} - \Psi_t\right] F\left((\phi_t - 1)N_t\right) + \sigma_t^2 f\left((\phi_t - 1)N_t\right)\right\} \tag{1-33}$$

$$Ex_t = \Psi_t\left[1 - (1-\xi_t^f)F\left((\phi_t - 1)N_t\right)\right] + (1-\xi_t^f)\left[\left((\phi_t - 1)N_t - D_{t+1}\right)\left(1 - F\left((\phi_t - 1)N_t\right)\right) - \sigma^2 f\left((\phi_t - 1)N_t\right)\right] \tag{1-34}$$

We will define Ω_E as the share of deposits held in the form of excess reserves in the banking system's balance sheet:

$$\Omega_{Et} = \frac{Ex_t}{D_{t+1}}$$

Net capital for period t+1 will therefore depend on the 4 different situations affecting intermediaries, so that:

$$N_{t+1} = N_{t+1}^p + N_{t+1}^{np}$$

where N_{t+1}^p is the total net worth of participating banks and N_{t+1}^{np} is the total net worth of non-participating banks, and:

$$\begin{aligned} N^{p}_{t+1} &= \xi^{f}_{t+1}\{\left[\left(R_{kt+1}-R^{d}_{t+1}\right)\phi_t+R^{d}_{t+1}\right]N_t+\left(R^{d}_{t+1}-R^{x}_{t+1}\right)\Psi_t+ \\ &\quad +\left(R^{E}_{t+1}-R^{d}_{t+1}\right)Ex^{p}_{t}\} \end{aligned}$$

While for non-participating banks, we have

$$\begin{aligned} N^{np}_{t+1} &= \left(1-\xi^{f}_{t+1}\right)\{\left[\left(R_{kt+1}-R^{d}_{t+1}\right)\phi_t+R^{d}_{t+1}\right]N_t+\left(R^{E}_{t+1}-R^{x}_{t+1}\right)\Psi_t+ \\ &\quad +\left(R^{E}_{t+1}-R^{d}_{t+1}\right)Ex^{np}_{t}\}-\left(R_{kt+1}-R^{d}_{t+1}\right)ID_t \end{aligned}$$

so that net worth evolution will now depend also on the cost of the central bank credit line as well as the return on excess reserves:

$$\begin{aligned} N_{t+1} = \ & \left[\left(R_{kt+1}-R^{d}_{t+1}\right)\phi_t+R^{d}_{t+1}\right]N_t+\left(R^{E}_{t+1}-R^{x}_{t+1}\right)\Psi_t-\left(R_{kt+1}-R^{d}_{t+1}\right)ID_t- \\ & -\left(1-\xi^{f}_{t+1}\right)\{\left(R^{E}_{t+1}-R^{d}_{t+1}\right)\Psi_t F\left((\phi_t-1)N_t\right)\}- \\ & \left((\phi_t-1)N_t-D_{t+1}\right)\left(1-F\left((\phi_t-1)N_t\right)\right) \\ & +\sigma^2 f\left((\phi_t-1)N_t\right) \end{aligned} \tag{1-35}$$

1.2.3
Credit policy

During a crisis in credit markets, the CB is allowed to intervene in order to normalize bank participation in the market and minimize the displacement of investment opportunities, according to the following rule:

$$\Psi_t = \nu(1-\xi^{f}_{t})D_{t+1} \tag{1-36}$$

where $\Psi_t = \int \psi_{jt} d_j$ is aggregate volume of Central Bank lending to banks. But this Government intermediation comes with an inefficiency cost τ per unit supplied, which limits the intervention only to crisis periods. This cost can be motivated as the costs related to decision-making relative to which assets to buy and of bond issuance.

1.2.4
Intermediate Goods Firms

In the productive side of the economy, the setup is quite standard: competitive non-financial firms produce intermediate goods which will then be sold to and repackaged by retail firms. In order to produce in each period t, intermediate goods firms use labor L_t and capital K_t, which is obtained from capital producing firms (next section) in period t-1 and funded through the issuance of financial asset S_t to financial intermediaries. Once production in period t is over, the firm can either keep this capital or sell it in the open market. Firms obtain financing for the purchase of capital by issuing assets S_t such that:

$$Q_t K_{t+1} = Q_t S_t \tag{1-37}$$

Production technology:

$$Y_t^I = A_t (U_t \xi_t K_t)^\alpha L_t^{1-\alpha} \tag{1-38}$$

Where At is total factor productivity and $\xi_t$is the quality of capital. Call Pm_t the intermediary good´s price. So, at each period t, the firm will choose L_t and U_t according to:

$$Pm_t \alpha \frac{Y_t}{U_t} = \delta'(U_t) \xi_t K_t$$

$$Pm_t (1-\alpha) \frac{Y_t}{L_t} = W_t$$

Firms earn zero profits: ex-post return to capital payed out to the intermediary. Replacement price of capital depreciated is 1, so the capital stock left over at the end of t+1 is $(1 - \delta(U_{t+1}))\xi_{t+1} K_{t+1}$. Therefore, the debt repayment, $Rk_{t+1} K_{t+1} Q_t$, which is the value borrowed at t plus the interest, should equal the return of capital to the firm (the marginal product of capital plus the value of capital after depreciation). So:

$$Rk_{t+1} K_{t+1} Q_t = Pm_{t+1} \alpha Y_{t+1} + Q_{t+1} \xi_{t+1} K_{t+1} - \delta(U_{t+1}) \xi_{t+1} K_{t+1}$$

$$\Rightarrow Rk_{t+1} = \frac{1}{Qt} \left[\frac{Pm_{t+1} \alpha Y_{t+1}}{K_{t+1}} + Q_t \xi_{t+1} - \delta(U_{t+1}) \xi_{t+1} \right] \tag{1-39}$$

1.2.5 Capital Producing Firms

At the end of each period t, competitive capital producing firms buy depreciated capital and refurbish it (at the cost of 1 per unit) or make new capital. The value of one unit of new capital is Q_t. Call I_t the gross capital created, I_{ss} its value in the steady-state, and In_t the net capital created, or:

$$In_t \equiv I_t - \delta(U_t) \xi_t K_t$$

Then the problem of this firm is to maximize its discounted profits:

$$Max\ E_t \left\{ [\tau = t]\infty\sum \beta^{\tau-t}\Lambda_{t,\tau} \left\{ (Q_\tau - 1)In_\tau - f\left(\frac{In_\tau + Iss}{In_{\tau-1} + Iss}\right)(In_\tau + Iss) \right\} \right\}$$

Gertler and Karadi (2011) use $f\left(\frac{In_t+Iss}{In_{t-1}+Iss}\right) = \frac{\eta_i}{2}\left(\frac{In_t+Iss}{In_{t-1}+Iss} - 1\right)^2$ in their calculation, which we follow. From the above, we get the "Q relation" for net investment:

$$Q_t = 1 + f(.) + f^{'}\left(\frac{In_t + Iss}{In_{t-1} + Iss}\right) . \left(\frac{In_t + Iss}{In_{t-1} + Iss}\right) - E_t\left\{\beta\Lambda_{t,t+1} f^{'}\left(\frac{In_{t+1} + Iss}{In_t + Iss}\right) . \left(\frac{In_{t+1} + Iss}{In_t + Iss}\right)^2\right\} \quad (1\text{-}40)$$

1.2.6 Retail Firms

Retail firms repackage the goods from intermediate goods firms to create the final consumption good Y_{ft}. The final output is produced according to a CES composite:

$$Y_t = \left([0]1 \int Y_{ft}^{(\varepsilon-1)/\varepsilon} df \right)^{\varepsilon/\varepsilon-1} \quad (1\text{-}41)$$

Where:

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\varepsilon} Y_t^I \quad (1\text{-}42)$$

$$P_t = \left[\int_0^1 P_{ft}^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \quad (1\text{-}43)$$

The marginal cost of the retail firm is the price of the intermediate goods, Pm_t. We follow Gertler and Karadi (2011) and assume nominal price rigidities as in the Christiano et al. (2005) paper: each firm faces, in each period, a probability $1-\gamma$ of adjusting its price, and, in between periods of price adjustment, they index their prices to lagged inflation. The retailers pricing problem is:

$$\underset{P_t^*}{Max}\ E_t\left\{ [i = 0]\infty\sum \gamma^i \beta^i \Lambda_{t,t+i}\left[\frac{P_t^*}{P_{t+i}} [K = 1]i\prod (1+\pi_{t+k-1})^{\gamma p} - Pm_{t+i}\right] Y_{f_{t+i}} \right\}$$

The FOC is:

$$E_t \left\{ [i = 0]\infty \sum \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} [K = 1]i \prod (1 + \pi_{t+k-1})^{\gamma p} - \mu P m_{t+i} \right] Y_{f_{t+i}} \right\} = 0 \tag{1-44}$$

where :

$$\mu = \frac{1}{1 - {}^{1}\!/_{\varepsilon}}$$

Which results in the equation for the evolution of the price level, using the law of large numbers:

$$P_t = \left[(1-\gamma) \left(P_t^*\right)^{1-\varepsilon} + \gamma \left(\pi_{t-1}^{\gamma p} P_{t-1}\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{1-45}$$

Notice that total final output Y_t and the total intermediate output Y_t^I will be related by:

$$Y_t^I = Disp_t Y_t \tag{1-46}$$

where $Disp_t$ is the level of price dispersion.

1.2.7 Resource constraint, Fiscal and Monetary Policies

The economy´s resource constraint is given by

$$Y_t = C_t + I_t + f\left(\frac{In_t + Iss}{In_{t-1} + Iss} \right) (In_t + Iss) + G + \tau \psi_t Q_t K_{t+1} \tag{1-47}$$

where G are Government expenditures, which we assume exogenously fixed, and the last term represents government intermediation in credit markets. Capital evolves according to:

$$K_{t+1} = \xi_t K_t + In_t \tag{1-48}$$

The government's budget constraint is given by:

$$G + \tau\psi_t + R_{t+1}^E EX_t = T_t + \left(R_t^x - R_t\right)\psi_{t-1} + m_t - \frac{m_{t-1}}{\pi_t} \tag{1-49}$$

We maintain the original model's simple Taylor rule with an interest rate smoothing parameter $\rho \epsilon [0, 1]$. Let i_t be the nominal net interest rate, and i its steady state value and ε_t an exogenous shock to monetary policy. Then the interest rate rule followed by the Central Bank is:

$$i_t = (1-\rho)\left[i + \kappa_\pi \pi_t + \kappa_y (logY_t - logY_t^*)\right] + \rho i_{t-1} + \varepsilon_t \tag{1-50}$$

Where Y_t^* is the level of output in the flexible price equilibrium. Real interest rates are linked to i_t by the Fisher equation:

$$i_t = R_{t+1} \frac{E_t P_{t+1}}{P_t} \tag{1-51}$$

1.2.8 Equilibrium and steady-state

The equilibrium is comprised of the sequences $\{C_t\}$, $\{L_t\}$, $\{m_t\}$, $\{W_t\}$, $\{K_t\}$, $\{\lambda_t\}$, $\left\{Y_t^I\right\}$, $\{Y_t\}$, $\{P_t\}$,$\{S_t\}$, $\{N_t\}$, $\{D_t\}$, $\{Ex_t\}$, $\{\phi_t\}$,$\{Q_t\}$, $\{I_t\}$, $\{Rk_t\}$, $\{R_t\}$ and $\{i_t\}$ for t = 1,....,∞ such that equations 5, 7, 9, 24,25, 27, 33,34,35,37,38, 46, 49, 51 and the household's budget constraint are respected, and the interbank and output markets clear.

In the steady-state equilibrium, where $\xi_t^f = 1$, the model boils down to the original Gertler and Karadi (2011) steady-state framework, with zero excess reserves and no friction in the interbank market. All banks are equal at all points in the steady-state.

1.3 Calibration

Most of our parameters are taken from Gertler and Karadi (2011), with the exception of the demand elasticity of money to interest rate deviation (which we take from Christiano et al. (2005)), σ_t and ξ_t^f, which we follow from Guntner (2015). Note that the benchmark interest rate R_t is 4% aa, while R_{kt} is 5%. We note that some slight adjustments were made to two particular parameters from the original: bank's survival rate and the transfer to new banks were respectively slightly lowered and augmented, in order to implement the steady-state equilibrium. We calibrate the nominal interest rate on the CB credit line as being equal to the benchmark interest rate, implying no spread by the part of the central bank, as seen in the US and EU - and which might imply a subsidy to borrowing when the rate is negative (as was implemented by the ECB since March 2017). On our benchmark calibration, we set τ_t, the difference between the benchmark nominal interest rates and the nominal rate paid by the BC on excess reserves, to 1 pp - above what most countries practice, but close enough to the 75 bps spread used in the recent past by the ECB.

1.4 Scenarios

For estimation of the scenarios, given the non-linearity of the ZLB on deposit rates, we use the methodology developed by Guerrieri and Iacoviello (2015), OccBin. Most scenarios start with a negative shock of 5% in capital quality (ξ_t), as in Gertler and Karadi (2011), which is necessary to drive the economy into a recession compatible with the need to use NIRP, associated

Table 1.1: Parameters

Parameter	Description	Value
β	Consumer discount rate	0.99
h	habit parameter	0.815
χ	Relative utility weight of labor	3.409
φ	Inverse Frisch elasticity of labor	0.276
λ	Share of assets that can be diverted	0.381
ω	Transfer to new banks	0.005
θ	Bank's survival rate	0.957
σ_q	Elasticity of money demand to interest rate	10.62
α	Effective capital share	0.33
$\delta(U)$	Steady state depreciation rate	0.025
ζ	Elasticity of depreciation to utilization rate	7.200
η_i	Inverse elasticity of net investment to Q	1.728
ε	Elasticity of substitution	4.167
γ	Probability of fixed prices each period	0.779
γ_p	Measure of price indexation	0.241
κ_π	Inflation coefficient in the Taylor rule	1.50
κ_y	Output gap coefficient in the Taylor rule	0.125
ρ_i	Smoothing parameter of the Taylor rule	0.8
υ	Intervention weight	1.0
m^*	Steady-state cash/GDP	0.128
ξ^f	Steady-state proportion of participating banks	1.0

with a decrease of 10 or 25 percentage points in the level of participation in the interbank market (i.e. $\xi_t^f = 0.90$ or $\xi_t^f = 0.75$).

1.4.1 Different ZLBs and no CB intervention

The usual case is the one in which the Central Bank doesn't intervene. Usually in the literature, the ZLB applies to the CB nominal interest rate, while in our new set up it resides in the nominal interest rate on deposits paid by the financial intermediaries, R^d_{t+1}. Figure 2.1 plots both scenarios against a "no ZLB" counterfactual (but using $\xi_t^f = 0.90$). As it becomes clear, transferring the ZLB to the banks leads to slightly worse effects on economic aggregates as in the usual ZLB case, since in the end the economy doesn't benefit from a lower level of interest rates because the transmission from monetary policy to loans has broken down, and banks net capital takes slightly longer to recover. But the biggest differences between the two ZLB scenarios come from the nominal and real CB interest rate, which can react very sharply to the ensuing deflation when not restricted by the ZLB, and the loan spread, which is a lot higher in the scenario where the rate on deposits is subject to the ZLB - explaining why the negative impact on bank's net wealth is very similar for both ZLB scenarios.

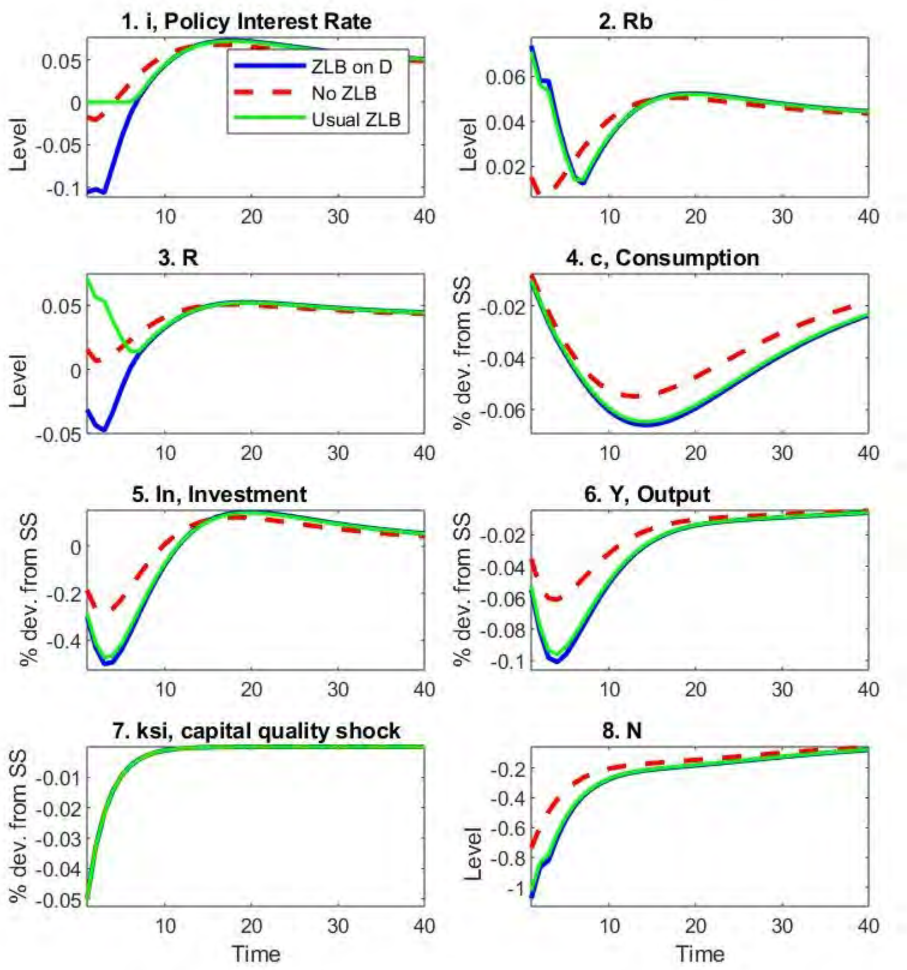
According to this model, if the economy is not subject to any ZLB, monetary policy would venture into negative interest rates for a very short period, and only at marginally negative levels, which would be passed on to depositors. The model suggests that a 1% aa rate would be enough to accommodate the effects of the shock, and for less than a year. Notice that this level is consistent with the NIRP seen in some countries (such as Switzerland), which so far seem feasible. But in the scenario where bank's deposit rates hit their ZLB, and the CB is free to react to the very negative economic consequences but banks are not, CB interest rates would need to go substantially lower into the negative camp (10% aa) for a while, although finding no transmission to the economy. The very deep negative rates reflect the Taylor rule's response to the sharp deflation in the economy, in a scenario where central banks are free to act. Loan spreads rise to compensate banks for their higher cost, and therefore the amount of capital financed does not react to the lower CB rates. And most importantly, the whole banking system's net capital is wiped out right at the start of the crisis, and recovers a lot more slowly (when compared to a no-ZLB scenario).

This exercises in itself is interesting in that it signals the break in monetary policy transmission once banks are hesitant to pass on NIRP to its deposit rates, effectively keeping all the problems that accompany the traditional ZLB - including the wipe out of the entire banking system's net capital as the shock hits. On the other hand, it suggests that, even faced with a conjunction of strong shocks such as the ones suggested, mild levels of negative interest rates would alleviate significantly not only the negative effects on the economy but also on banks, actually being of help. The "mild" levels suggested by the model (around 1% aa) are close to the implemented levels seen so far in countries adopting NIRP, which suggest that they might be expansionary in theory but - with the effective ZLB imposed by banks - could lose their stimulative power and require even lower levels. A similar result is reached in Eggertsson et al. (2017), namely, that there is a break in monetary policy transmission when NIRP are in place, but through a model different from ours, where the restriction on bank's pass-through of NIRP arises from the bank's cost function.

1.4.2 Central Bank intervention

What is the effect of an untargeted intervention by the Central bank? As in Guntner (2015), an untargeted injection of liquidity serves to accommodate part of the disruption from interbank stress but will also generate large amount

Figure 1.5: Usual ZLB vs ZLB on Deposit rates on a scenario of no intervention

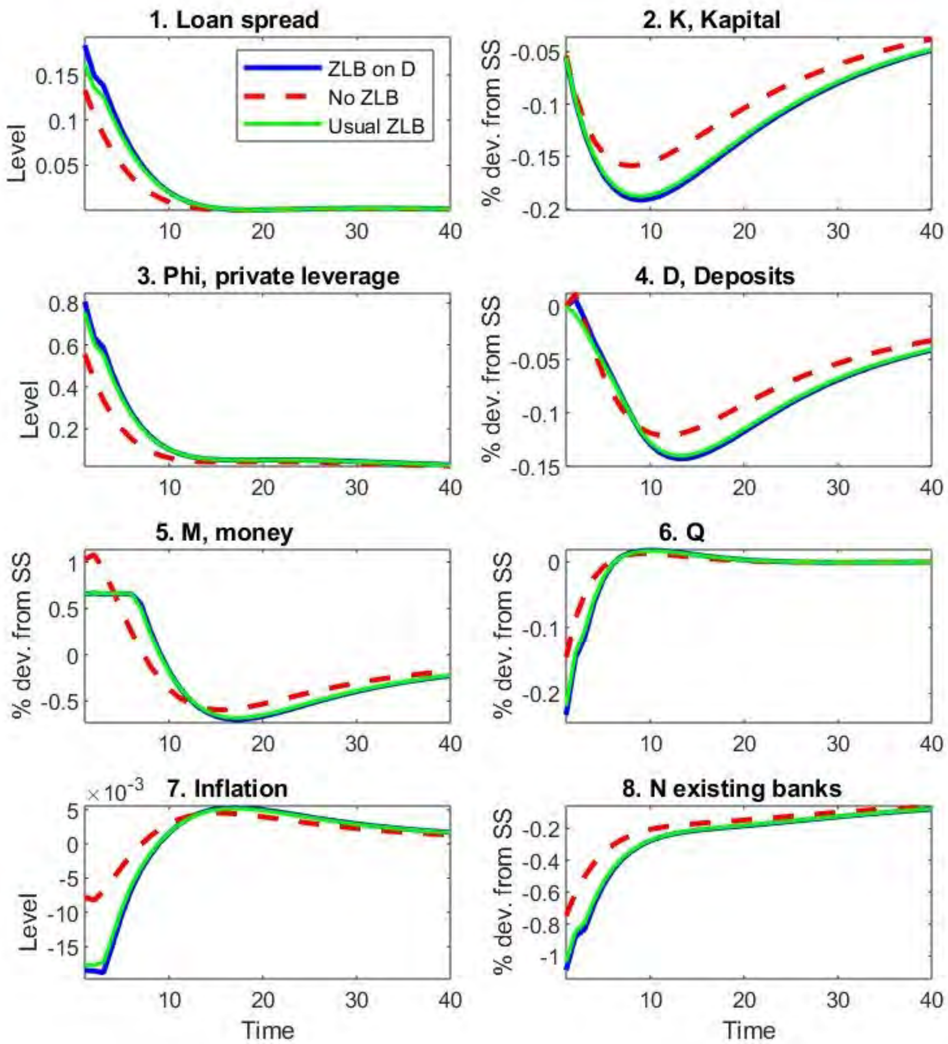


of excessive reserves in the system[4]. We therefore try a different experiment: given a smaller capital quality shock that pushes benchmark interest rates to zero (instead of negative), in a scenario of stress in interbank markets, how does bank's net worth react to different levels of interest rates on excess reserves? By construction, in our framework CB intervention will imply in excess reserves on bank's balance sheets, so the difference between the benchmark rates and the excess reserves rate (the variable τ_t in the model) will imply a growing cost for banks with excess reserves as well as for banks that lend in the interbank market.

We begin with a study of a scenario where the benchmark interest rate doesn't have to dip into negative territory: a capital quality shock of around 3.5%, taking it naturally to zero nominal interest rates, so that the ZLB on deposits in not relevant. But we trace two very different scenarios for the value

[4]In the Guntner (2015) model, a targeted intervention will completely eliminate the economic disruption while implying very limited rise in excessive reserves

Figure 1.6: Usual ZLB vs ZLB on Deposit rates on a scenario of no intervention-additional IRFs



of τ_t, one in which the negative spread to the benchmark interest rate is of 1pp (annum) and, in a more radical scenario, one in which it is 4 pps. We start with the minimal 10% cut in bank participation, but as Figure 1.9 in the Appendix shows, the difference in those scenarios is negligible. The brunt of the economic downfall is a result on the capital quality shock. Because the disruption represented by a 10% dislocation of banks is minimal, specially when the CB intervenes with cheap credit to undo some of the capital dislocation presented by the participation shock - then the accumulation of excess reserves is also not big enough to become a burden to banks.

Therefore we increase the stress in interbank markets to cut out 25% of banks ($\xi_t^f = 0.75$), which is probably closer to the distress seen in the EU, and helps to motivate higher excess reserves in the model. We present the main indicators in figures 1.7 and 1.10. With big enough excess reserves, it becomes clear how the rising cost of NIRP affects the recovery path in the banking system's net capital. The decrease in the price of capital is smaller on the onset in the -4 pps spread scenario, which implies in a smaller decrease also in the banking system's net wealth. But as time goes by and the growing burden of excess reserves reveals itself, the recovery in bank´s net capital becomes clearer, with resulting negative effects in output. It is interesting to see that the spread charged by banks is also lower in the -4pps scenario, adding to a decrease in bank's interest margin and therefore to lower profits and net wealth. The decrease in bank's net interest margin has repeatedly being pointed as the main concern regarding the effects of NIRP on the banking system, as it induces possibly reckless behavior and also leaves banks more exposed to negative shocks (in defaults or capital quality, for example).

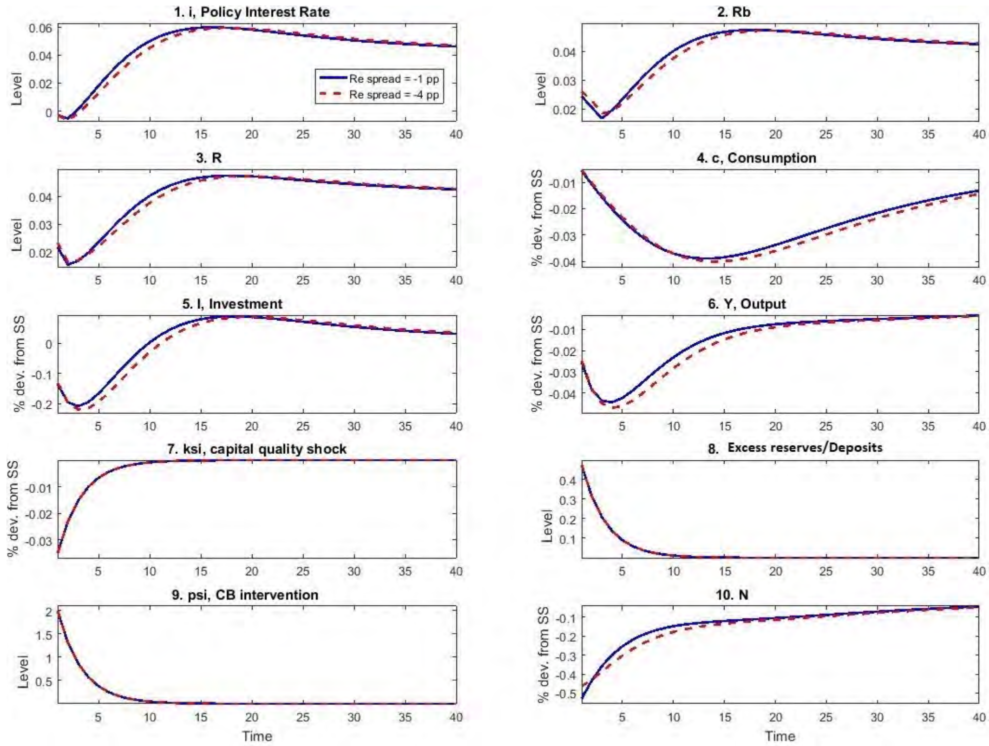
Finally, although dragging the recovery of the banking system in the long run, we would point out that the impact of very large negative rates on reserves (-4% aa) on the banking system net wealth is not drastically different from the moderate scenario, suggesting, at least in this exercise, that an incapacitation of banks during the NIRP period doesn't happen as long as it is for a short period. In part that has to do with the fact that by seventh quarter nominal benchmark interest rates are well into positive territory, and R^E_{t+1} is back at 0% in the -4 pps case. Still, the exercise suggests that interest rates on excess reserves might have some further room to fall in developed countries in the current environment of benchmark rates at or near zero, although it is not clear whether such decreases will have any extra expansionary effect.

1.4.3 ZLB on R^d_{t+1} and CB intervention

Finally, we study the effects of the conjunction of CB intervention with a dive into NIRP through benchmark interest rates. In some countries adopting NIRP, the effective benchmark interest rate has actually become the rate paid on excess reserves, as interbank rates converged to the lower bound of their traditional range and public opinion and banks focus more closely on movements in such rates - this is the case in the EU and Japan, for example. So the exercise of the previous section will reflect better that sort of environment. In this section we return to the 5% drop in capital quality - implying the need by the economy to have negative rates, but in an environment of the ZLB in R^d_{t+1}- and to $\tau_t = 1$ pp, along with heavy CB credit extension in a scenario of distress for bank's funding ($\xi^f_t = 0.75$).

The results are presented in figure 1.8. Overall, under this benchmark scenario with a ZLB on R^d_{t+1}, we can see that credit intervention serves to

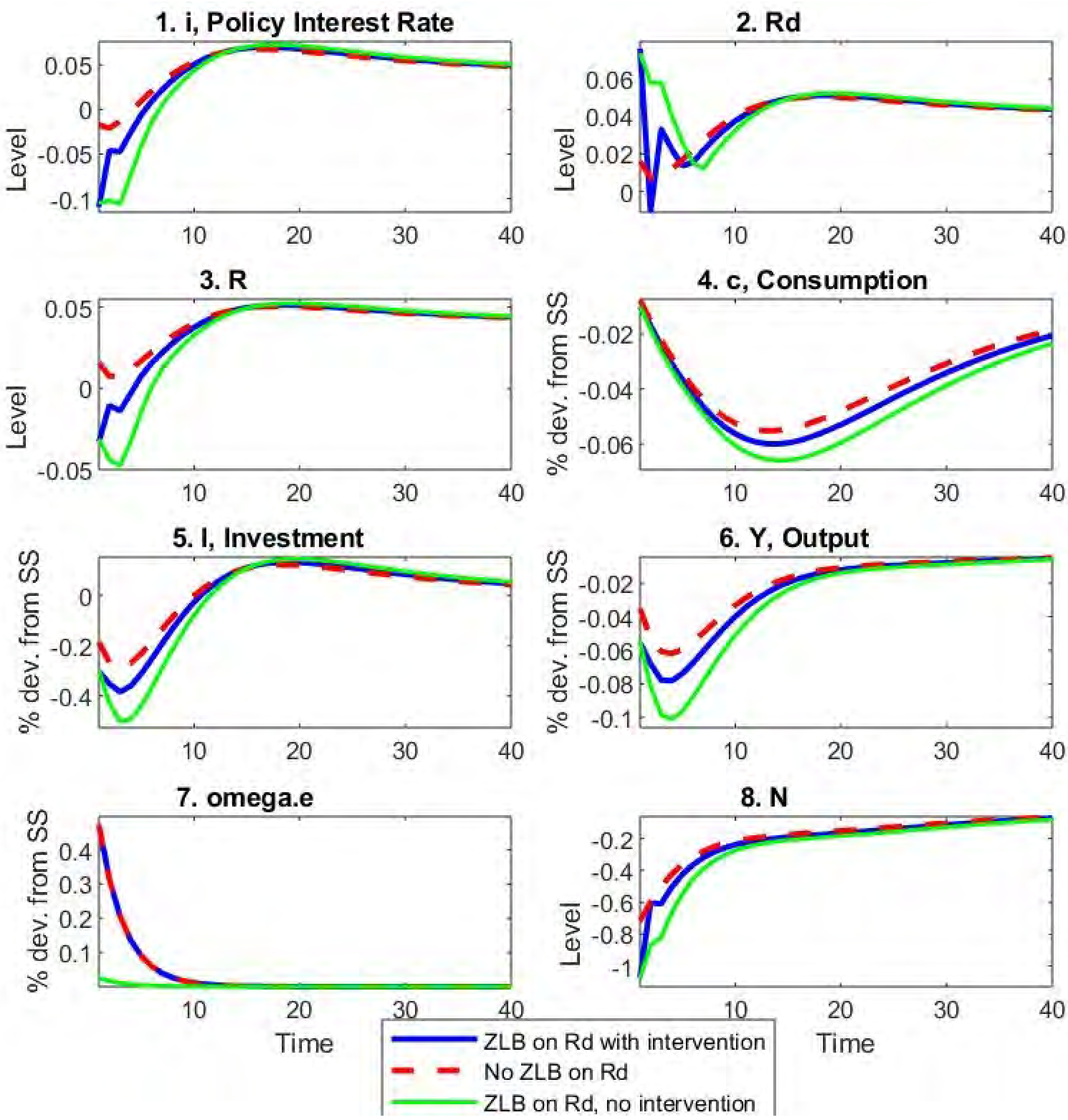
Figure 1.7: Intervention scenarios, $\xi_t^f = 0.75$



alleviate the impacts of the distress in interbank markets, as would be expected, and is an improvement to the no intervention scenario with a moderate cost of 1 pp to R^E_{t+1}. The CB, following the Taylor rule, drops nominal rates i_{t+1} further into the negative territory as a reaction to the sharp deflation ensued by the shock, but only for a quarter, with rates ranging from -5% to 0% over the following three quarters. The real interest rate on deposits R^d_{t+1}fluctuates sharply over this first year, reflecting initially the reaction on i_{t+1} and afterwards the comeback from deflation while nominal rates slowly normalize. Although the banking system goes bust at the onset of the crisis in both cases, it recovers a lot faster under the intervention case, as asset prices recover faster and banks are able to finance more assets in the economy given the CB credit injection in a scenario of high spreads, which decrease slowly. The reasonably moderate “cost”of the intervention is more than offset by the improvement in the price of assets and the gained profit from the additional assets financed relative to the no-intervention case. The price of capital also recovers faster, as the demand for capital is higher and therefore there is no need for a “fire sale” of assets, helping to shield a bit bank’s balance sheets.

Note also that initially R^x_{t+1}, the rate charged on the central bank’s credit line, will also be negative, implying in a subsidy to banks through the credit line and therefore lower costs from the point of view of banks. We highlight that under the scenario with intervention, the economy remains under NIRP for close to 5 quarters, period during which the recovery of the economy is hindered by the slower recovery of bank's balance sheets and the failure of private banks to pass negative rates along. After nominal interest rates return to zero, the difference between the first-best scenario of no-ZLB whatsoever and a ZLB on R^d_{t+1}diminishes substantially - suggesting that giving a lot of credit under a moderate spread helps to approximate the restricted case to this first-best case. Where the NIRP policy to last longer, the hit to bank's net worth would be higher, and the recovery slower still, due to the growing costs associated with excess reserves.

Figure 1.8: NIRP and Central Bank Intervention



1.5 Conclusion

In this paper we put together a framework that would replicate the breakdown in policy transmission under NIRP as well as the rise in excess reserves of late and allow the study of such policies and its effects in the banking system after a shock. We find that the mild negative rates being adopted in some countries might be expansionary and a rational response to a very negative shock but, even in a rich DSGE model, as long as banks are not passing on to depositors this cost, it is as if these economies remain bounded by the ZLB. The credit lines that some Central Banks have extended since 2008 (TAF in the US, LTRO and etc in the EU) have acted to provide security in funding and liquidity to banks but, in a NIRP environment, have also implied growing costs for the banking system as heightened volumes of excess reserves developed. Our model suggests that rates could go well into negative territory and have the potential to be very helpful, but whether they will have an expansionary effect will depend on how long the NIRP are in place, whether these rates are passed on by banks to depositors, the level of excess reserves accumulated and the spreads charged by the CB both on the credit line and on excess reserves relative to the benchmark rate.

We caution though that this new use of the interest rate instrument has dimensions that are not deeply modeled in our work and which suggest that the levels found here should be viewed with caution. We assume throughout our model that the interbank market clears at the rate R^I_{t+1}, which is indeterminate in the model. But as R^I_{t+1} approaches the excess reserves interest rate R^E_{t+1}, this will have consequences in the interbank market that we do not attempt to model here and add an extra layer of complexity to ever more negative rates, be they in the benchmark central bank rate or in R^E_{t+1}. Rising exposure to risk might also entail a bigger drag on banks margins as time develops. On the other hand, we used a very low leverage ratio, following the calibration in Gertler and Karadi (2011), and therefore the model warns that the effects on banks presented in this paper could be even deeper.

One natural extension of our work is the study of the evolution of the heterogeneity in banks, and how it reacts under NIRP - in our set up, all banks are "re-normalized" each period to the same net capital in order to keep the model tractable. But since the funding profile and risk aversion of banks seems to change under NIRP, the systemic risk and effects induced by NIRP might be significantly higher than our model predicts and therefore the lower bounds higher than the model suggests.

Figure 1.9: Intervention scenarios, $\xi_t^f = 0.90$

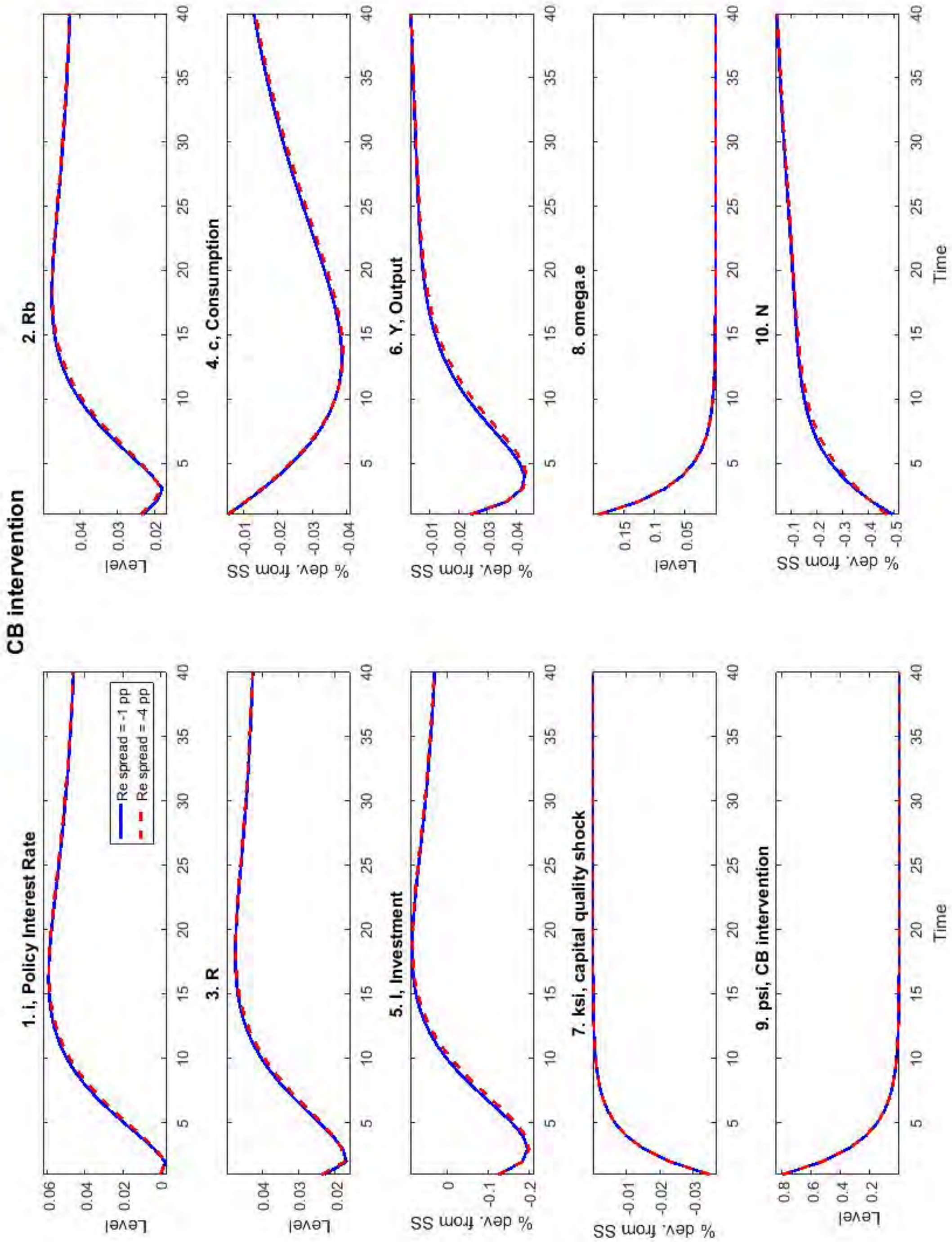


Figure 1.10: Intervention scenarios, $\xi_t^f = 0.75$, additional variables

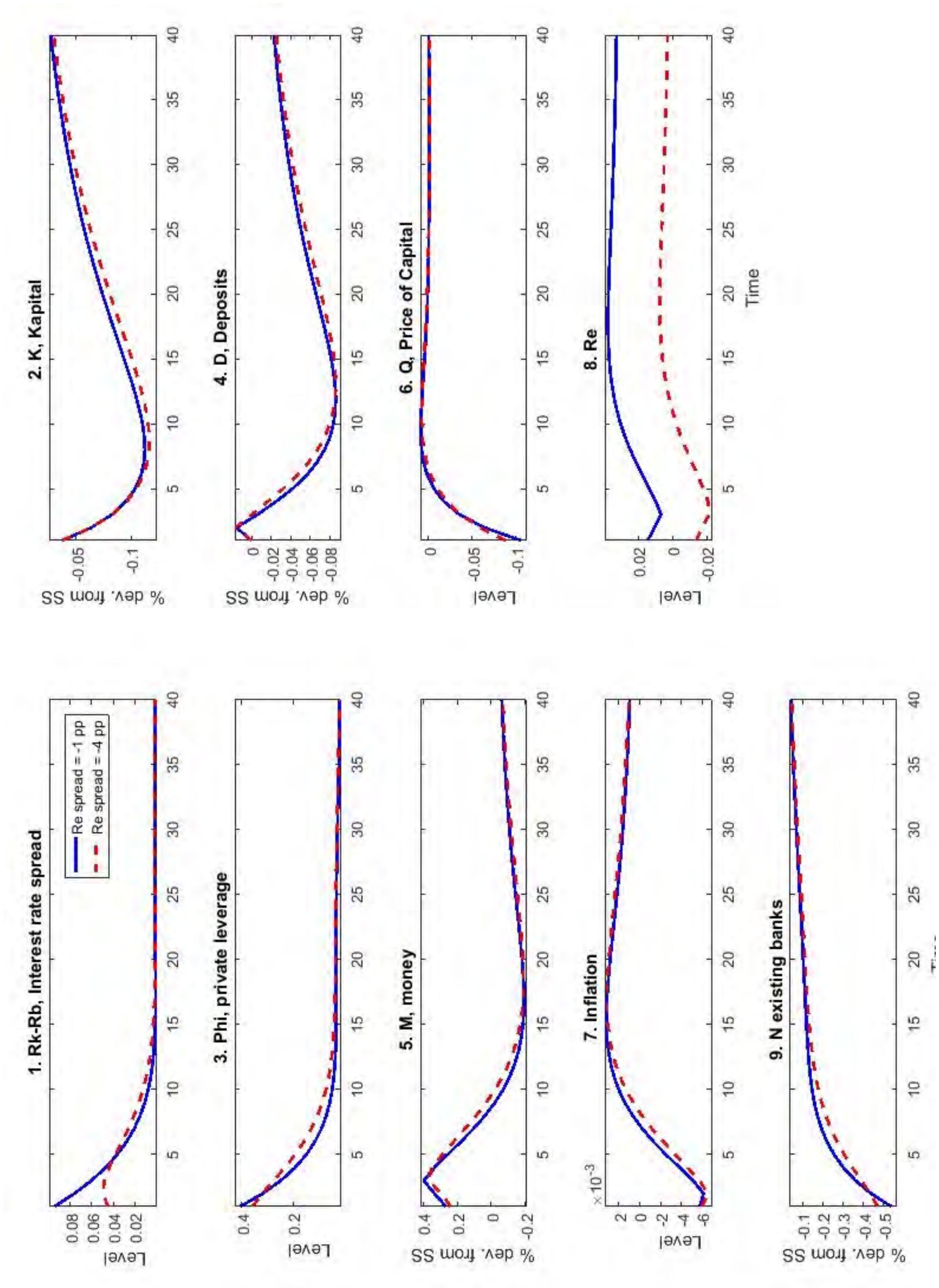
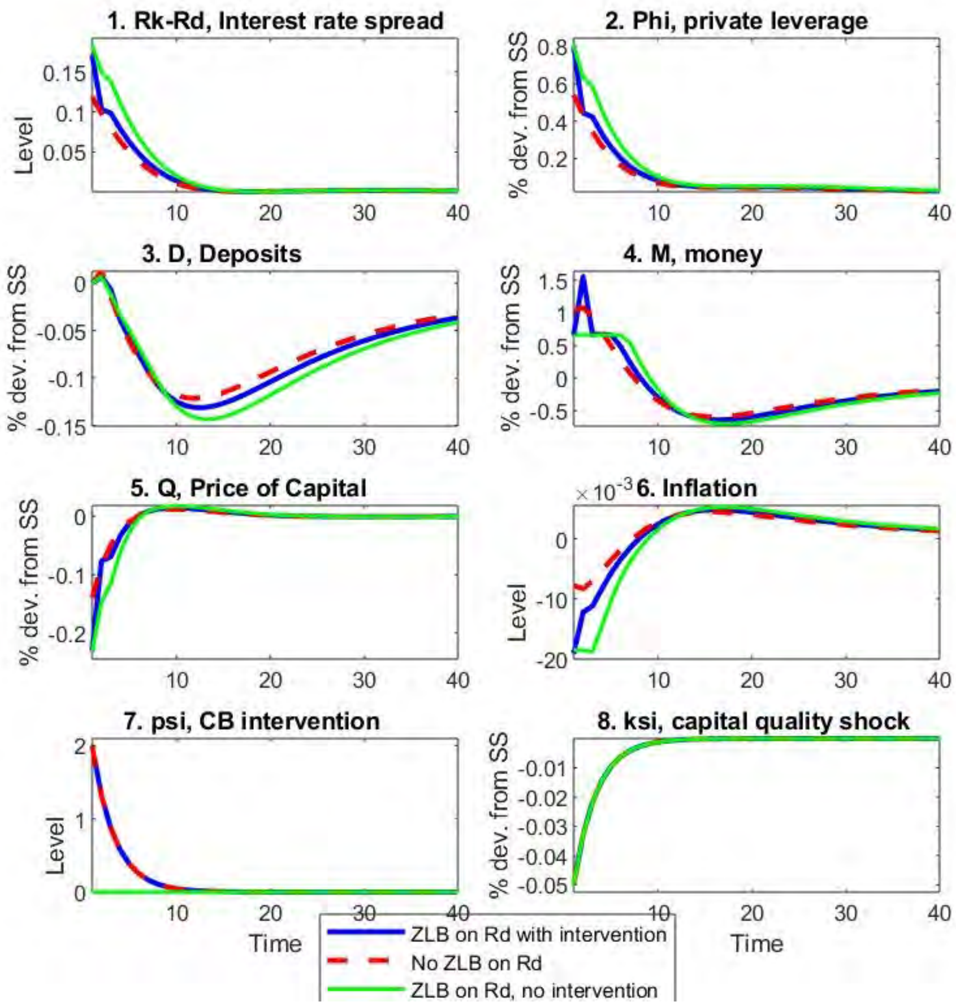


Figure 1.11: NIRP and Central Bank Intervention - additional variables



Chapter 2
CBDCs and NIRP: New tools for monetary policy

2.1
Introduction

> "It is with peculiar diffidence and even apprehension that one ventures to open one's mouth on the subject of money" - John Hicks

The past few years have witnessed a growing tendency towards digitalization in payments and monetary transfers, as well as the development of the distributed ledger technology, which allows the decentralization of trade settlements with improved security and private digital currencies in its tail. In this context, the issuance of private digital currencies such as Bitcoin and Ethereum has attracted increased attention and demand, posing additional concerns for policy makers and Governments. As a subproduct of these developments, and reflecting the fact that some countries are in the process of becoming effectively cashless economies, as in Sweden [1], has emerged the idea of issuance of Central Banks digital currencies (henceforth, CBDC) as a way to tackle not only the supply of a risk-free liquid asset in substitution for cash, but also as a way to improve efficiency in payment transactions and possibly monetary policy.

To be clear, as in Kumhoff, when we refer to CBDCs "(...) we refer to a central bank granting universal, electronic, 24x7, national-currency-denominated and interest-bearing access to its balance sheet" [2]. In other words, central banks would be extending to citizens the possibility of access to the monetary authority's balance sheet in the form of digital transactions, as is already usual for financial institutions, which can deposit their excess reserves at the central bank and receive interest on them, as well as exchange Government bonds for reserves in open market operations. This access would be in substitution of the general public's current access to the monetary authority's, which is through cash. An interest-bearing CBDC would mean a stable real value for the risk-free asset issued by the central bank. The

[1] see Riksbank (2017)
[2] Barrdear and Kumhof (2016), pg 3

positive implications from CBDCs range from gains in efficiency and costs in the payments systems (Barrdear and Kumhof (2016)) to better settlement and security in cross-border transactions (Leckow et al. (2017)), as well as better monitoring and rule enforcement of financial activities. On the other hand, CBDCs would be a more powerful competitor to private bank´s deposits, and also carry a lot of uncertainty around their implementation and demand (see BIS (2018)), especially during financial crises.

In another important development of the past decade, Central Banks have been experimenting with different tools of monetary policy, including bypassing the so called zero lower bound in interest rates (henceforth ZLB). Negative interest rates, usually depicted in economic textbooks as an impossibility due to the prospect of infinite demand for money, are now a reality in several countries due to different reasons, but mostly as a result of the continued effort to restart economic activity since the Great Recession (or contain currency depreciation). Since 2014, Denmark, Switzerland, Japan, Sweden and the Euro Area have experimented with negative interest rate policies (NIRP) of different flavors: while in Sweden and Switzerland the monetary policy benchmark interest rates are outright negative (-0.5% and -0.75% aa, respectively), in Denmark, Japan and the Euro Area benchmark interest rates are set at zero but the monetary authorities have steadily deepened rates on deposits at the Central Bank into negative territory, effectively setting a negative rate for these countries' interbank markets (deposit rates at -0.65%, -0.10% and -0.40% aa respectively at the time of writing).

Notwithstanding, these countries have not, so far, witnessed an explosion in money demand, which is the usual excuse for the existence of the ZLB, as money pays a zero nominal interest rate. Instead, while there has been some recovery in lending as Central Bank (CB) interest rates have transcended the ZLB (see Arteta et al. (2016)), a surprising phenomenon has occurred: commercial banks have not passed on to their deposit rates the fall into negativity of CB rates. Jobst and Lin (2016) document the different paths of policy rates and bank's lending and deposit rates for different NIRP countries, while Eggertsson et al. (2017) not only present the fact but also document a breakdown in the pass-through of monetary policy to lending rates in Sweden and other countries, as noticed previously in Heider et al. (2016). In a few countries such as France, negative interest rates on savings deposits are actually forbidden by law (as the bank must repay "at least" the sum deposit by the client), automatically capping the pass-through, but competition and regulation issues as well as costs to intermediation in an environment of falling spreads have been suggested as causes for this phenomenon (see ESBG

(2016)). As Figure 1.1 in the previous chapter has suggested, the paths of average deposit rates at credit institutions for non-financial corporations and households have not dropped as much as the relevant interest rates of national Central Banks - the “benchmark” interest rate and the deposit facility interest rate. It is evident how deposit rates, especially for households, have not droppped into negative territory - especially in places where the Central Bank has been most aggressive, such as Switzerland.

This fact therefore suggest that the transmission mechanism of monetary policy partially “breaks down” once NIRP are introduced, but now because of the non-responsiveness of private banks, effectively keeping in place the original ZLB problems and restrictions, despite the CB’s bold dive into negative interest rates. Therefore, adopting NIRP in an environment of limited pass-through to deposit rates actually means that the hurdles of NIRP remain in place, and diving further into the negative side of interest rates can actually hurt the recovery in the banking side rather than help - which we also tackled in Berriel and Guardado (2017). Eliminating cash and introducing some sort of liquid, interest-bearing risk free asset in its place, has been cited as one of the possible ways to circumvent the obstacles posed by the ZLB (Barrdear and Kumhof (2016); BIS (2018)), although such option hasn’t been explored in a model. Such an asset could, in particular, be a CBDC, in the spirit of the definition proposed above. In a sense, the introduction of CBDC in a NIRP environment would be tantamount to the nominal devaluation of cash proposed by Kimball and Agarwal (2013), with the added benefit of becoming a permanent monetary policy tool and bringing possible efficiency gains to the economy and policy making.

In this paper, we focus on how CBDCs might help to solve the ZLB problem, and study some of its properties as a monetary policy tool. We argue that introduction of CBDC could remove the obstacles from the point of view of banks to the pass-through of NIRP, turning this policy more effective, while providing the policy maker an additional tool for monetary policy. We find that while monetary policy could work in its usual manner if the central bank focuses only on its benchmark interest rate, changing the spread between the benchmark rate and the CBDC rate might not lead to reliable and expected results from the point of view counter-cyclical policy, due to wealth effects. Therefore, changes only to the CBDC rate, which would be equivalent to the usual rate paid on bank’s excess reserves, might not lead to the usual effects in the economy- a response that is bound to be stronger the higher is the volume of CBDC in the economy. We believe that this work helps to fill the theoretical and modeling space that surrounds the current debate on NIRPs, using a simpler framework than Barrdear and Kumhof (2016), while helping to study the levels and associated effects of such policies on the economy wide and on banks in particular. We do not aim to discuss the many subtleties surrounding the design and implementation of CBDCs, which are well tackled in Barrdear and Kumhof (2016) and Bordo and Levin (2017).

Our paper builds mainly on the literature of DSGE models with financial frictions and the ZLB, with main contributions from Kiyotaki and Moore (1997); Bernanke et al. (1999); Eggertsson and Woodford (2003); Woodford (2010); Christiano et al. (2005). We derive specially from Gertler and Karadi (2011), from where most of our model comes, as well as Gertler and Kiyotaki (2010), which develops a framework with financial frictions. Another important strand to which we relate has to do with the growing literature surrounding NIRP, with early contributions from Buiter (2009), Rogoff and Kimball and Agarwal (2013). But most of these papers discuss strategies for dealing with a heightened demand for paper currency in an environment of NIRP, a subject we avoid in our model through a constant semi-elasticity of demand, in line with the so far behaved response seen in NIRP countries. Rognlie (2016) shows that negative interest rates can be welfare enhancing, although not touching on the subject of the limited pass-through in the banking system. Arteta et al. (2016) discusses the developments witnessed after the adoption of NIRP. We point out that recently Eggertsson et al. (2017) also modeled the effects of NIRP on the economy and banking system in a simpler framework, with results similar to ours, and also documented the break in monetary transmission. Brunnermeier and Koby estimates a "reversal" interest rates which turns accommodative monetary policy contractionary, although such reversal rate might be either negative or positive. In Berriel and Guardado (2017) we develop a similar framework to study NIRP transmission under the accumulation of large excess reserves from central bank intervention.

The literature on CBDCs is more recent, and we single out the contributions of Barrdear and Kumhof (2016); BIS (2015,0). The first paper builds a large DSGE model with financial frictions aimed at studying the properties and dynamics both during the transition as well as in the new equilibrium in the event of an introduction of CBDC in the UK economy. The model is cashless and there isn´t therefore much discussion over the gains under NIRP. Bordo and Levin (2017) reviews the literature and discusses the desirable properties and design of a CBDC, also suggesting its benefits for policy around the ZLB. BIS (2018) discusses extensively the benefits and dangers of the adoption of CBDCs, with a focus on the impact on the banking side of the economy. Although it recognizes the possible benefits for

NIRP, the discussion is swift and without any modeling behind it[3]. Riksbank (2017) focuses on the designs of a possible digital currency in Sweden.
This paper is organized as follows: section 2.2 presents the model; section 2.3 turns to the model's calibration, while section 2.4 reports several scenarios and the model's results; section 2.5 concludes.

2.2 The Model

The model closely follows Gertler and Karadi (2011), but we make three important distinctions: First, we introduce money following the strategy in Schmitt-Grohe and Uribe (2004); secondly, we transfer the ZLB from CB interest rates to the banking sector, creating a non-linearity in interest rates paid by the banking sector on time and saving deposits; and third, we introduce the interest rate that CBs usually pay on excess reserves, and later also on the CBDC .

2.2.1 The Benchmark model

2.2.1.1 Households

There is a continuum of identical households of measure unity. Each household is at any moment in time composed of two types of agents: a percentage f of bankers, which manage financial intermediaries and transfer their profits back to the household, who owns the banks; and a (1-f) percentage of workers, which supply labor (L_t) to firms and kick back to the household any wages it earns. Each household holds its savings in banks not owned by it. In each period, bankers face a probability θ, independent of history, of remaining bankers, and a $(1-\theta)$ chance of becoming workers, in which case they transfer the financial intermediary's net worth back to the household. As pointed in Gertler and Karadi (2011), $(1-\theta)\,f$ bankers become workers every period, being replaced by a similar amount of randomly chosen workers - therefore, proportions remain constant throughout time.
Following Schmitt-Grohe and Uribe (2004), the household is subject to proportional transaction costs $s(\nu_t)$ during the purchase of the consumption bundle, reflecting, as is usual in the literature, the services provided by money in goods's transactions. But, not only does it facilitate transactions: money also might serve as a substitute for time deposits, especially when

[3] see page 12 of the report

nominal interest rates are negative. Since it pays a zero nominal interest rate, it will seldom be the preferred means of saving in this model under positive interest rates, but below the ZLB its demand might increase substantially.

$$v_t = \frac{C_t}{m_{t+1}} \tag{2-1}$$

Where the transaction cost technology's functional form is:

$$s(v_t) = Av_t + \frac{B}{v_t} - 2\sqrt{AB}$$

There exists a satiation level $m*$ such that transaction costs are zero - so that $s(\bar{\nu}) = s(\nu(C, m*)) = s'(\bar{\nu}) = 0$.

Households can choose to save through time/savings deposits D_t in the financial intermediaries or by holding real money balances m_t - which pays zero interest rate. Our definition of money includes both cash (whose representation in actual economies is quite small) and checking deposits - liquid non-interest bearing assets, while D_t would be more related to all sorts of short-term interest-paying funding by banks. Time and savings deposits are paid the gross real interest rate R^d_{t+1}, which is equal to the real risk free interest rate set by the Central Bank R_{t+1}:

$$R^d_{t+1} = R_{t+1} \tag{2-2}$$

Therefore, the household problem is:

$$\max_{C_t, D_{t+1}, M_{t+1}, L_t} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right] \right\} \tag{2-3}$$

subject to the budgetary restriction:

$$C_t\left(1 + s(\nu_t)\right) = (1 - \tau_l)\, W_t L_t + \frac{m_t}{\pi_t} + R^d_t D_t - m_{t+1} - C(\frac{m_t}{\pi_t}) - D_{t+1} - T_t + profits_t$$

where T_t are taxes net of transfers, π_t is inflation accumulated in the period and all variables are in real terms. The term $C(\frac{m_t}{\pi_t})$ represents the cost of holding money, as in Eggertsson et al. (2017), and effectively set the lower bound in interest rates from the consumer side. As in that paper, we will consider that this cost is different, but close to zero, and that it is proportional to the volume of real money holdings, such that $C(\frac{m_t}{\pi_t}) = \alpha_m \frac{m_t}{\pi_t}$.

From the FOCs, we get:

$$\frac{1}{C_t - hC_{t-1}} - \frac{\beta h}{C_{t+1} - hC_t} = \lambda_t \tag{2-4}$$

$$\frac{\chi}{\lambda_t} L_t^{\varphi} = W_t \tag{2-5}$$

$$E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\beta R^d_{t+1}\right\} = 1 \tag{2-6}$$

$$m^{-2}_{t+1} = \frac{1}{AC^2_t}\left[B + 1 - \frac{(1-\alpha_m)}{i^d_{t+1}}\right] \tag{2-7}$$

Where λ_t is the Lagrange multiplier of the maximization problem and i^d_{t+1} is the nominal deposit interest rate, which is usually equal to i_{t+1}, except when under NIRP. Define:

$$\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \tag{2-8}$$

Notice that, if B approaches zero, cost are linear in ν_t and the demand for money becomes the usual square-root money demand with respect to the opportunity cost of holding real cash balances $({}^{i_t-1-\alpha}/{}_{i_t})$, similar to the formats obtained when using money in the utility function[4].

We will follow Christiano et al. (2005) and rewrite 2-7 by log-linearizing it around its steady state value m^*, where transaction costs are zero and steady state interest rate $\bar{R}$ equals ${}^1/{}_\beta$. We get:

$$ln(m_{t+1}) = ln(m^*) + \hat{C}_t - \frac{(1-\alpha_m)\beta\hat{i}^d_{t+1}}{2\left[B+1-\beta(1-\alpha_m)\right]} \tag{2-9}$$

where all variables with a hat denote deviations from steady state values. From 2-9, we have that the semi-elasticity of money demand to changes in the interest rate is:

$$\frac{\partial ln\,(m_{t+1})}{\partial \hat{i^d}_{t+1}} = -\frac{(1-\alpha_m)\beta}{2\left[B+1-\beta(1-\alpha_m)\right]} \tag{2-10}$$

Equation 2-10 tells us that a 1 percentage point change in i^d_{t+1} relative to its steady-state level will lead to a $-\frac{(1-\alpha_m)\beta}{B+1-\beta(1-\alpha_m)}\%$ change in money demand relative to the optimum level m^*, as noted in Rognlie (2016).

When i_{t+1} dips into negative territory, the bank's nominal deposit rates remains at zero, as a strategy by the financial intermediary to contain losses of deposits when rates become negative - banks fear that they might be out of funding should households divert savings massively towards money, which pays zero, and therefore they will have lower profits. Therefore, real interest rates will amount to the negative of the period's inflation, π_t :

$$R^d_{t+1} = \begin{cases} R_{t+1} & if\ i_t \geq 0 \\ -\pi_t & if\ i_t < 0 \end{cases} \tag{2-11}$$

[4]see Christiano et al. (2005)

2.2.1.2
Financial Intermediaries

The financial intermediaries are a unit mass continuum of bankers, each running an individual banks. Let N_{jt} be the net worth of intermediary j at the end of period t; S_{jt} its quantity of financial claims on non-financial firms (or assets on its balance sheet); and Q_t the relative price of each claim. D_{jt+1} is the long term deposits from households in intermediary j (since checking deposits are equivalent to money in the model), which earn the non-contingent gross real rate R^d_{t+1} at t+1, and make up the liabilities side of the intermediary's balance sheet. Remember that, from 2-11, this rate is equal to R_{t+1} if i_{t+1} is non-negative. Banks participate in an interbank market for reserves, and can therefore even out differences in financing needs that might eventually arise. The following equality must hold in the intermediary´s balance sheet at each period:

$$Q_t S_{jt} = N_{jt} + D_{jt+1} - Ex_{jt+1} + b_{jt+1} \tag{2-12}$$

Ex_{jt} is the amount of excessive reserves , which are determined endogenously at each period t. A financial intermediary might choose to hold excessive reserves at times when the return on assets is momentarily below the rate paid by the Central Bank on reserves, or for precautionary reasons. At the beginning of each period t, aggregate deposits become available by the representative household (D_{t+1}). The amount of investment projects (S_t) in the economy are disclosed in a competitive market with the same stochastic rate of return of R_{kt+1}, which will be determined endogenously.

The intermediary's net worth will change over time as a result of the different paths on its assets and liabilities sides, as a result of the different returns on them:

$$N_{jt+1} = R_{kt+1} Q_t S_{jt} - R^d_{t+1} D_{jt+1} + R^E_{t+1} Ex_{jt} \tag{2-13}$$

Notice that, in the usual situations where the return on assets is higher than the cost of liabilities, banks will wish to lend as much as they can, or most of their deposits - up to the maximum permitted by the regulatory leverage ratio. As in Gertler and Karadi (2011), we define: $\beta^i \Lambda_{t,t+i}$ as the stochastic discount of the banker at time t over earnings in the future t+i. Given the non-contingent cost R_{t+1} that if faces on its liabilities, banks will not fund asset that earn less than said rate on a discounted basis. Therefore, the bank's condition to operate in period i is:

$$E_t \left\{ \beta^i \Lambda_{t,t+1+i} (R_{kt+1+i} - R^d_{t+1+i}) \right\} \geq 0 \tag{2-14}$$

While the risk adjusted return on assets is larger than the cost of its funding, it pays for the intermediary to build assets, as long as it remains in the industry. The banker's objective is to maximize it expected terminal wealth, given by:

$$V_{jt}\left(S_{jt-1}, Ex_{jt}, D_{jt}\right) = \underset{N_{jt+1+i}}{max}\; E_{t-1}\left\{\textstyle\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^{i}\beta^{i+1}\varLambda_{t,t+1+i}N_{jt+i}\right\} =$$
$$= \max E_{t-1}\varLambda_{t,t-1}\beta E_t\left\{\left(1-\theta\right)N_{jt} + \theta\underset{\psi_{jt}}{\max}\left[\underset{s_{jt},b_{jt}}{\max}V_{jt}\left(S_{jt}, Ex_{jt}, D_{jt+1}\right)\right]\right\} \tag{2-15}$$

As in Gertler and Karadi (2011), we assume that the intermediary can divert a fraction λ of its assets, but not the funds borrowed from the interbank market. Such a possibility will limit the amount of net capital that each financial intermediary can extract from the representative household. The intermediary will refrain from diverting assets as long as:

$$V_{jt} \geq \lambda\left(Q_t S_{jt} - b_{jt}\right) \tag{2-16}$$

Equation 2-16, which imposes a limit to much funds financial intermediaries can obtain from households, will pose the main constraint in banking activity. From the bank's optimal behavior, we can rewrite equation 2-15 as:

$$V_{jt} = \upsilon_t Q_t S_{jt} + \eta_t N_{jt+1} \tag{2-17}$$

where:

$$\upsilon_t = E_t\left\{(1-\theta)\beta\varLambda_{t,t+1}\left(R_{kt+1} - R^d_{t+1}\right) + \theta\beta\varLambda_{t,t+1}x_{t+1}\upsilon_{t+1}\right\} \tag{2-18}$$

$$\eta_t = E_t\left\{(1-\theta)\beta\varLambda_{t,t+1}\left(R_{kt+1} - R^d_{t+1}\right) + \theta\beta\varLambda_{t,t+1}z_{t+1}\upsilon_{t+1}\right\} \tag{2-19}$$

$$x_{t+1} \equiv \frac{Q_{t+1}S_{t+1}}{Q_t S_t} \tag{2-20}$$

$$z_{t+1} \equiv \frac{N_{jt+1}}{N_{jt}} \tag{2-21}$$

And the bank's asset allocation will respect::

$$Q_t S_{jt} = \phi_{jt} N_{jt} \tag{2-22}$$

where:

$$\phi_{jt} = \frac{\eta_t}{\lambda - \upsilon_t} \tag{2-23}$$

Notice that all banks will choose in the middle of period t the same ex-ante leverage ratioϕ_{jt}. Since is not made up of any bank specific parameter, we can sum up over the whole industry to find the optimal ex-ante relationship between assets and net worth:

$$Q_t S_t = \phi_t N_t \tag{2-24}$$

Where $\phi_t = \phi_{jt}$ is the aggregate banking sector leverage ratio and S_t is the total amount of banking assets. We can therefore rewrite equations 2-13 and 2-20 as:

$$N_{jt+1} = \left[\left(R_{kt+1} - R^d_{t+1}\right)\phi_t + R^d_{t+1}\right] N_{jt} \tag{2-25}$$

$$x_{t+1} = \frac{\phi_{t+1} N_{jt+1}}{\phi_t N_t} = \left(\frac{\phi_{t+1}}{\phi_t}\right) z_{t+1}$$

And it follows that $z_{t+1} = \left(R_{kt+1} - R^d_{t+1}\right)\phi_t + R^d_{t+1}$. Total bank net worth in the economy will be a result of the net worth of surviving banks (N_{et}) plus the net worth of entering banks (N_{nt}) in each period: $N_t = N_{nt} + N_{et}$, where

$$N_{et} = \theta\left[\left(R_{kt} - R^d_t\right)\phi_{t-1} + R^d_t\right] N_{t-1}$$

The fraction of banks leaving intermediation and becoming workers is $1-\theta$, which is aimed at a constant net worth in the steady state, and implies an exit of $(1-\theta)\, Q_t S_{t-1}$ assets from the economy in each period. Households a fraction $^{\omega}/_{(1-\theta)}$ of those assets to entering intermediaries as initial capital, so that, in aggregate:

$$N_{nt} = \omega Q_t S_{t-1}$$

and therefore we can rewrite equation 2-25 for the aggregate of the economy as:

$$N_t = \left[\left(R_{kt} - R^d_t\right)\phi_{t-1} + R^d_t\right] N_{t-1} + \omega Q_t S_{t-1} \tag{2-26}$$

2.2.1.3
Intermediate Goods Firms

In the productive side of the economy, the setup is quite standard: competitive non-financial firms produce intermediate goods which will then be sold to and repackaged by retail firms. In order to produce in each period t, intermediate goods firms use labor L_t and capital K_t, which is obtained from capital producing firms (next section) in period t-1 and funded through the issuance of financial asset S_t to financial intermediaries. Once production in period t is over, the firm can either keep this capital or sell it in the open market. Firms obtain financing for the purchase of capital by issuing assets S_t such that:

$$Q_t K_{t+1} = Q_t S_t \tag{2-27}$$

Production technology:

$$Y_t = A_t (U_t \xi_t K_t)^{\alpha} L_t^{1-\alpha} \tag{2-28}$$

Where At is total factor productivity and ξ_t is the quality of capital. Call Pm_t the intermediary good´s price. So, at each period t, the firm will choose L_t and U_t according to:

$$Pm_t \alpha \frac{Y_t}{U_t} = \delta'(U_t) \xi_t K_t \tag{2-29}$$

$$Pm_t (1-\alpha) \frac{Y_t}{L_t} = W_t \tag{2-30}$$

Firms earn zero profits: ex-post return to capital payed out to the intermediary. Replacement price of capital depreciated is 1, so the capital stock left over at the end of t+1 is $(1-\delta(U_{t+1}))\xi_{t+1}K_{t+1}$. Therefore, the debt repayment, $Rk_{t+1}K_{t+1}Q_t$, which is the value borrowed at t plus the interest, should equal the return of capital to the firm (the marginal product of capital plus the value of capital after depreciation). So:

$$Rk_{t+1}K_{t+1}Q_t = Pm_{t+1}\alpha Y_{t+1} + Q_{t+1}\xi_{t+1}K_{t+1} - \delta(U_{t+1})\xi_{t+1}K_{t+1}$$
$$\Rightarrow Rk_{t+1} = \frac{1}{Qt}\left[\frac{Pm_{t+1}\alpha Y_{t+1}}{K_{t+1}} + Q_t\xi_{t+1} - \delta(U_{t+1})\xi_{t+1}\right] \tag{2-31}$$

2.2.1.4 Capital Producing Firms

At the end of each period t, competitive capital producing firms buy depreciated capital and refurbish it (at the cost of 1 per unit) or make new capital. The value of one unit of new capital is Q_t. Call I_t the gross capital created, I_{ss} its value in the steady-state, and In_t the net capital created, or:

$$In_t \equiv I_t - \delta(U_t)\xi_t K_t$$

Then the problem of this firm is to maximize its discounted profits:

$$Max\ E_t\left\{ \overset{[}{\tau}= t]\infty \sum \beta^{\tau-t}\Lambda_{t,\tau}\left\{(Q_\tau - 1)In_\tau - f\left(\frac{In_\tau + Iss}{In_{\tau-1}+Iss}\right)(In_\tau + Iss)\right\}\right\}$$

Gertler and Karadi (2011) use $f\left(\frac{In_t+Iss}{In_{t-1}+Iss}\right)=\frac{\eta_i}{2}\left(\frac{In_t+Iss}{In_{t-1}+Iss}-1\right)^2$ in their calculation, which we follow. From the above, we get the "Q relation" for net investment:

$$Q_t = 1 + f(.) + f'\left(\frac{In_t + Iss}{In_{t-1} + Iss}\right) \cdot \left(\frac{In_t + Iss}{In_{t-1} + Iss}\right) - E_t\left\{\beta\Lambda_{t,t+1}\, f'\left(\frac{In_{t+1} + Iss}{In_t + Iss}\right) \cdot \left(\frac{In_{t+1} + Iss}{In_t + Iss}\right)^2\right\} \quad (2\text{-}32)$$

2.2.1.5
Retail Firms

Retail firms repackage the goods from intermediate goods firms to create the final consumption good Y_{ft}. The final output is produced according to a CES composite:

$$Y_t = \left([0]1\int Y_{ft}^{(\varepsilon-1)/\varepsilon} df\right)^{\varepsilon/\varepsilon-1} \quad (2\text{-}33)$$

Where:

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\varepsilon} Y_t \quad (2\text{-}34)$$

$$P_t = \left[\int_0^1 P_{ft}^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \quad (2\text{-}35)$$

The marginal cost of the retail firm is the price of the intermediate goods, Pm_t . We follow Gertler and Karadi (2011) and assume nominal price rigidities as in the Christiano et al. (2005) paper: each firm faces, in each period, a probability $1-\gamma$ of adjusting its price, and, in between periods of price adjustment, they index their prices to lagged inflation. The retailers pricing problem is:

$$\underset{P_t^*}{Max}\ E_t\left\{[i=0]\infty\sum\gamma^i\beta^i\Lambda_{t,t+i}\left[\frac{P_t^*}{P_{t+i}}[K=1]i\prod\left(1+\pi_{t+k-1}\right)^{\gamma p} - Pm_{t+i}\right]Y_{f_{t+i}}\right\}$$

The FOC is:

$$E_t\left\{[i=0]\infty\sum\gamma^i\beta^i\Lambda_{t,t+i}\left[\frac{P_t^*}{P_{t+i}}[K=1]i\prod\left(1+\pi_{t+k-1}\right)^{\gamma p} - \mu Pm_{t+i}\right]Y_{f_{t+i}}\right\} = 0 \quad (2\text{-}36)$$

where :

$$\mu = \frac{1}{1-1/\varepsilon}$$

Which results in the equation for the evolution of the price level, using the law of large numbers:

$$P_t = \left[(1-\gamma)\left(P_t^*\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma p} P_{t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{2-37}$$

2.2.1.6 Resource constraint, Fiscal and Monetary Policies

The economy's resource constraint is given by

$$Y_t = C_t + I_t + f\left(\frac{In_t + Iss}{In_{t-1} + Iss}\right)(In_t + Iss) + G + \alpha_m \frac{m_t}{\pi_{t+1}} + s(\nu_t)C_t \tag{2-38}$$

where G are Government expenditures, which we assume exogenously fixed, and the last term represents the costs associated with cash. Capital evolves according to:

$$K_{t+1} = \xi_t K_t + In_t \tag{2-39}$$

The government's budget constraint is given by:

$$G_t + \frac{m_t}{\pi_{t+1}} = T_t + \tau_t w_t L_t + m_{t+1} \tag{2-40}$$

We maintain the original model's simple Taylor rule with an interest rate smoothing parameter $\rho \epsilon [0,1]$. Let i_t be the nominal net interest rate, and i its steady state value (which is equal to $1/\beta$ and implies a zero inflation rate) and ε_t an exogenous shock to monetary policy. Then the interest rate rule followed by the Central Bank is:

$$i_t = (1-\rho)\left[i + \kappa_\pi \pi_t + \kappa_y(logY_t - logY_t^*)\right] + \rho i_{t-1} + \varepsilon_t \tag{2-41}$$

Where Y_t^* is the level of output in the flexible price equilibrium. Real interest rates are linked to i_t by the Fisher equation:

$$1 + i_t = R_{t+1}\frac{E_t P_{t+1}}{P_t} \tag{2-42}$$

2.2.2 CBDC regime

We consider a scenario after the full transition to a CBDC regime[5], where the digital currency has fully substituted cash[6]. The household problem therefore becomes:

$$\max_{C_t, D_{t+1}, M_{t+1}, L_t} E_t\left\{\sum_{t=0}^{\infty}\beta^t\left[\ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right]\right\} \tag{2-43}$$

[5]We assume that CBDCs do not hold every property that deposits might have. For example, households are not able, to receive pay

[6]For dynamics during the transition period and with different functional forms for transaction costs under the new regime, please see **?**

subject to the budgetary restriction:

$$C_t\left(1+s(\nu_t)\right)=\left(1-\tau_l\right)W_tL_t+R_t^e dc_t+R_t^d D_t-dc_{t+1}-D_{t+1}-T_t+profits_t$$

where now ν_t is:

$$v_t=\frac{C_t}{dc_{t+1}} \tag{2-44}$$

And R_{t+1}^e is the rate paid on CBDC deposits, which will usually be below R_{t+1} due to a negative spread τ_t, and subject to monetary policy shocks ε_t^e:

$$R_{t+1}^e=R_{t+1}-\tau_t+\varepsilon_t^e$$

After maximization, only equation 2-7 changes, becoming:

$$dc_{t+1}^{-2}=\frac{1}{AC_t^2}\left[B+1-\frac{R_{t+1}^e}{R_{t+1}^d}\right] \tag{2-45}$$

Notice that, if B approaches zero, cost are linear in ν_t and the demand for digital currency becomes:

$$dc_{t+1}^{-2}=\frac{1}{AC_t^2}\left[\frac{R_{t+1}^d-R_{t+1}^e}{R_{t+1}^d}\right]$$

and in that case, if $R_{t+1}^e \rightarrow R_{t+1}^d \Rightarrow dc \rightarrow \infty$. Otherwise (if 0 < B <1), if $\frac{R_{t+1}^e}{R_{t+1}^d} \rightarrow 1+B \Rightarrow dc \rightarrow \infty$. But that would mean that $R_{t+1}^e > R_{t+1}^d$, which wouldn't make economic sense since the central bank would be deliberately making CBDC more attractive and in effect diminishing deposits and intermediation in the economy. So, while $R_{t+1}^e \leq R_{t+1}^d$ (when 0 < B <1), dc_{t+1} demand is finite.

Equation 2-9 transforms into:

$$log(DC_{t+1})=log(DC^*)+\hat{C}_t-\frac{\beta\bar{R}^e}{2\left(1+B-\beta\bar{R}^e\right)}\left(\hat{R}_{t+1}^d-\hat{R}_{t+1}^e\right) \tag{2-46}$$

where all variables with a hat denote deviations from steady state values. From 2-46, we have that the semi-elasticity of money demand to changes in the interest rate *spread* is:

$$\frac{\partial ln\left(dc_{t+1}\right)}{\partial\left(\hat{R}_{t+1}^d-\hat{R}_{t+1}^e\right)}=-\frac{\beta\bar{R}^e}{2\left(1+B-\beta\bar{R}^e\right)} \tag{2-47}$$

The Government's budget constraint incorporates the cost of CBDC emission and remuneration:

$$G_t+R_t^e dc_t=T_t+\tau_t w_t L_t+dc_{t+1} \tag{2-48}$$

Notice that our steady state and inflation rule implies a zero inflation rate target. That alleviates the possible dilemma of switching from a positive inflation target to a zero inflation target once in the CBDC regime, as the abolition of the ZLB would eliminate the need for a positive inflation target[7].

2.2.3 Equilibrium and steady-state

The equilibrium is comprised of the sequences $\{C_t\}$, $\{L_t\}$, $\{m_t\}$, $\{dc_t\}$, $\{W_t\}$, $\{K_t\}$, $\{\lambda_t\}$, $\{Y_t\}$, $\{P_t\}$,$\{S_t\}$, $\{N_t\}$, $\{D_t\}$, $\{\phi_t\}$,$\{Q_t\}$, $\{I_t\}$, $\{Rk_t\}$, $\{R_t\}$ and $\{i_t\}$ for t = 1,....,∞ such that equations 2-1, 2-5, 2-9, 2-11,2-12 ,2-29, 2-30, 2-31, 2-32, 2-37, 2-42, 2-46 and the household's and Government's budget constraints are respected, and the outputs and labor markets clear.

In the steady-state equilibrium, the model boils down to the original Gertler and Karadi (2011) steady-state framework. All banks are equal at all points in the steady-state.

2.3 Calibration

Most of our parameters are taken from the original Gertler and Karadi (2011) model, with the exception of A and B, which we follow from Schmitt-Grohe and Uribe (2004). These values imply m_t at 20% of GDP, which is somewhat higher than the M1 measure for the US. Note that the benchmark interest rate R_t is 4% aa, while R_{kt} is 5% aa. On our benchmark calibration, we set τ_t, the difference between the benchmark nominal interest rates and the nominal rate paid by the BC on excess reserves, to 1 pp - above what is current practice in countries such as the US and the EU, but close enough to the 75 bps spread used in the recent past by the ECB. The steady state value of Government expenditures is 20% of GDP.

2.4 Scenarios

For estimation of the scenarios, given the non-linearity of the ZLB on deposit rates, we use the methodology developed by Guerrieri and Iacoviello (2015), OccBin. Sections 4.1 and 4.3 start with a negative shock of 5% in capital quality (ξ_t), as in Gertler and Karadi (2011), which is necessary to drive the economy into a recession compatible with the need to adopt a NIRP.

[7] see Bordo and Levin (2017)

Table 2.1: Parameters

Parameter	Description	Value
β	Consumer discount rate	0.99
h	habit parameter	0.815
χ	Relative utility weight of labor	3.409
φ	Inverse Frisch elasticity of labor	0.276
λ	Share of assets that can be diverted	0.381
ω	Transfer to new banks	0.005
θ	Bank's survival rate	0.957
α	Effective capital share	0.33
$\delta(U)$	Steady state depreciation rate	0.025
ζ	Elasticity of depreciation to utilization rate	7.200
η_i	Inverse elasticity of net investment to Q	1.728
ε	Elasticity of substitution	4.167
γ	Probability of fixed prices each period	0.779
γ_p	Measure of price indexation	0.241
κ_π	Inflation coefficient in the Taylor rule	1.50
κ_y	Output gap coefficient in the Taylor rule	0.125
ρ_i	Smoothing parameter of the Taylor rule	0.0
α_m	Proportional cost of holding money	0.01%
A	Parameter of $s(\nu_t)$	0.0111
B	Parameter of $s(\nu_t)$	0.07524

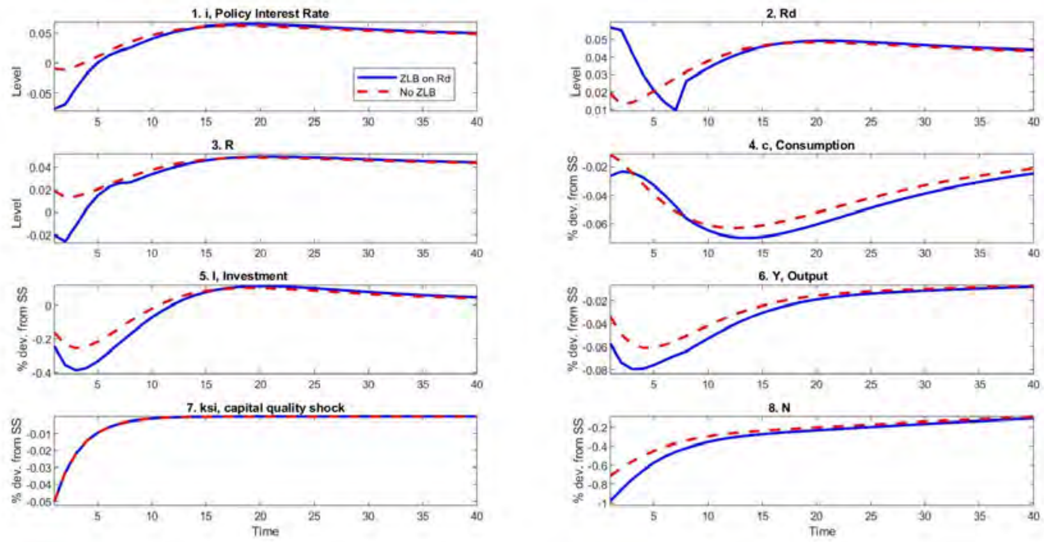
Table 2.2: Steady State values

i_t	R_{kt}	m_t	D_t	G_t	L_t	τ_t	ξ_t
4%	5%	0.20	4.246	0.20	0.333	1 pp	1.0

2.4.1 Different ZLBs

In this section, we compare the model´s predictions to the 5% shock in capital quality, that would drive the economy into a recession. The loss of output reaches 8% in the first year, and the recession lasts for over 20 quarters, because of the failed transmission of monetary policy as nominal interest rates paid on deposits by private banks are stuck at zero. Central bank nominal interest rates reach -5% aa in the first year, as a result of the stronger recession and deflation that arise because nominal interest rates in the banking system do not go below 0%, and therefore the real interest rate paid by banks in long term deposits rise substantially. Rates then reach zero by the second year. Notice that, if unconstrained by any ZLB, the policy maker would need to set rates only slightly negative over the first year - around 1% aa, returning to zero before year end and implying a recession through 2 pp above the constrained scenario.

Figure 2.1: Usual ZLB vs ZLB on Deposit rates

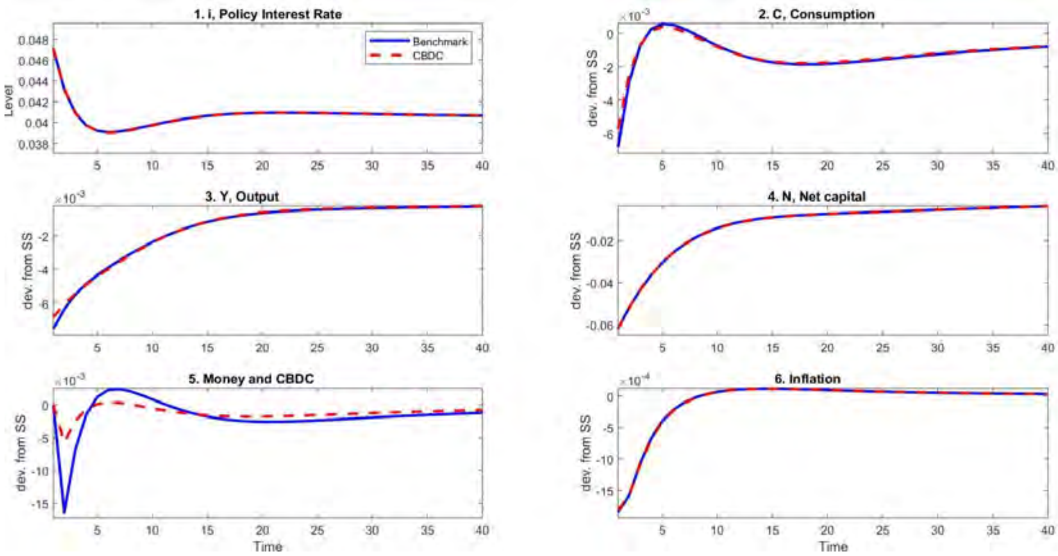


As also noted in Berriel and Guardado (2017) and Eggertsson et al. (2017), the macroeconomic results are the same as the ones that would be observed under the usual ZLB (in i_t) . In this simpler model, with no Government bonds and yield curve, monetary policy transmission channels are reduced to the bank lending, savings and Q channels. But we would note that an aggressive policy maker, driving the economy temporarily into negatives rates as low as 5% aa as suggested by the model, could induce stronger responses in expectations about the future from agents, as well as in (lower) longer term bonds and risk premium, stoking more risk taking in lending and incentives for consumption. On the other hand, such developments would mean diminishing returns on bank's activity and net capital, as margins would be thinner, inducing a slower recovery of the banking system - especially if banks remain constrained on how much to pay for their funding. So, while a ZLB in central banks or private banks could roughly be the same thing from the point of view of the model, we believe it could imply in a better recovery path when compared to the usual ZLB due to the additional support to risky assets prices.

2.4.2 Usual monetary policy shock

Our model incorporates an additional interest rate to the standard monetary policy toolkit, namely, the CBDC rate R^e_{t+1}, which now impacts directly asset choices of the households. In this section we test the two regimes (benchmark and CBDC) after two different monetary policy shocks, during times of "normal" monetary policy (meaning far from the ZLB). The first shock is the traditional shock to the benchmark interest rate, of 1 pp for only quarter. But due to the persistence in the Taylor rule, higher - although declining- rates persist for one year. Differences between the benchmark and CBDC regimes are limited to the different reactions of money and CBDC, and for consumption and output, which are slightly lower in the benchmark regime. The deeper decline in the last two variables are due to the costs of holding money associated with higher transactions costs, as money declines as much as 1.6%, while CBDC demand declines by a third of that amount.

Figure 2.2: IRFs of a 1 p.p. shock to i_t



The second exercise comprises the response to changes to the rate paid by the central bank on the CBDC, R^e_{t+1} , of close to 1 pp (annualized) for only one quarter - again, starting from the equilibrium level of 4% aa and well away from the ZLB. This rate would determine the floor in the market for reserves in the current monetary policy framework (our benchmark model), but most importantly, it determines the relative demand between CBDC and deposits. As figure 2.3 shows, this shock would imply a rise in demand for CBDC of close to 2% relative to steady state levels, that would even out over 2 years. Deposits would barely move, falling only 0.02%, reflecting the much higher volume relative to the steady state level of CBDC (close to 20x). Still, this shock would lead to some loss of investment in this economy (-0,06%) as both deposits and bank's net capital are lower for a long period of time. But, since there is a substitution effect between deposits and CBDC, the tightening of monetary policy induced by CBDC is actually reversed: consumption and output witness a slight increase over the first year, of respectively 0.15% and 0.1% over the steady state values, as a result of the higher return on the household's existing holdings of digital currency, while investment declines slightly as the available amount of deposits for intermediation decreases. The savings decision is not impacted, only the portfolio choice between deposits and CBDC holdings. In this model, the wealth effect dominates when the tightening is done exclusively by higher interest rates paid on holdings of the digital currency, suggesting that the manipulation of the deposit facility (and CBDC) interest rate alone might not be a reliable counter cyclical instrument in a CBDC regime. The small magnitudes of movements in this exercise reflect not only the small rise in interest rates but also the very short duration of the shock.

Although Barrdear and Kumhof (2016) do acknowledge that the countercyclicality of CBDC policy is subject to its design[8], their results do not show such a strong wealth effect, possibly due to the existence of the liquidity taxes in their model. After a credit risk shock followed by discretionary policy, in their model investment reacts faster as the spread narrows. To be fair, our exercise is different from theirs, in the sense that we are looking exclusively at a discretionary shock to the spread of the excess reserves rate to the benchmark rate, which is already endogenous in their model. Still, our results, in a much simpler framework, point to the importance of the design of the CBDC regime as well as the less reliable nature of it as a countercyclical tool.

2.4.3 NIRP in the CBDC regime

While Bordo and Levin (2017) argues that the introduction of a CBDC would actually make the central bank's balance sheet more transparent and eliminate the need of policies such as QE (because the monetary accommodation could now come exclusively from the fall in interest rates), Barrdear and Kumhof (2016) go in the opposite direction and argue that CBDC would allow for more efficient central bank interventions in the market for reserves and therefore the need for NIRP would be diminished.

Figures 2.4 and 2.5 present the IRFs for a 5% capital quality shock on capital under the benchmark regime and the CBDC regime - for the benchmark regime, the exercise is the same as the one in section 4.1. The substitution of money with CBDC allows the economy to recover the first-best recovery path of no ZLBs, with the added benefit of no cost of holding money for the economy as a whole (which would amount to 0.02% of GDP under our parametrization).

[8] According to the authors: "The implication is that under a CBDC regime policymakers need to anticipate technological, institutional or legal changes that might affect this elasticity, because these changes can materially change the countercyclicality of a policy rule, away from what may be desired by the policymaker.", page 63

Negative interest rates would prevail for less than a year and be very shallow, but the economy would still endure a large contraction on the onset of the shock. Still, overall paths would be less negative than under the limited pass-through scenario with money.

The main contribution of CBDCs in this model is therefore to reclaim the efficacy in the pass-through of monetary policy after a crisis that requires nominal interest rates to go negative - such as the Great Financial crisis in 2008 and its repercussions in the European crisis since 2010. The main monetary policy instrument then becomes benchmark interest rates again, diminishing the need for unconventional policies by the central bank. Of course, interest rate reductions alone might not be able to undo all the distortions surrounding rising spreads and credit contraction in a financial crisis, as noted in Woodford (2010). Distortions - such as flights to safe assets, freezes in the trading of risky securities and bank runs - generated by heightened fear and mistrust over certain types of assets could actually increase with the introduction of a more liquid Government-backed safe asset such as CBDC. Still, we would note that such fears could be tackled by the proper design of the CBDC, with limits to individual transfers to the accounts, for example, and/or with a sufficiently negative spread (or outright return, under NIRP) to CBDC deposits in those situations.

2.4.4 Different parametrization

Our benchmark parametrization for the coefficients in $s(\upsilon_t)$ are derived, as mentioned before, from Schmitt-Grohe and Uribe (2004). Those authors, in turn, reach those values by regressing the ratio of non-durable consumption and services expenditures to M1 ($=\upsilon_t$), in the 3-month Treasury bill rate over almost 40 years. In this section, we test a different parametrization and the sensitivity of results to the change in the value of the parameter B, by setting it to zero. Notice that the elasticity of money demand to interest rate is:

$$\varOmega_t^m = \frac{-\left(1-\alpha_m\right)}{2i_{t+1}^d\left[1+B-\frac{(1-\alpha_m)}{i_{t+1}^d}\right]} \tag{2-49}$$

while for CBDC it is:

$$\varOmega_t^{dc} = \frac{-R_{t+1}^e}{2R_{t+1}^d\left[1+B-\frac{R_{t+1}^e}{R_{t+1}^d}\right]} \tag{2-50}$$

This change would approximate the money demand function to the usual format obtained from money in the utility function specifications, and increases the elasticity of both money and CBDC demand to their relevant interest rates as well as their steady state values (taxes are raised too, to counter higher Government spending).

Figure 2.6 compares both specifications after a 1 pp (annualized) shock to the deposit facility rate. As expected, setting B to zero increases the reactions to the shock on R^e_{t+1}, driving the demand for CBDC up almost 50% over the first year, while deposits fall twice as much as observed in the benchmark scenario. Consumption and output rise 3 times as much as in the benchmark specification, despite the sharper decline in investment as a result of lower deposits and bank's net worth, as the volume of CBDC earning higher returns is more than 5 times higher in the alternative specification. While the responses go in the same direction, this exercises underscores the uncertainty surrounding the effective magnitudes of responses to a changed CBDC regime, especially should the steady state level of digital currency demanded by households be higher than it is for the benchmark regime with money.

2.5 Conclusion

In this paper we studied the adoption of central bank digital currencies in a theoretical framework, and discussed some of the possible outcomes and benefits of the adoption of CBDCs especially in an environment of negative interest rates policies. We find that the adoption of this new regime could improve the efficiency in the transmission of monetary policy, particularly under NIRP, as the efficacy of monetary policy is recovered and the ZLB circumvented. Additionally, the interest rate paid on the CBDC gives the central bank further leeway in adjusting monetary policy. On the other hand, wew find that monetary policy shocks to the deposit facility rate by itself does not seem to guarantee the expected counter-cyclical properties of usual instruments, due to wealth effects of consumers, and therefore might not be a reliable tool.

The design of the CBDC by itself carries many possibilities, and they were not tackled in this work. But we believe that a carefully designed digital currency, that has the desirable properties of liquidity, could serve as a useful tool in periods that require negative interest rates or unusual intervention policies such as QE. But we acknowledge the risks, stressed in previous works[9], that CBDC may pose as an additional source of financing instability for financial intermediaries in a context of financial crisis and elevated uncertainty, and act as a pro-cyclical force in times of stress. This balance can be reached by limiting the services that the CBDC can perform, and which distinguish

[9] Broadbent (2016); Bordo and Levin (2017); BIS (2018)

bank deposits and the client-bank relationship, and in that context, our work suggests that the trade-off with deposits can be limited.

Figure 2.3: IRFs of a 1 p.p. shock to R_t^e

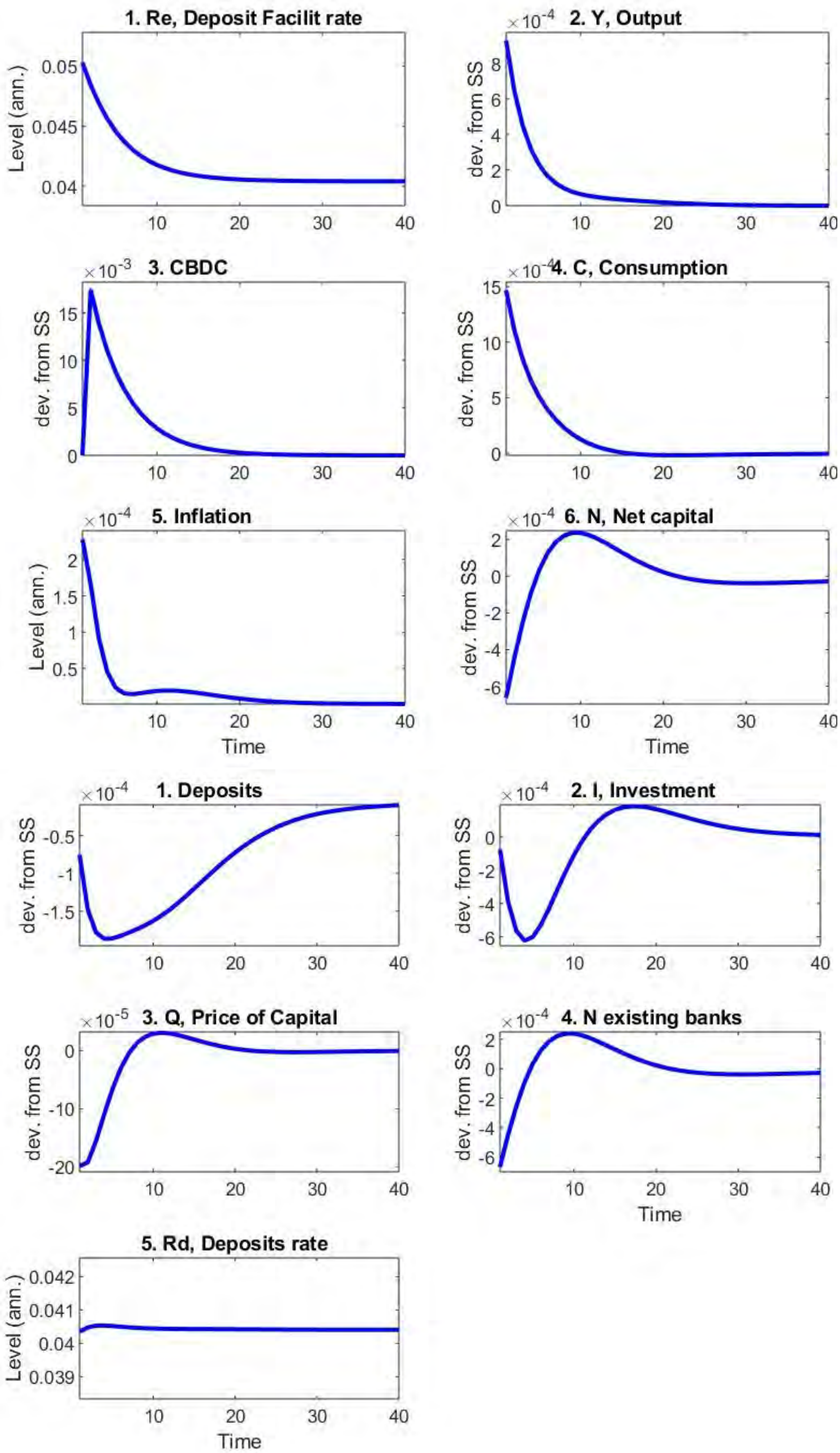


Figure 2.4: IRFs after a 5% shock in capital quality - benchmark vs CBDC regimes

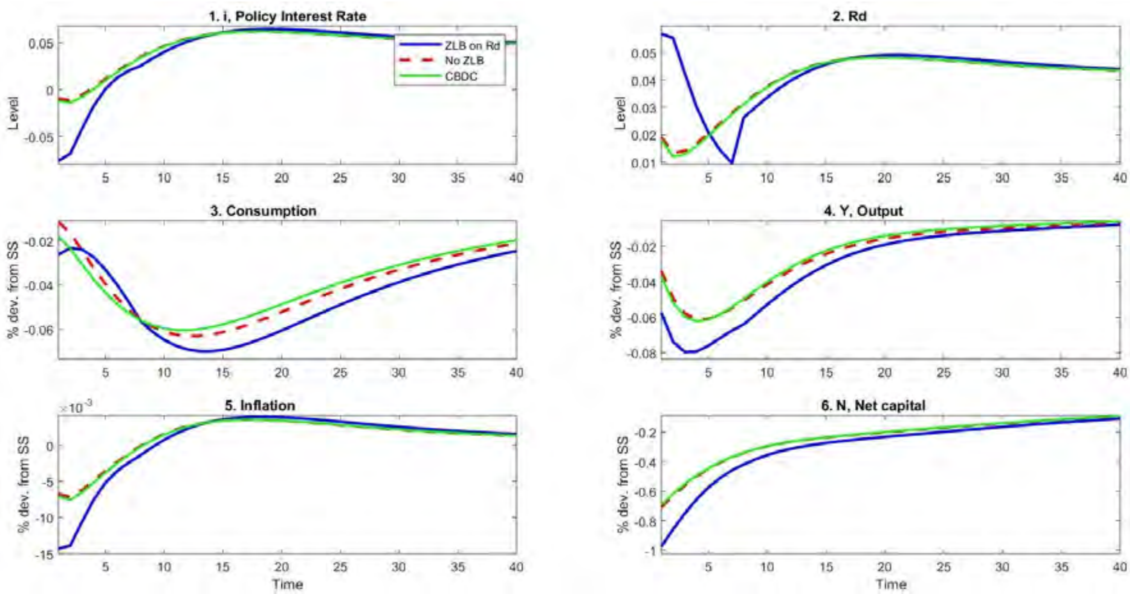


Figure 2.5: Additional IRFs after a 5% shock in capital quality - benchmark vs CBDC regimes

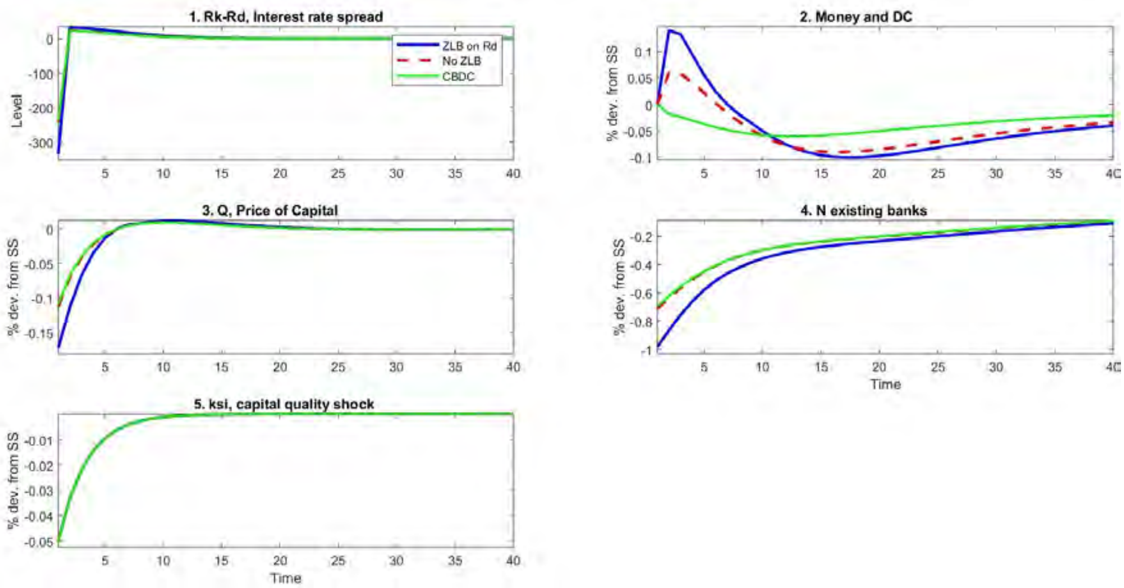
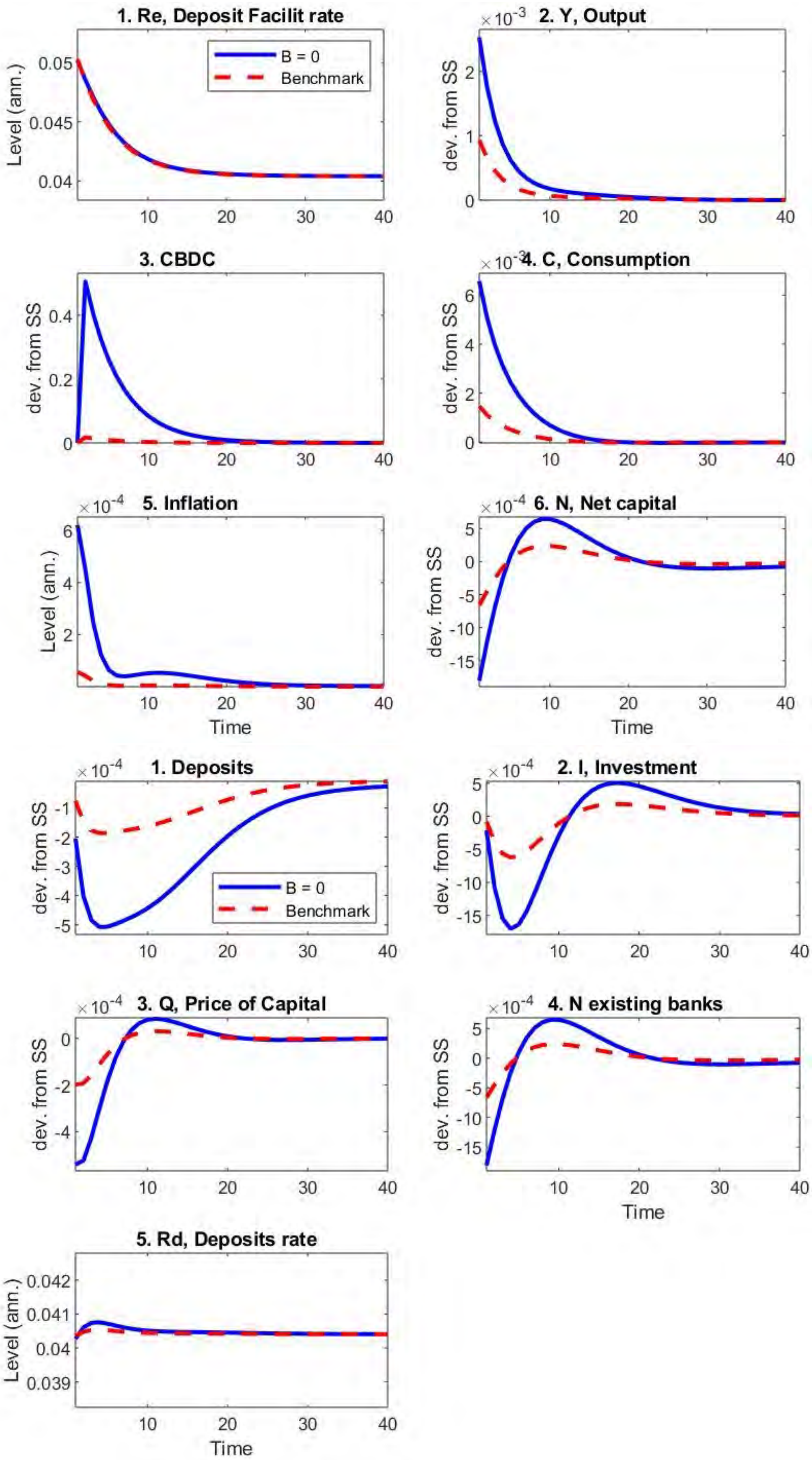


Figure 2.6: IRFs, Benchmark vs B=0 scenarios



Chapter 3
Long-term Output Forecasting with Factors and Big Data Sets

3.1
Introduction

Forecasting output growth many quarters in advance is a difficult, but vital task for a long list of sectors and Government bodies, as well as private sector economists and financial institutions. Such forecasts are the base for inflation forecasts at Central Banks, for fiscal policies and Government budgets over the long run, and debt sustainability measures tracked by financial and international institutions, such as the IMF. On the private sector, longer term output forecasts serve as a basis for the analysis of project returns. One high-profile example of long term forecasting is the yearly World Economic Outlook by the IMF [1] and the OECD economic projections[2], whose GDP forecasts for the current and next year usually draw a lot of attention from the countries researched and are in cases the basis for relevant policies by these institutions. Still, given the breadth on the shocks to which any economy is subject over the course of a multi-year period, as well as the uncertainty surrounding the eventual course of policies that will react to those developments, forecasts for two to three year periods usually carry very low trust and very high uncertainty, making the exercises that need them especially challenging and prone to error.

But with the increase in the supply of data, and the new methods developed to deal with their usage in estimation and forecasting, there is a promising terrain for improving predictions of economic variables, in particular output. For example, uncertainty indexes such as the ones developed by Baker et al. (2016) and Jur (2015) have been shown to bring information about future developments in investment and output.

In this work, we test alternative ways of forecasting longer term GDP growth, by combining a large data set of economic variables and factors to forecast potential output, output gap, and overall GDP growth. To make better use of this data set, we test two methods for variable selection and

[1]see https://www.imf.org/en/Publications/WEO
[2]http://www.oecd.org/eco/outlook/

estimation: adaLASSO and Random Forest, against a simple auto-regressive model and a medium scale DSGE model from the FED. This model, called EDO, has shown a good forecasting performance in works such as Edge et al. (2010b), which shows that its predictions are more accurate than the ones from the Survey of Professional Forecasters or the Green Book. We find that random forest has superior accuracy especially for the 2 years window, although DSGE modeling performs better on the very short term. But when output is decomposed between its potential (trend) growth and output gap, the results are mixed, with better results by either Random Forest or an autoregression depending on the variable and the time frame.

Chauvet and Potter (2013) surveys the literature surrounding output forecasting and finds that most models have very different performances depending on the phases of the business cycles. The literature surrounding DSGE forecasts usually suggests that DSGE model perform reasonably well when it come to forecasting growth. Baldo et al. (2017) study the forecasts for the Euro Area against reduced-form models such as VAR and BVARs, and find that they are competitive. Chung et al. (2010) shows that DSGE model forecasts (the EDO model in particular) are competitive with, and indeed often better than, others methods, although Edge et al. (2010a) suggests that despite their competitiveness on relative terms to professional forecasters, for example, they still perform badly in absolute terms - especially on longer terms. Negro and Schorfheide (2012) also point to mixed results. Meanwhile, a growing literature that incorporate new (and large) data sets and methods has suggested the superiority of those for nowcasting and forecasting over more traditional methods. Stock and Watson (2002) show that industrial productions forecasts with principal components largely outperform VARs and autoregressions, and from there a long literature in forecasting and nowcasting developed. Dynamic factor models are surveyed in Stock and Watson (2011), which point out the improvement in the accuracy of nowcasting and short-term forecasts for real activity indicators from dynamic factor models. Varian (2014) analyses tools for manipulating and analyzing big data, with particular focus on decision trees. Works such as Medeiros et al. (2019), for example, show that random forests have better predictive performance of inflation over simpler forecasting methods, such as VAR, and suggests it is a promising method to deal with forecasting in an ever richer data environment.

This paper is organized as follows: section 3.2 presents and explains the methodologies we will be comparing; section 3.3 turns to the data and model estimation, while section 3.4 reports and compares the different methods; section 3.5 concludes.

3.2 The Methods

In this section we present and explain briefly the two competing methods for estimation and forecasting that we will use. We will start out by estimating models for forecasting on the total GDP growth rate, and then follow on to estimating the potential and cyclical components of GDP.

Both methods (EDO and the adaLASSO/Random Forests) will be compared also to a simple and naive auto-regressive (AR) model with 9 lags.

3.2.1 The EDO model

As our benchmark model, we use the EDO (Estimated Dynamic Optimization) model from the Federal Reserve Board[3]. It is a medium-scale DSGE model with bigger disaggregation in both the productive and consumer sectors. It divides the productive sector into a fast-growing and a slow-growing sectors, which make up for the technology and business capital sectors and the traditional manufacturing and housing sector, respectively. It also separates consumer spending into durable and non-durable spending, and investment into residential and non-residential expenditures.

The model consists of 97 equations (including measurement equations). For the sake of space we refer the reader seeking for more details of the model to Chung et al. (2010). The model's Dynare codes are also available for download at the FED website.

3.2.2 Alternative estimations

An alternative way of forecasting GDP could potentially come from a decomposition of output according to its components, and the evaluation of the best method for forecasting on each of them. In economic models, real output is usually decomposed into its trend component T_t , usually associated with potential output, and the cycle C_t, economically associated with the output gap:

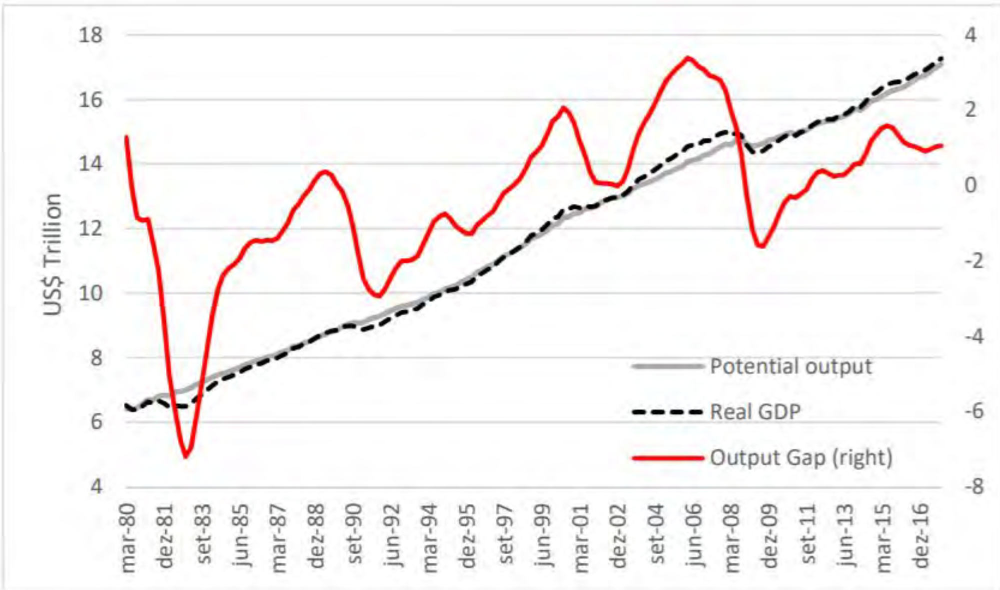
$$Y_t = T_t + C_t \tag{3-1}$$

In their model, Laubach and Williams (2003) create a state-space representation of the economy and use the Kalman filter to estimate both the level and growth of potential output, the output gap, and the level of the neutral interest rate. We will be interested in just the first two outputs.

[3]More information at https://www.federalreserve.gov/econres/EDO-models-about.htm

Figure 3.1 plots the series from 1980Q1 to 2017Q3 using the 2017Q4 vintage. Estimating and forecasting these two components of output separately might have economic importance for economists that are trying to assess changes in, for example, the structural rate of growth in an economy after some kind of shock.

Figure 3.1: Real GDP and its components according to the LW model - 2017Q4 vintage



We will use two different methods to forecast separately the trend and the cycle component of GDP, as well as the overall rate of output growth: adaLASSO and Random Forest. The first method is adaLASSO, which was proposed by Zou (2006), and is a form of shrinkage method with a penalty function that depends on a first round LASSO estimation of the coefficients, β_i^*:

$$\hat{\beta}_h = arg \min_{\beta} \left[\sum_{t=1}^{T-h} (y_{t+h} - \beta' x_t)^2 + \lambda \sum_{i=1}^{n} p(\beta_i; w_i, \alpha) \right] \tag{3-2}$$

where

$$\lambda \sum_{i=1}^{n} p(\beta_i; w_i, \alpha) = \lambda \sum_{i=1}^{n} w_i |\beta_i| \tag{3-3}$$

$$w_i = |\beta_i^*|^{-1} \tag{3-4}$$

According to Hastie et al. (2005), the adaLASSO "(...) yields consistent estimates of the parameters while retaining the attractive convexity property of the lasso”[4], and has the oracle property, according to Zou (2006). As with LASSO, it is a useful method for variable selection and forecasting, especially

[4]pg. 92

in the context of big data sets, but more consistent (as Meinshausen and Buhlmann (2004) point out, variable selection can be very inconsistent under LASSO). The first round of estimation provides a non-zero subset of variables for the second round, where larger coefficients are penalized less than smaller ones.

Meanwhile Random Forest (RF) is an estimation method initially proposed by Breiman (2001), and consists of the averaging out of predictions from a large number of de-correlated regression trees, each specified in a bootstrapped sub-sample of the original data. Trees predict outcomes by a partitioning of the input space (explanatory variables) into regions that minimize the sum of square errors for each chosen variable. Regression trees allow the capture of non-linear and complex relations in the data, and allow the use of a large number of explanatory variables, but are also very unstable. Random Forests therefore use both boosting and bagging (bootstrap aggregation) to improve accuracy, as according to Hastie et al. (2005) "bagging can dramatically reduce the variance of unstable procedures like trees, leading to improved prediction". An example from Hastie et al. (2005) [5]helps to better understand the method: Consider a regression problem in which X1 and X2 are explanatory variables, each taking values in some given interval, and Y is the dependent variable. As per Figure 3.2, we first split the space into two regions, at X1 = s1, then the region to the left (right) of X1 = s1 is split at X2 = s2 (X1 = s3). Finally, the region to the right of X1 = s3 is split at X2 = s4. The space X is split in five regions: R_m, m = 1,...,5. In each region R_m, the model predicts Y as a constant c_m, that could be estimated, for example, as the sample average of realizations of Y that "fell" within region R_m. Therefore, each region corresponds to a terminal node of the single tree created by the space, as illustrated in the left plot of Figure 3.2. To choose the splitting points for each variable X, you seek for the value in variable j that minimizes the sum of squared errors of the values of Y relative to the predictions c_m of each region created after the split. Once the best split is found, we proceed iteratively repeating this process on each of the resulting regions. On choosing how large should a tree be (number of nodes), Random Forest applies the essence of bagging, as explained in Medeiros et al. (2019): "A Random Forest is a collection of regression trees, each specified in a bootstrapped sub-sample of the original data. Suppose there are B bootstrapped sub-samples. For each sub-sample, obtain a prediction for Y by applying a modified version of the aforementioned splitting iterative process until a pre-specified minimum number of observations, say five, is reached in

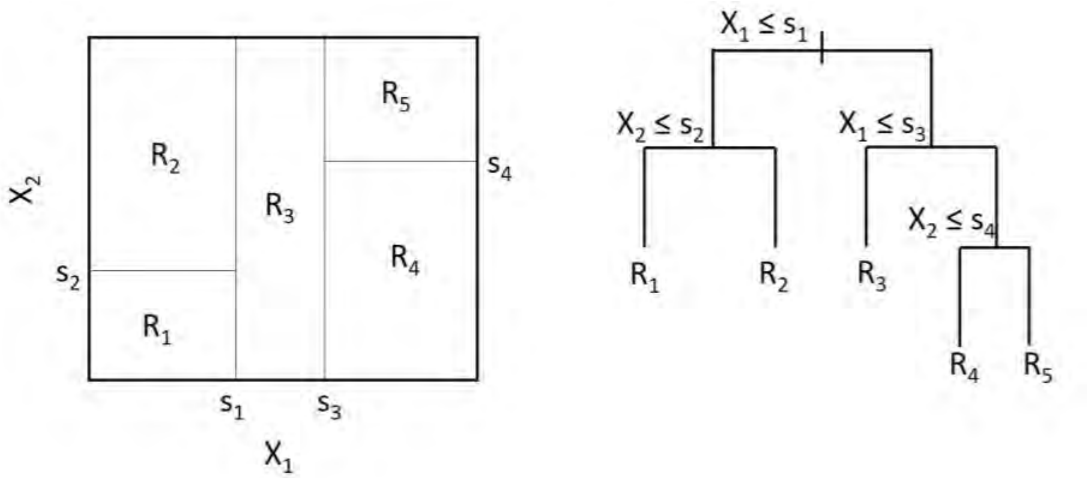
[5]page 305

any resulting region. In particular, the modification is to select q variables at random from the p explanatory variables at each step of the process. Finally, simply average the predictions of Y across the B bootstrapped sub-samples". Bootstrap samples are calculated using block bootstrap, since we are dealing with time series. See also Varian (2014).

3.3 Data and estimation

We set out to forecast quarterly GDP yoy growth over the next two years using three different methods: a simple AR(9) regression with sequential forecasting for each step, the FED's EDO model, and the adaLASSO or Random Forest for overall GDP growth or its components as estimated by the Laubach and Williams (2003) model. The data used in the two last models are explained below.

Figure 3.2: A regression tree - (Figure 9.2 in Hastie et al. (2008))



3.3.1 FED's EDO model

For the estimation of the EDO model, we followed the methodology at Chung et al. (2010) and the observed time series in the model's documentation[6]. We used 12 quarterly time series for this estimation:

1. Growth rate of Real GDP (2012 prices - detrended)

2. Effective Federal Funds rate

3. The growth rate of real consumption expenditure on non-durables and non-housing services (detrended)

[6]https://www.federalreserve.gov/econres/EDO-model-package.htm

4. The growth rate of real consumption expenditure on durables (detrended)

5. The growth rate of real residential investment expenditure (detrended)

6. The growth rate of real business investment expenditure

7. Growth of Real Compensation Per Hour

8. Growth rate of the Personal Consumption Expenditure (PCE) price index

9. Growth rate of the Core PCE price index (excluding food and energy prices)

10. Hours, which equals hours of all persons in the non-farm business sector from the Bureau of Labor Statistics (detrended)

11. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods

12. Interest rates of the 2-year Treasury bonds

The series start at 1985 Q1 and end in 2018Q4. The model was then estimated for each quarter between 1999Q3 and 2017Q4, with data up to the previous quarter of each vintage. After the estimation of the posterior distributions for the relevant parameters and unobserved variables, the model forecasted 9 quarters ahead (the current vintage quarter plus the quarters of the next two years) of the observed variables. The estimation and prediction are done over the final series, collected at the beginning of 2018, therefore setting a higher benchmark as forecasts will be less prone to forecasting errors due to data revision.

We collect the model´s predictions for overall GDP quarter over quarter detrended growth, and therefore, after reconstruction, compare only GDP´s year over year growth to the competing model.

3.3.2
Laubach-Williams model

We used the Laubach and Williams (2003) model to calculate potential output, its trend growth rate g and the output gap for quarterly vintages of data from 99 Q1 to 2017 Q4. The inputs into the estimation are only 5 time series: log real GDP (yearly rate), import prices variation, PCE inflation, oil prices variation and real interest rates. The final series range from 1962Q1 to the quarter prior to the vintage's name (reflecting the available data up to the end of the respective quarter).

Note, as shown in Figure 3.3 below (only for the 2017Q4 vintage), that the potential output has a performance that differs from the series implied by the trend growth rate of potential output (g) especially during the years 2000s. Although the LW algorithm allows g to suffer shocks, the potential output time series also incorporates level shifts that will impact the yoy growth rate and therefore the forecasts of output growth and errors. Therefore we adopted the yoy growth rate of the estimated potential GDP from the LW model for each quarter as the preferred object of estimation for potential output, as it is the relevant input into the calculation of overall output growth.

3.3.3
The data

Our main source of data is the dataset from McCracken and Ng (2015), and whose vintages are compiled by Michael McCraken at the St Louis FED[7]. The series are described at tables B.1 to B.8, taken from that article, in the Appendix. The McCraken set, which is monthly, was turned quarterly by averaging out some variables over the quarter up to the last observation and by use of the last observation for price indexes variations. We adopted two sets of principal component variables into the set of regressors: principal components from the McCracken data and the uncertainty indexes estimated by Jur (2015). We estimate the first 5 principal components for the McCracken and Ng (2015) monthly dataset, and then computed quarterly averages. The uncertainty indexes are divided into "financial uncertainty" and "macroeconomic uncertainty".

The data set therefore comprised of: the 360 variables in the original McCracken set, 6 first principal components from that set, the 6 uncertainty indexes from Jur (2015), plus the quarter over quarter and year over year growth for each of the variables. On top of that we added 8 lags for each of these variables and of the fitted variable, in a total of over 3000 regressors.

[7]https://research.stlouisfed.org/econ/mccracken/fred-databases/

We used the same GDP series for every method, namely real GDP (chained 2012 dollars). For the LW, its only change is a log transformation, with the algorithm resulting in the series of potential GDP (from where we extract the growth rate) and output gap (in percentage points). For the EDO, real GDP must be divided by the level of total hours worked, as a proxy for GDP per worker. We then reconstruct the original GDP series and its growth rate by extrapolating the hours worked series.

In RF, we use 500 trees for each estimation, and the BIC (Bayesian information criterion) for model selection in adaLASSO.

3.4 Results

3.4.1 Total GDP growth forecasts

We start by testing models and predictions for the time series (vintages) of real output growth year over year (henceforth yoy), which is the growth rate of a quarter relative to the same quarter of the previous year. This measure allows seasonal variations in growth to be evened out, as the comparison is between the same period of the year.

Our first forecasting strategy is a simple AR regression with 9 lags of the yoy quarterly growth series, compared to estimations using adaLASSO or Random Forests for the same series with the whole regressor data set, and to the forecasts 2 years on from the EDO model. Table 3.1 presents the mean square and absolute errors for each method (MSE and MAE, respectively). Notice that the EDO model's MSE is the smallest over the first year, while RF outperforms over the second - which is the time frame of interest for us.

Table 3.1: MSEs and MAEs - YoY GDP growth rate

	MSE				MAE			
	AR	EDO	RF	Ada	AR	EDO	RF	Ada
t0	0.00013	0.00004	0.00011	0.00015	0.006	0.004	0.008	0.008
t1	0.00023	0.00010	0.00014	0.00030	0.007	0.007	0.009	0.013
t2	0.00034	0.00018	0.00022	0.00046	0.009	0.009	0.010	0.017
t3	0.00042	0.00027	0.00029	0.00044	0.011	0.012	0.011	0.015
t4	0.00044	0.00028	0.00029	0.00045	0.012	0.012	0.011	0.014
t5	0.00043	0.00029	0.00028	0.00036	0.013	0.012	0.011	0.014
t6	0.00042	0.00031	0.00028	0.00033	0.013	0.012	0.011	0.013
t7	0.00040	0.00032	0.00030	0.00028	0.013	0.013	0.011	0.012
t8	0.00037	0.00033	0.00029	0.00027	0.012	0.013	0.011	0.011
1st year	*0.00036*	*0.00021*	*0.00024*	*0.00041*	*0.00985*	*0.00986*	*0.01043*	*0.01479*
2nd year	*0.00041*	*0.00031*	*0.00029*	*0.00031*	*0.01262*	*0.01270*	*0.01110*	*0.01260*

Table 3.2 presents the rate of the MSE and MAEs for each forecast step of adaLASSO or Random Forest relative to the same measure from the AR

forecasts. Both alternative methods outperfom the simple AR regression, although the random forest approach seems more consistent than the adaLASSO over the whole forecasting period.

Table 3.2: MSE and MAE: RF or adaLASSO vs AR forecasts

	MSE		MAE	
	RF	Ada	RF	Ada
t0	0.571	0.778	0.749	0.835
t1	0.527	1.111	0.795	1.187
t2	0.620	1.265	0.835	1.389
t3	0.615	0.955	0.846	1.107
t4	0.814	1.256	0.869	1.076
t5	0.793	1.022	0.848	1.087
t6	0.837	0.974	0.883	1.076
t7	0.940	0.881	0.957	1.042
t8	1.016	0.923	1.005	0.966
1st year	*0.644*	*1.147*	*0.836*	*1.190*
2nd year	*0.896*	*0.950*	*0.923*	*1.043*

Table 3.3 below presents the ratios of MSEs and MAEs relative to the EDO forecasts. The predictions from the DSGE model are a lot superior over the first year - not only reflecting the possible superiority of the method thanks to the economic dynamics embedded in it but also possibly to the fact that the estimation was conducted over the final series for the 12 variables used. This naturally implicated in less error as the EDO wasn't as subject to revisions or changes to its time series and forecasts, creating a higher benchmark for the other models. On the other hand, over the longer time frame these treats were not enough to overcome the accuracy of RF, with the errors over the second year being roughly 10% below that of EDO - despite the already mentioned higher benchmark.

Plots of the forecasts under RF, EDO and the AR models are presented in figures 3.4 and 3.5. Given these results, for a straightforward estimation of longer term output growth, random forest seems as the best (more accurate) option.

3.4.2 Decomposed forecasting

We repeat the strategy from the previous section, but now on each component of output as calculated according to the LW approach. We compare the outcomes for each component and total GDP growth (calculated from the

Table 3.3: MSE and MAE: RF or adaLASSO vs EDO forecasts

	MSE		MAE	
	RF	Ada	RF	Ada
t0	3.078	4.238	1.734	1.943
t1	1.448	3.083	1.276	1.920
t2	1.250	2.569	1.134	1.892
t3	1.044	1.629	0.968	1.272
t4	1.038	1.607	0.956	1.187
t5	0.953	1.237	0.891	1.146
t6	0.915	1.073	0.887	1.082
t7	0.923	0.869	0.869	0.946
t8	0.880	0.802	0.851	0.819
1st year	*1.195*	*2.222*	*1.084*	*1.568*
2nd year	*0.918*	*0.995*	*0.875*	*0.999*

components) from the AR, adaLASSO and RF only - as the EDO model only estimates total GDP.

When GDP is decomposed into potential and output gap, with each component being forecasted separately and final GDP as a result of the two, accuracy declines relative to the straightforward estimation of growth. Table 3.4 presents the MSE ratios from the decomposed forecasts relative to the forecasts from the previous sections (using AR, EDO and RF). Except for the autoregressive decomposed model over the second year, the decomposition underperforms in all comparisons. Figures 3.6 and 3.7 plot the forecasts from the autoregression and Random Forest methods against actual values for potential output growth and output gap.

Table 3.4: MSE and MAE: Decomposed estimates vs AR, RF and EDO (total GDP)

	MSE				
	AR LW/AR	AR LW/RF	RF e ada LW/AR	RF e ada LW/RF	RF e ada LW/EDO
t0	1.470	1.751	2.086	2.484	7.647
t1	1.178	1.899	1.798	2.899	4.198
t2	1.064	1.612	1.506	2.282	2.851
t3	1.096	1.625	1.412	2.094	2.187
t4	0.816	1.228	1.039	1.564	1.624
t5	0.817	1.261	1.028	1.588	1.514
t6	0.801	1.195	1.045	1.558	1.426
t7	0.780	1.064	1.071	1.463	1.350
t8	0.777	0.985	1.028	1.302	1.145
1st year	*1.038*	*1.591*	*1.439*	*2.210*	*2.715*
2nd year	*0.794*	*1.126*	*1.043*	*1.478*	*1.359*

Table 3.5 presents the results for the estimation of potential GDP using

Table 3.5: MSE and MAE: RF (potential) and adaLASSO (output gap) vs AR forecasts - Decomposed estimates

	MSE			MAE		
	YoY Potential	Output Gap	GDP yoy	YoY Potential	Output Gap	GDP yoy
t0	1.078	1.379	1.419	1.006	1.054	1.474
t1	0.981	1.583	1.526	0.973	1.075	1.457
t2	0.870	1.772	1.415	0.904	1.178	1.392
t3	0.762	1.700	1.288	0.840	1.203	1.373
t4	0.782	1.761	1.274	0.822	1.175	1.140
t5	0.774	1.801	1.259	0.840	1.206	1.147
t6	0.831	1.930	1.304	0.908	1.156	1.308
t7	0.876	1.954	1.374	0.950	1.121	1.370
t8	0.864	1.911	1.322	0.956	1.207	1.577
1st year	*0.848*	*1.704*	*1.376*	*0.885*	*1.158*	*1.340*
2nd year	*0.836*	*1.899*	*1.315*	*0.913*	*1.172*	*1.350*

RF and adaLASSO for the output gap - they were the best performing methods for each respective output component. While RF predictions performs a lot better than the AR over almost all time frames for an average gain of 15% in accuracy, both methods perform very poorly when it comes to the output gap - adaLASSO, which is a little better than RF, still has a squared error 35% above the AR model over the 2 year horizon. As a result of this performance, the resulting YoY GDP growth forecasts are less precise than the ones resulting from the auto-regressive framework, in particular for the longer horizons. We tested the removal of the factor and uncertainty indexes from the regressors, but the MSEs increased, suggesting that these indexes are a useful source of information. Another test was estimating the output gap (which has the largest error) with RF using only the principal components and the uncertainty indexes. As shown in table 3.6, MSEs for the output gap decrease relative to the previous exercise in most time frames, but increase very much for the predictions 7 and 8 quarters ahead, weighing on the performance of final output growth. In the end, there is not much gain for longer term final output growth prediction, but it seems like a better strategy for forecasting the output gap for the shorter term.

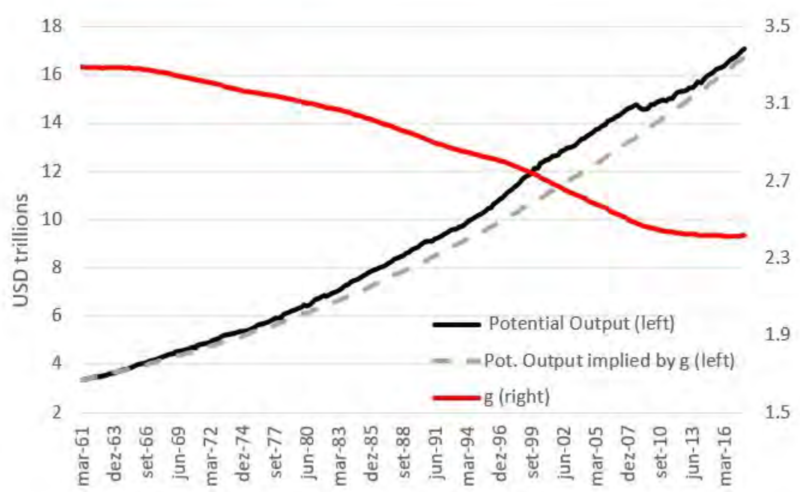
3.5 Conclusion

In this paper we compared alternative methods for forecasting long term (8 quarters) GDP output growth. We incorporated a large data set, as well as uncertainty indexes and factors, and tested a simple autoregression against richer methods, such as random forest. Our results suggest that RF is a more accurate way of forecasting growth over 5 and 8 quarters in advance, although the predictions from a DSGE model are quite competitive. But if the researcher is interested in the components of output, such as the output gap, Random

Table 3.6: MSE and MAE: RF with principal components only vs AR forecasts - Decomposed estimates

	MSE			MAE		
	YoY Potential	Output Gap	GDP yoy	YoY Potential	Output Gap	GDP yoy
t0	1.078	0.360	1.318	1.006	0.621	1.102
t1	0.981	0.507	1.320	0.973	0.726	1.133
t2	0.870	1.323	1.106	0.904	1.322	1.126
t3	0.762	0.600	0.982	0.840	0.778	1.217
t4	0.782	0.256	1.262	0.822	0.636	1.271
t5	0.774	0.403	1.420	0.840	0.599	1.335
t6	0.831	0.235	1.610	0.908	0.448	1.396
t7	0.876	1.444	1.875	0.950	1.416	1.525
t8	0.864	2.340	1.833	0.956	1.830	1.568
1st year	*0.848*	*0.671*	*1.167*	*0.885*	*0.865*	*1.187*
2nd year	*0.836*	*1.106*	*1.685*	*0.913*	*1.073*	*1.456*

Figure 3.3: LW model (2017 Q4 vintage): YoY growth of potential output vs trend growth rate g



Forest is a good strategy, although not superior to a simple autoregression in some time frames and depending on the component.

Figure 3.4: Actual vs Forecasted YoY Growth - RF vs AR

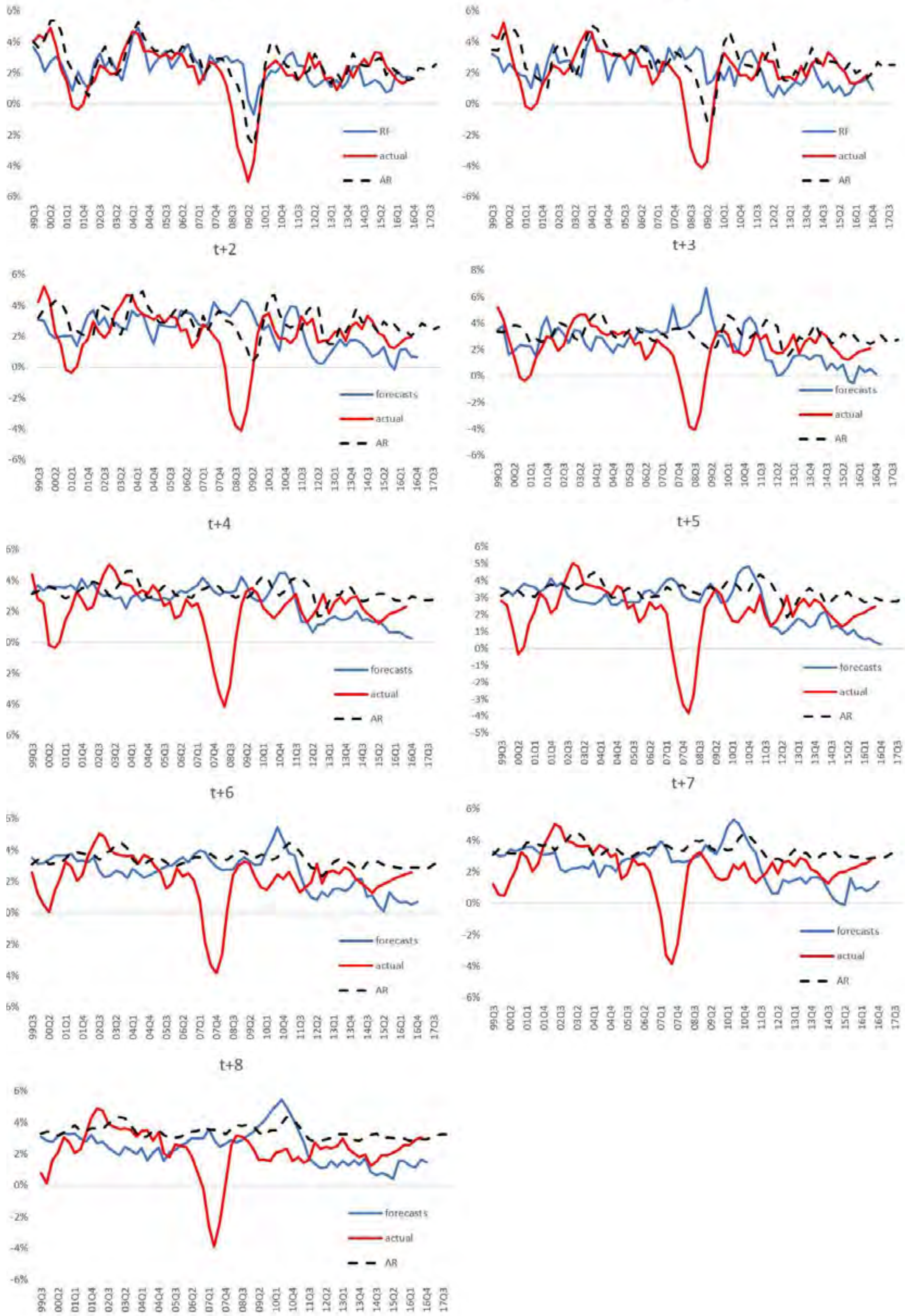


Figure 3.5: Actual vs Forecasted YoY Growth - RF vs EDO

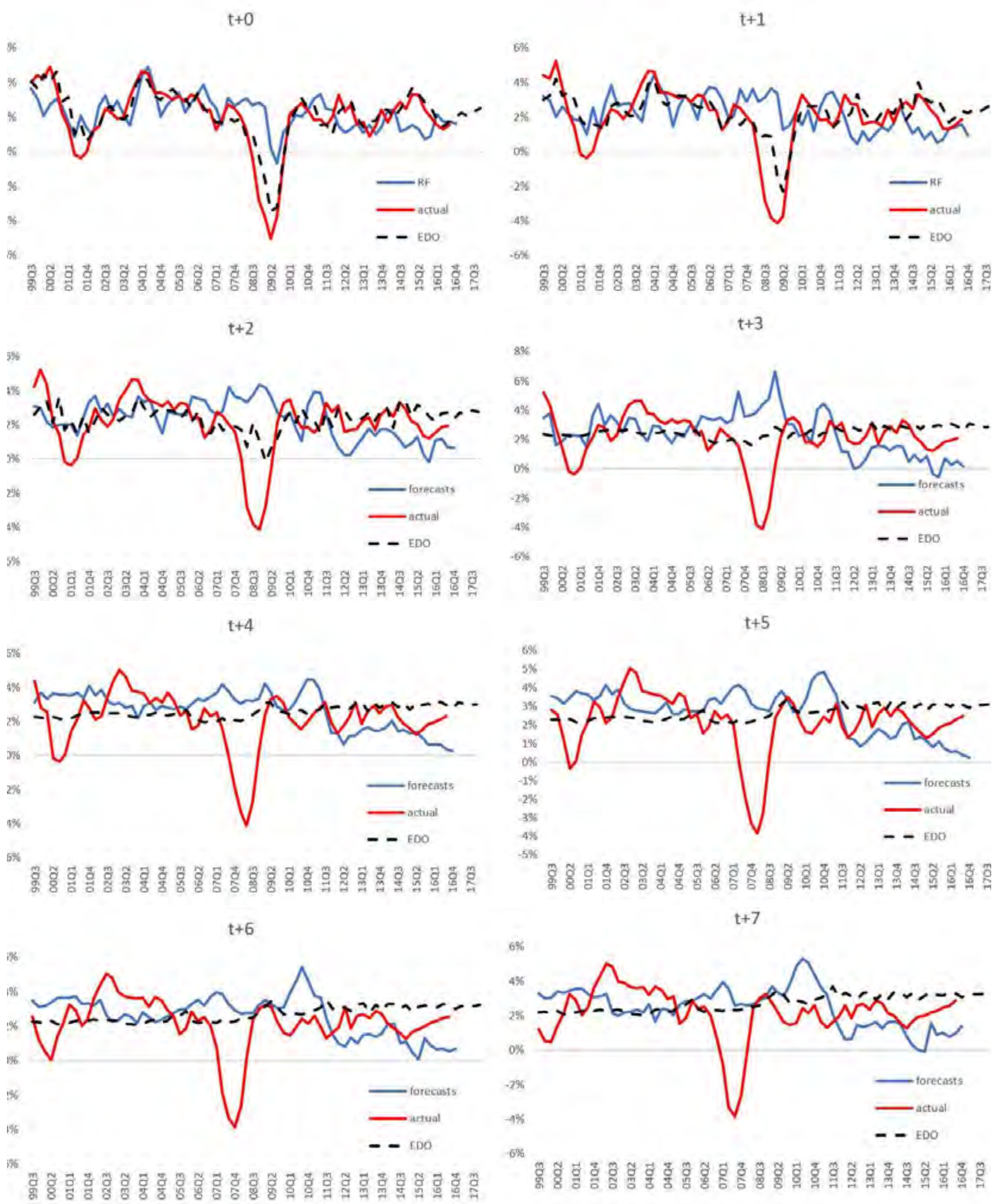


Figure 3.6: Potential output YoY growth: Actual vs AR vs RF

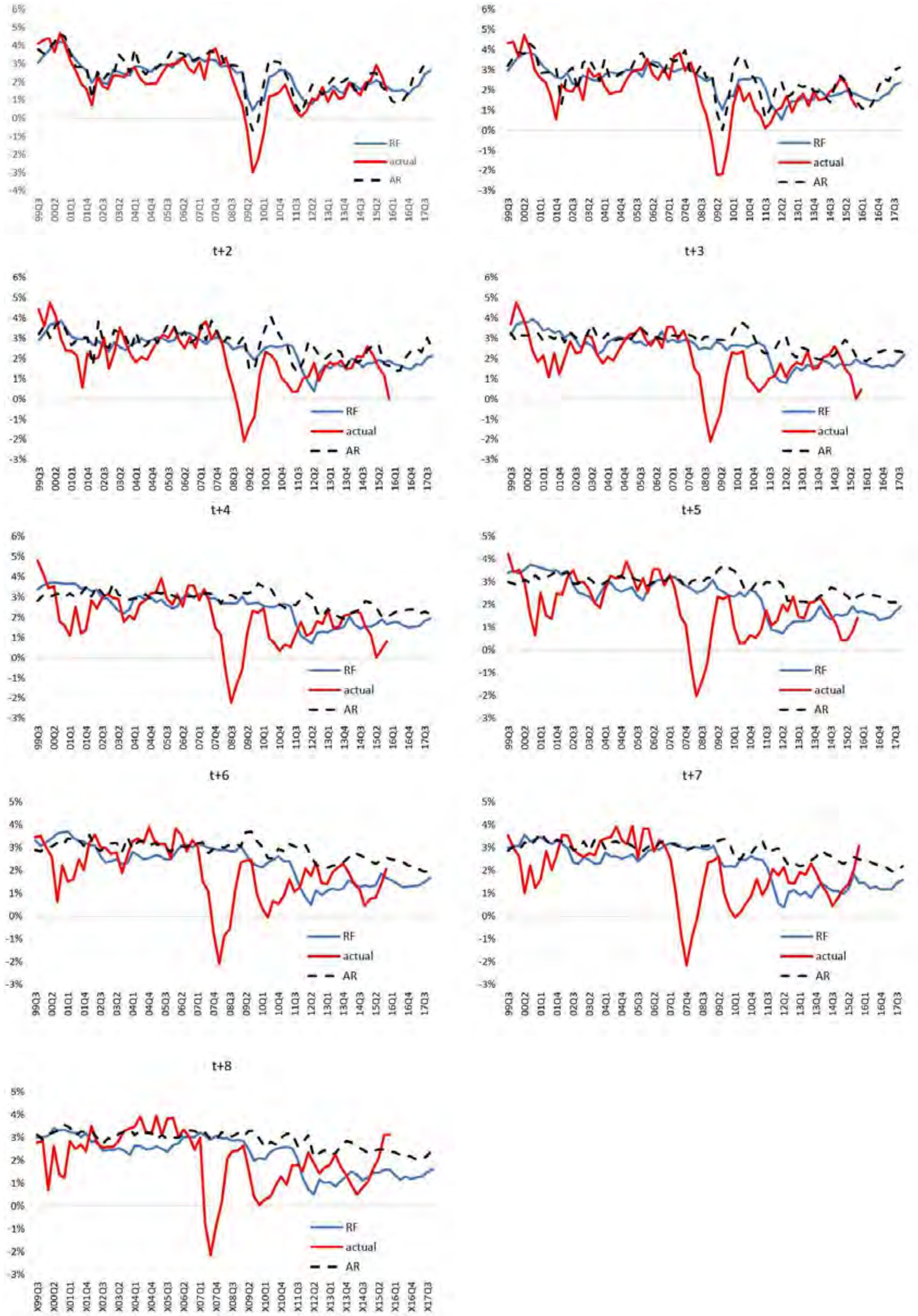
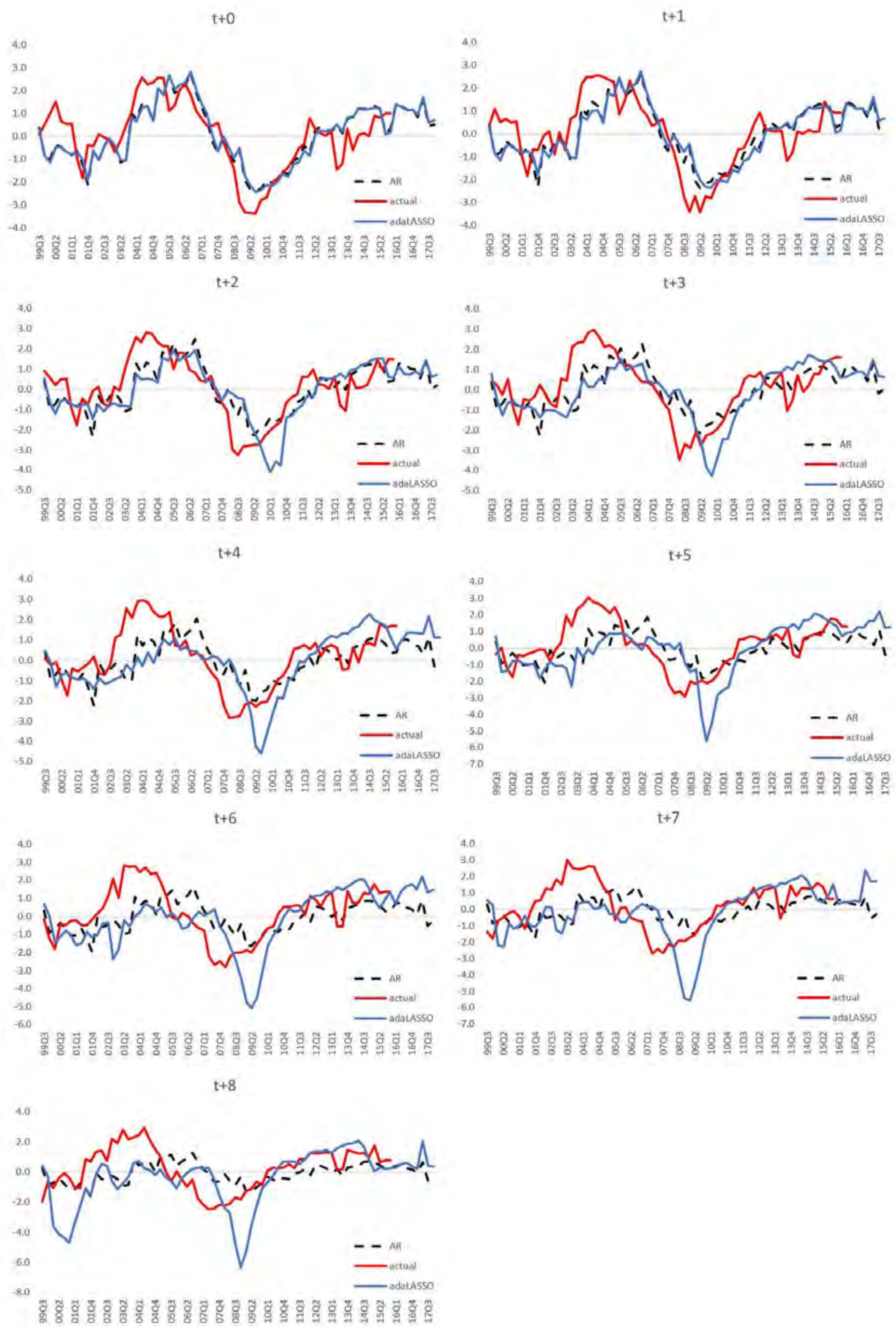


Figure 3.7: Output gap (p.p.): Actual vs AR vs adaLASSO



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Appendix A

The financial intermediary maximization problem

This section relies heavily on the methodology described in Gertler and Kiyotaki (2010).

While the risk adjusted return on assets is larger than the cost of its funding, it pays for the intermediary to build assets, as long as it remains in the industry. The banker's objective is to maximize it expected terminal wealth, given by:

$$V_{jt}\left(S_{jt-1}, b_{jt-1}, Ex_{jt}, D_{jt}\right) = \underset{N_{jt+1+i}}{max}\, E_{t-1}\left\{\textstyle\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^{i}\beta^{i+1}\Lambda_{t,t+1+i}N_{jt+i}\right\} =$$

$$= \max E_{t-1}\Lambda_{t,t-1}\beta E_{t}\left\{\left(1-\theta\right)N_{jt} + \theta\left[\max_{s_{jt},b_{jt}} V_{jt+1}\left(S_{jt}, b_{jt}, Ex_{jt}, D_{jt+1}\right)\right]\right\}$$

As in Gertler and Karadi (2011), we assume that the intermediary can divert a fraction λ of its assets, but not the funds borrowed from the interbank market. Such a possibility will limit the amount of net capital that the financial intermediaries can extract from the representative household. The intermediary will refrain from diverting assets as long as:

$$V_{jt} \geq \lambda\left(Q_{t}S_{jt} - b_{jt}\right) \tag{A-1}$$

We will guess that the value function is linear:

$$V_{jt} = \upsilon_{st}Q_{t}S_{jt} + \upsilon_{xt}Ex_{jt} - \upsilon_{t}D_{jt+1} - \upsilon_{bt}b_{jt} \tag{A-2}$$

The maximization problem then becomes:

$$\max \mathcal{L} = V_{jt} + \lambda_{t}^{i}\left[V_{jt} - \lambda\left(Q_{t}S_{jt} - b_{jt}\right)\right] + \lambda_{2t}Ex_{jt}$$

Where λ_{t}^{i} is the Lagrange multiplier for the asset diversion limitation and λ_{2t} refers to the restriction that excess reserves are non-negative. We can rewrite the above equation, after substituting for D_{t+1}, as:

$$\mathcal{L} = \left(1+\lambda_t^i\right)\left[\left(v_{st}-v_t\right)Q_tS_{jt}-\left(v_{bt}-v_t\right)b_{jt}+v_tN_{jt}+\left(v_{xt}-v_t\right)Ex_{jt}\right]+\lambda_{2t}Ex_{jt}-\lambda\lambda_t^i\left(Q_tS_{jt}-b_{jt}\right)$$

From the FOCs we get:

$$\left(1+\lambda_t^i\right)\left(v_{st}-v_t\right)=\lambda\lambda_t^i \tag{A-3}$$

$$\left(1+\lambda_t^i\right)\left(v_{bt}-v_t\right)=\lambda\lambda_t^i \tag{A-4}$$

$$\left(1+\lambda_t^i\right)\left(v_{xt}-v_t\right)=-\lambda_{2t} \tag{A-5}$$

$$v_{st}Q_tS_{jt}+v_{xt}Ex_{jt}-v_tD_{jt+1}-v_{bt}b_{jt}\geq\lambda\left(Q_tS_{jt}-b_{jt}\right) \tag{A-6}$$

$$Ex_{jt}\geq 0 \tag{A-7}$$

Where equation A-6 is the FOC of λ_t^i , and the last equation is the FOC for λ_{2t} . Notice that A-5 and A-7 imply that $\lambda_{2t}Ex_{jt}=0$.

We can rewrite A-6 as:

$$\left(\lambda+v_t-v_{st}\right)Q_tS_{jt}\leq\left(v_t-v_{bt}+\lambda\right)b_{jt}+v_tN_{jt}+\left(v_{xt}-v_t\right)Ex_{jt} \tag{A-8}$$

which becomes:

$$\left(v_t-v_{st}+\lambda\right)Q_tS_{jt}\leq\left(v_t-v_{bt}+\lambda\right)b_{jt}+v_tN_{jt}-\frac{\lambda_{2t}}{1+\lambda_t^i}Ex_{jt}$$

and finally:

$$\left(v_t-v_{st}+\lambda\right)Q_tS_{jt}\leq\left(v_t-v_{bt}+\lambda\right)b_{jt}+v_tN_{jt} \tag{A-9}$$

We can rewrite the value function with the FOCs, which yields:

$$V_{jt}=\left(1+\lambda_t^i\right)v_tN_{jt}-\lambda_{2t}Ex_{jt} \tag{A-10}$$

But we know that the last term is zero, as noticed above. Substituting this expression for date t+1 into the Bellman equation, we learn that:

$$V_{jt}\left(S_{jt},b_{jt},Ex_{jt},D_{jt+1}\right)=E_t\left\{\left(1-\theta\right)\Lambda_{t,t+1}\beta N_{jt+1}+\theta\Lambda_{t,t+1}\beta V_{jt+1}\right\}$$
$$=E_t\left\{\left(1-\theta\right)\Lambda_{t,t+1}\beta N_{jt+1}+\theta\Lambda_{t,t+1}\beta\left(\xi_{t+1}^f\left(1+\lambda_{t+1}^i\right)v_{t+1}N_{jt+1}+\left(1-\xi_{t+1}^f\right)\left(1+\lambda_{t+1}^i\right)v_{t+1}\tilde{N}_{jt+1}\right)\right\}$$

where $\tilde{N}_{jt+1}=N_{jt+1}+R_{t+1}^Ib_{jt}+R_{t+1}^EEx_{jt}$. After some manipulation, the above equation becomes:

$$V_{jt}\left(S_{jt}, b_{jt}, Ex_{jt}, D_{jt+1}\right) =$$
$$= E_t\left\{\Lambda_{t,t+1}\beta\Omega_{t+1}N_{jt+1}\right\} + E_t\{\Lambda_{t,t+1}\theta\left(1+\lambda^i_{t+1}\right)\left(1-\xi^f_{t+1}\right)\nu_{t+1}\left(R^I_{t+1}b_t + R^E_{t+1}Ex_{jt}\right)$$

where:

$$\Omega_{t+1} = \beta\left[1-\theta+\theta\left(1+\lambda^i_{t+1}\right)v_{t+1}\right] \tag{A-11}$$

By using the method of undetermined coefficients, we learn that:

$$v_t = E_t\left\{(1-\theta)\beta\Lambda_{t,t+1}R^d_{t+1} + \theta\beta\Lambda_{t,t+1}\left(1+\lambda^i_{t+1}\right)R^d_{t+1}v_{t+1}\right\} \tag{A-12}$$

$$v_{st} = \frac{R_{kt+1}}{R^d_{t+1}}v_t \tag{A-13}$$

$$v_{xt} = E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R^E_{t+1} + \Lambda_{t,t+1}\theta\beta\Lambda_{t,t+1}\left(1+\lambda^i_{t+1}\right)\left(1-\xi^f_t\right)R^E_{t+1}v_{t+1}/D_{jt+1} > D_{t+1}\right\} \tag{A-14}$$

$$v_{bt} = E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R^I_{t+1}\right\} + E_t\left\{\Lambda_{t,t+1}\theta\beta\Lambda_{t,t+1}\left(1+\lambda^i_{t+1}\right)\left(1-\xi^f_t\right)R^I_{t+1}v_{t+1}\right\} \tag{A-15}$$

Which we can apply to A-9, to reach:

$$Q_tS_{jt} = \phi_{jt}N_{jt} + b_{jt+1}$$

Lets call ϕ_{jt} the bank's leverage ratio, where:

$$\phi_{jt} = \frac{v_t}{\lambda - v_{st} + v_t} \tag{A-16}$$

Appendix B

Figure B.1: Group 1: Output and income

	id	tcode	fred	description	gsi	gsi:description
1	1	5	RPI	Real Personal Income	M_14386177	PI
2	2	5	W875RX1	Real personal income ex transfer receipts	M_145256755	PI less transfers
3	6	5	INDPRO	IP Index	M_116460980	IP: total
4	7	5	IPFPNSS	IP: Final Products and Nonindustrial Supplies	M_116460981	IP: products
5	8	5	IPFINAL	IP: Final Products (Market Group)	M_116461268	IP: final prod
6	9	5	IPCONGD	IP: Consumer Goods	M_116460982	IP: cons gds
7	10	5	IPDCONGD	IP: Durable Consumer Goods	M_116460983	IP: cons dble
8	11	5	IPNCONGD	IP: Nondurable Consumer Goods	M_116460988	IP: cons nondble
9	12	5	IPBUSEQ	IP: Business Equipment	M_116460995	IP: bus eqpt
10	13	5	IPMAT	IP: Materials	M_116461002	IP: matls
11	14	5	IPDMAT	IP: Durable Materials	M_116461004	IP: dble matls
12	15	5	IPNMAT	IP: Nondurable Materials	M_116461008	IP: nondble matls
13	16	5	IPMANSICS	IP: Manufacturing (SIC)	M_116461013	IP: mfg
14	17	5	IPB51222s	IP: Residential Utilities	M_116461276	IP: res util
15	18	5	IPFUELS	IP: Fuels	M_116461275	IP: fuels
16	19	1	NAPMPI	ISM Manufacturing: Production Index	M_110157212	NAPM prodn
17	20	2	CUMFNS	Capacity Utilization: Manufacturing	M_116461602	Cap util

Source: McCraken and Ng (2015)

Figure B.2: Group 2: Labor market

	id	tcode	fred	description	gsi	gsi:description
1	21*	2	HWI	Help-Wanted Index for United States		Help wanted indx
2	22*	2	HWIURATIO	Ratio of Help Wanted/No. Unemployed	M_110156531	Help wanted/unemp
3	23	5	CLF16OV	Civilian Labor Force	M_110156467	Emp CPS total
4	24	5	CE16OV	Civilian Employment	M_110156498	Emp CPS nonag
5	25	2	UNRATE	Civilian Unemployment Rate	M_110156541	U: all
6	26	2	UEMPMEAN	Average Duration of Unemployment (Weeks)	M_110156528	U: mean duration
7	27	5	UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	M_110156527	U < 5 wks
8	28	5	UEMP5TO14	Civilians Unemployed for 5-14 Weeks	M_110156523	U 5-14 wks
9	29	5	UEMP15OV	Civilians Unemployed - 15 Weeks & Over	M_110156524	U 15+ wks
10	30	5	UEMP15T26	Civilians Unemployed for 15-26 Weeks	M_110156525	U 15-26 wks
11	31	5	UEMP27OV	Civilians Unemployed for 27 Weeks and Over	M_110156526	U 27+ wks
12	32*	5	CLAIMSx	Initial Claims	M_15186204	UI claims
13	33	5	PAYEMS	All Employees: Total nonfarm	M_123109146	Emp: total
14	34	5	USGOOD	All Employees: Goods-Producing Industries	M_123109172	Emp: gds prod
15	35	5	CES1021000001	All Employees: Mining and Logging: Mining	M_123109244	Emp: mining
16	36	5	USCONS	All Employees: Construction	M_123109331	Emp: const
17	37	5	MANEMP	All Employees: Manufacturing	M_123109542	Emp: mfg
18	38	5	DMANEMP	All Employees: Durable goods	M_123109573	Emp: dble gds
19	39	5	NDMANEMP	All Employees: Nondurable goods	M_123110741	Emp: nondbles
20	40	5	SRVPRD	All Employees: Service-Providing Industries	M_123109193	Emp: services
21	41	5	USTPU	All Employees: Trade, Transportation & Utilities	M_123111543	Emp: TTU
22	42	5	USWTRADE	All Employees: Wholesale Trade	M_123111563	Emp: wholesale
23	43	5	USTRADE	All Employees: Retail Trade	M_123111867	Emp: retail
24	44	5	USFIRE	All Employees: Financial Activities	M_123112777	Emp: FIRE
25	45	5	USGOVT	All Employees: Government	M_123114411	Emp: Govt
26	46	1	CES0600000007	Avg Weekly Hours : Goods-Producing	M_140687274	Avg hrs
27	47	2	AWOTMAN	Avg Weekly Overtime Hours : Manufacturing	M_123109554	Overtime: mfg
28	48	1	AWHMAN	Avg Weekly Hours : Manufacturing	M_14386098	Avg hrs: mfg
29	49	1	NAPMEI	ISM Manufacturing: Employment Index	M_110157206	NAPM empl
30	127	6	CES0600000008	Avg Hourly Earnings : Goods-Producing	M_123109182	AHE: goods
31	128	6	CES2000000008	Avg Hourly Earnings : Construction	M_123109341	AHE: const
32	129	6	CES3000000008	Avg Hourly Earnings : Manufacturing	M_123109552	AHE: mfg

Source: McCraken and Ng (2015)

Figure B.3: Group 3: Housing

	id	tcode	fred	description	gsi	gsi:description
1	50	4	HOUST	Housing Starts: Total New Privately Owned	M_110155536	Starts: nonfarm
2	51	4	HOUSTNE	Housing Starts, Northeast	M_110155538	Starts: NE
3	52	4	HOUSTMW	Housing Starts, Midwest	M_110155537	Starts: MW
4	53	4	HOUSTS	Housing Starts, South	M_110155543	Starts: South
5	54	4	HOUSTW	Housing Starts, West	M_110155544	Starts: West
6	55	4	PERMIT	New Private Housing Permits (SAAR)	M_110155532	BP: total
7	56	4	PERMITNE	New Private Housing Permits, Northeast (SAAR)	M_110155531	BP: NE
8	57	4	PERMITMW	New Private Housing Permits, Midwest (SAAR)	M_110155530	BP: MW
9	58	4	PERMITS	New Private Housing Permits, South (SAAR)	M_110155533	BP: South
10	59	4	PERMITW	New Private Housing Permits, West (SAAR)	M_110155534	BP: West

Source: McCraken and Ng (2015)

Figure B.4: Group 4: Consumption, orders, and inventories

	id	tcode	fred	description	gsi	gsi:description
1	3	5	DPCERA3M086SBEA	Real personal consumption expenditures	M_123008274	Real Consumption
2	4*	5	CMRMTSPLx	Real Manu. and Trade Industries Sales	M_110156998	M&T sales
3	5*	5	RETAILx	Retail and Food Services Sales	M_130439509	Retail sales
4	60	1	NAPM	ISM : PMI Composite Index	M_110157208	PMI
5	61	1	NAPMNOI	ISM : New Orders Index	M_110157210	NAPM new ordrs
6	62	1	NAPMSDI	ISM : Supplier Deliveries Index	M_110157205	NAPM vendor del
7	63	1	NAPMII	ISM : Inventories Index	M_110157211	NAPM Invent
8	64	5	ACOGNO	New Orders for Consumer Goods	M_14385863	Orders: cons gds
9	65*	5	AMDMNOx	New Orders for Durable Goods	M_14386110	Orders: dble gds
10	66*	5	ANDENOx	New Orders for Nondefense Capital Goods	M_178554409	Orders: cap gds
11	67*	5	AMDMUOx	Unfilled Orders for Durable Goods	M_14385946	Unf orders: dble
12	68*	5	BUSINVx	Total Business Inventories	M_15192014	M&T invent
13	69*	2	ISRATIOx	Total Business: Inventories to Sales Ratio	M_15191529	M&T invent/sales
14	130*	2	UMCSENTx	Consumer Sentiment Index	hhsntn	Consumer expect

Source: McCraken and Ng (2015)

Figure B.5: Group 5: Money and credit

	id	tcode	fred	description	gsi	gsi:description
1	70	6	M1SL	M1 Money Stock	M_110154984	M1
2	71	6	M2SL	M2 Money Stock	M_110154985	M2
3	72	5	M2REAL	Real M2 Money Stock	M_110154985	M2 (real)
4	73	6	AMBSL	St. Louis Adjusted Monetary Base	M_110154995	MB
5	74	6	TOTRESNS	Total Reserves of Depository Institutions	M_110155011	Reserves tot
6	75	7	NONBORRES	Reserves Of Depository Institutions	M_110155009	Reserves nonbor
7	76	6	BUSLOANS	Commercial and Industrial Loans	BUSLOANS	C&I loan plus
8	77	6	REALLN	Real Estate Loans at All Commercial Banks	BUSLOANS	DC&I loans
9	78	6	NONREVSL	Total Nonrevolving Credit	M_110154564	Cons credit
10	79*	2	CONSPI	Nonrevolving consumer credit to Personal Income	M_110154569	Inst cred/PI
11	131	6	MZMSL	MZM Money Stock	N.A.	N.A.
12	132	6	DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	N.A.	N.A.
13	133	6	DTCTHFNM	Total Consumer Loans and Leases Outstanding	N.A.	N.A.
14	134	6	INVEST	Securities in Bank Credit at All Commercial Banks	N.A.	N.A.

Source: McCraken and Ng (2015)

Figure B.6: Group 6: Interest and exchange rates

	id	tcode	fred	description	gsi	gsi:description
1	84	2	FEDFUNDS	Effective Federal Funds Rate	M_110155157	Fed Funds
2	85*	2	CP3Mx	3-Month AA Financial Commercial Paper Rate	CPF3M	Comm paper
3	86	2	TB3MS	3-Month Treasury Bill:	M_110155165	3 mo T-bill
4	87	2	TB6MS	6-Month Treasury Bill:	M_110155166	6 mo T-bill
5	88	2	GS1	1-Year Treasury Rate	M_110155168	1 yr T-bond
6	89	2	GS5	5-Year Treasury Rate	M_110155174	5 yr T-bond
7	90	2	GS10	10-Year Treasury Rate	M_110155169	10 yr T-bond
8	91	2	AAA	Moody's Seasoned Aaa Corporate Bond Yield		Aaa bond
9	92	2	BAA	Moody's Seasoned Baa Corporate Bond Yield		Baa bond
10	93*	1	COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS		CP-FF spread
11	94	1	TB3SMFFM	3-Month Treasury C Minus FEDFUNDS		3 mo-FF spread
12	95	1	TB6SMFFM	6-Month Treasury C Minus FEDFUNDS		6 mo-FF spread
13	96	1	T1YFFM	1-Year Treasury C Minus FEDFUNDS		1 yr-FF spread
14	97	1	T5YFFM	5-Year Treasury C Minus FEDFUNDS		5 yr-FF spread
15	98	1	T10YFFM	10-Year Treasury C Minus FEDFUNDS		10 yr-FF spread
16	99	1	AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS		Aaa-FF spread
17	100	1	BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS		Baa-FF spread
18	101	5	TWEXMMTH	Trade Weighted U.S. Dollar Index: Major Currencies		Ex rate: avg
19	102*	5	EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	M_110154768	Ex rate: Switz
20	103*	5	EXJPUSx	Japan / U.S. Foreign Exchange Rate	M_110154755	Ex rate: Japan
21	104*	5	EXUSUKx	U.S. / U.K. Foreign Exchange Rate	M_110154772	Ex rate: UK
22	105*	5	EXCAUSx	Canada / U.S. Foreign Exchange Rate	M_110154744	EX rate: Canada

Source: McCraken and Ng (2015)

Figure B.7: Group 7: Prices

	id	tcode	fred	description	gsi	gsi:description
1	106	6	WPSFD49207	PPI: Finished Goods	M110157517	PPI: fin gds
2	107	6	WPSFD49502	PPI: Finished Consumer Goods	M110157508	PPI: cons gds
3	108	6	WPSID61	PPI: Intermediate Materials	M_110157527	PPI: int matls
4	109	6	WPSID62	PPI: Crude Materials	M_110157500	PPI: crude matls
5	110*	6	OILPRICEx	Crude Oil, spliced WTI and Cushing	M_110157273	Spot market price
6	111	6	PPICMM	PPI: Metals and metal products:	M_110157335	PPI: nonferrous
7	112	1	NAPMPRI	ISM Manufacturing: Prices Index	M_110157204	NAPM com price
8	113	6	CPIAUCSL	CPI : All Items	M_110157323	CPI-U: all
9	114	6	CPIAPPSL	CPI : Apparel	M_110157299	CPI-U: apparel
10	115	6	CPITRNSL	CPI : Transportation	M_110157302	CPI-U: transp
11	116	6	CPIMEDSL	CPI : Medical Care	M_110157304	CPI-U: medical
12	117	6	CUSR0000SAC	CPI : Commodities	M_110157314	CPI-U: comm.
13	118	6	CUSR0000SAD	CPI : Durables	M_110157315	CPI-U: dbles
14	119	6	CUSR0000SAS	CPI : Services	M_110157325	CPI-U: services
15	120	6	CPIULFSL	CPI : All Items Less Food	M_110157328	CPI-U: ex food
16	121	6	CUSR0000SA0L2	CPI : All items less shelter	M_110157329	CPI-U: ex shelter
17	122	6	CUSR0000SA0L5	CPI : All items less medical care	M_110157330	CPI-U: ex med
18	123	6	PCEPI	Personal Cons. Expend.: Chain Index	gmdc	PCE defl
19	124	6	DDURRG3M086SBEA	Personal Cons. Exp: Durable goods	gmdcd	PCE defl: dlbes
20	125	6	DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods	gmdcn	PCE defl: nondble
21	126	6	DSERRG3M086SBEA	Personal Cons. Exp: Services	gmdcs	PCE defl: service

Source: McCraken and Ng (2015)

Figure B.8: Group 8: Stock market

	id	tcode	fred	description	gsi	gsi:description
1	80*	5	S&P 500	S&P's Common Stock Price Index: Composite	M_110155044	S&P 500
2	81*	5	S&P: indust	S&P's Common Stock Price Index: Industrials	M_110155047	S&P: indust
3	82*	2	S&P div yield	S&P's Composite Common Stock: Dividend Yield		S&P div yield
4	83*	5	S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio		S&P PE ratio
5	135*	1	VXOCLSx	VXO		

Source: McCraken and Ng (2015)